6. Real Signal Wavelet Transform

Instrumentation Systems Lab. Prof. Zhang Zhong

5. Image processing by Wavelet Transform

5.1 2D-Discrete Wavelet Transform

1) Calculation method 2) Characters

5.2 2D-Complex Wavelet Transform

Calculation method, 2D-Mother Wavelets, Translation invariance property of 2D-CDWT

5.3 Application on de-noising

Model Image, **Inspection of metallic surface**

5.4 Direction detection

Calculation method, Extracting fingerprint and crack in a grass Braun table

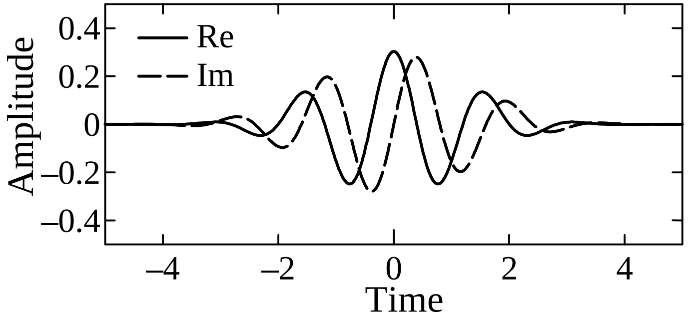
Important point in the CWT

1) Definition of the CWT

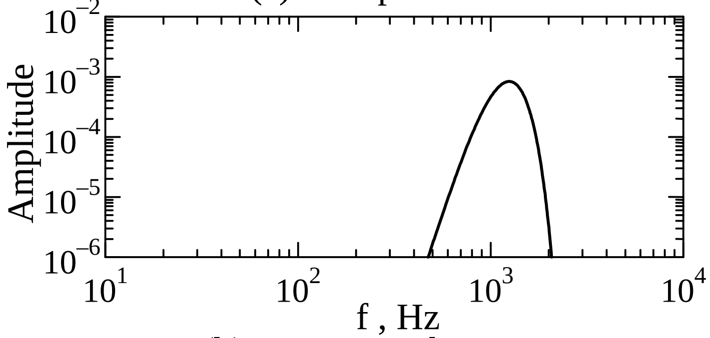
$$w(a,b) = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother Wavelet (MW)
a : Scale ($1/a$ Frequency)
b : Time



(a) RI–Spline wavelet



(b)Frequency characteristic

2) Example of the CWT

$$w(a,b) = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother Wavelet (MW)
a : Scale (1/a Frequency)
b : Time

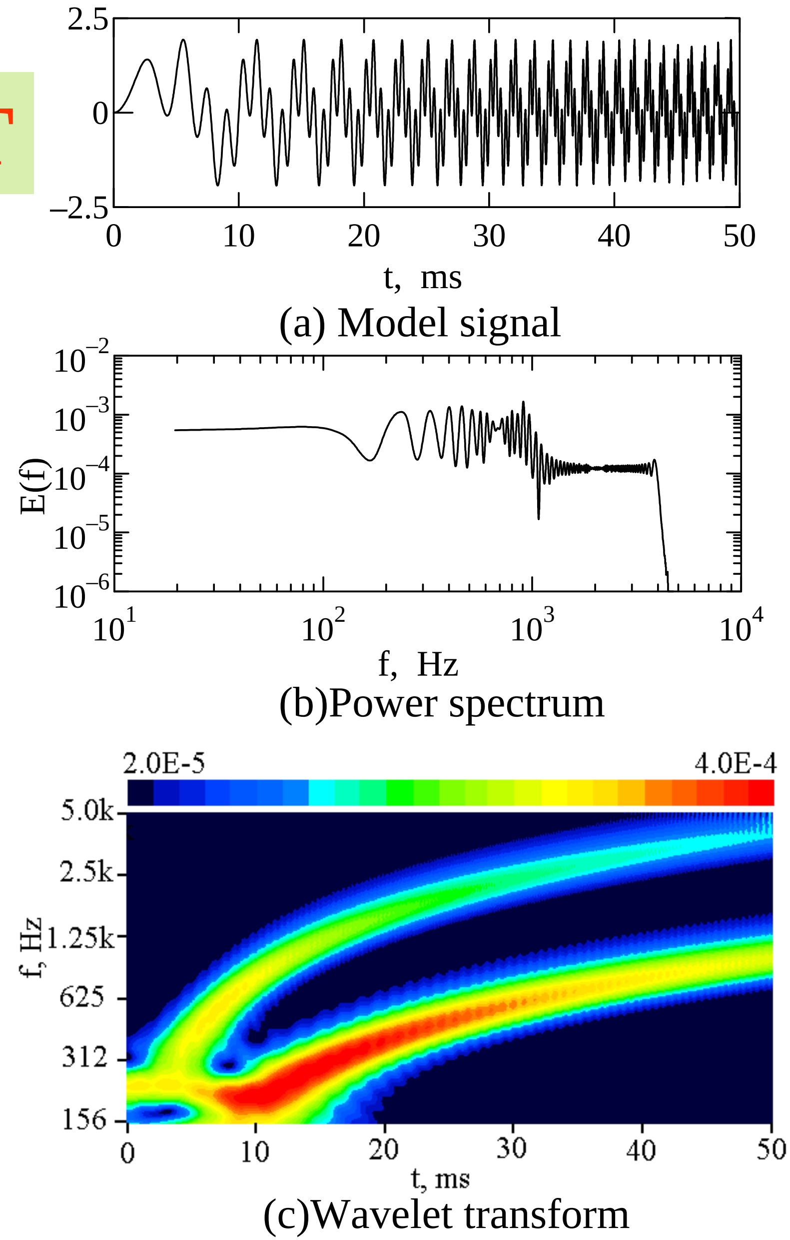
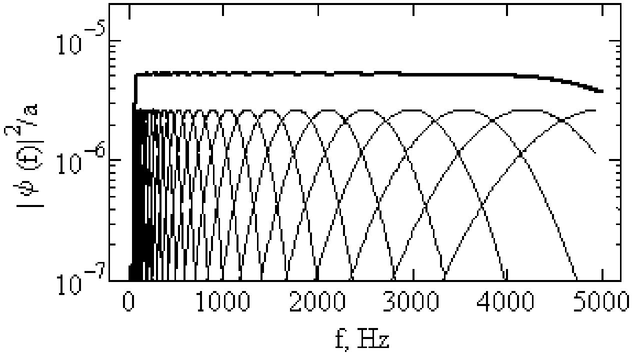


Fig.1 Continuance Wavelet Transform of Model Signal

3) Features of the CWT

Advantages:

- Transforming signal to time-frequency plane and making its nature clearly (illustrating)
- Time and frequency resolution can be changed with frequency



(b) Basis of CWT for 6 octave, 4 divided

Problems:

- Redundancy bases
- How do select Mother wavelet?
- There isn't a common fast algorithm for calculation.
- Can not detection of abnormal milt-components

4) Condition of the MW

$$w(a,b) = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother Wavelet (MW)
a : Scale (1/a Frequency)
b : Time

Condition of The MW (admissibility condition):

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$$

This condition can be sampled as following Equation:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

So many MW have been proposed and how do select them be comes problem.

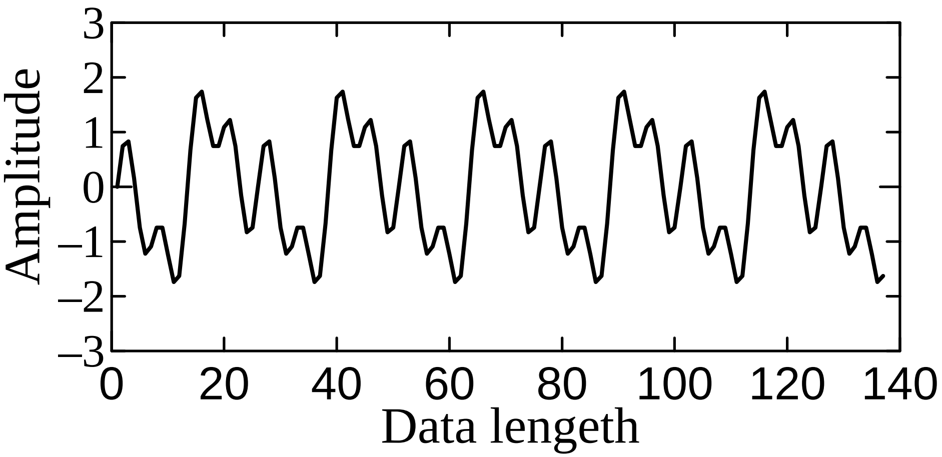
6.1 Construction of Real-Signal MW

1) Selection of window function

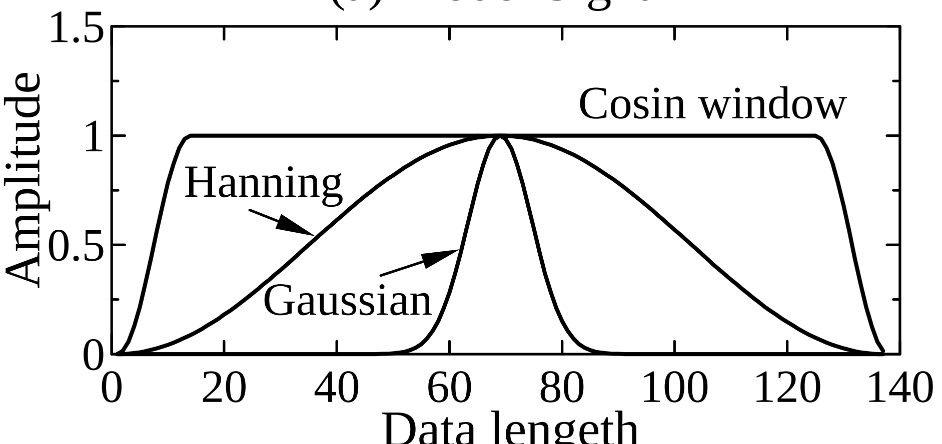
1) Model signal

$$f(k) = \sin(\frac{2\pi k}{25}) + 0.7\sin(\frac{2\pi k}{12}) + 0.7\sin(\frac{2\pi k}{6})$$

2) Multiplying the real signal with a window function and removing the average for making it becomes zero sufficiently quickly at the distant point



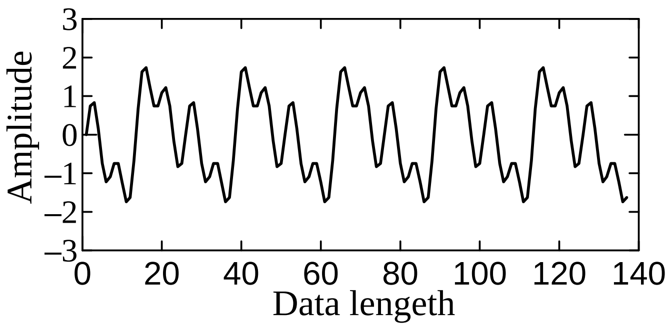
(a) Model Signal



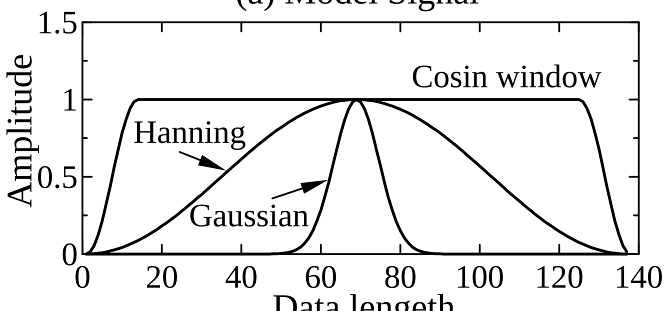
(b) Window functions

2) Example of the RMW

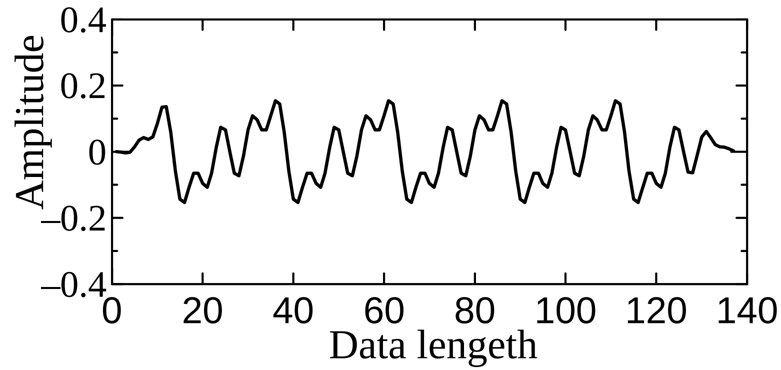
Influence of the Window function



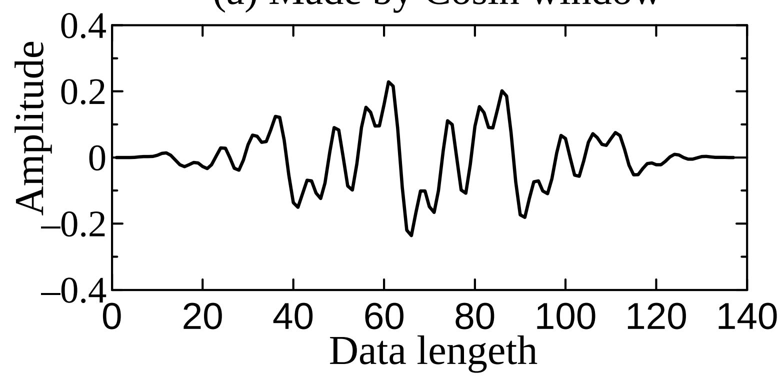
(a) Model Signal



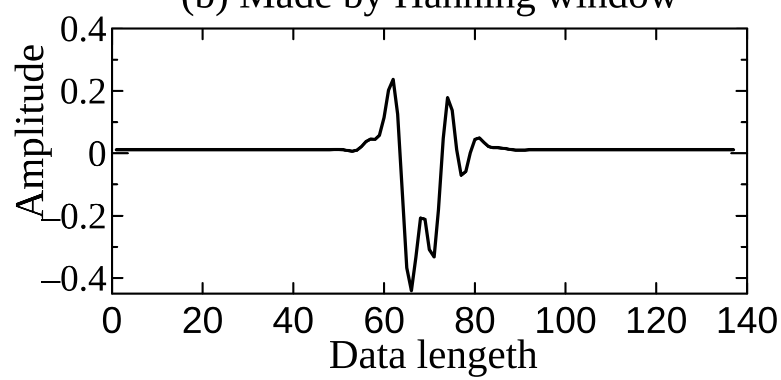
(b) Window functions



(a) Made by Cosin window

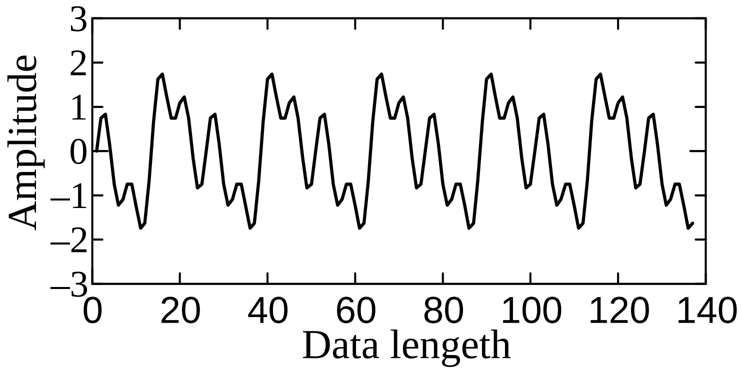


(b) Made by Hanning window

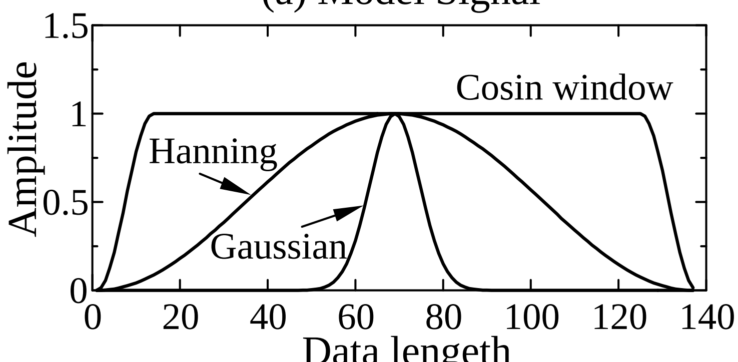


(c) Made by Gaussian function

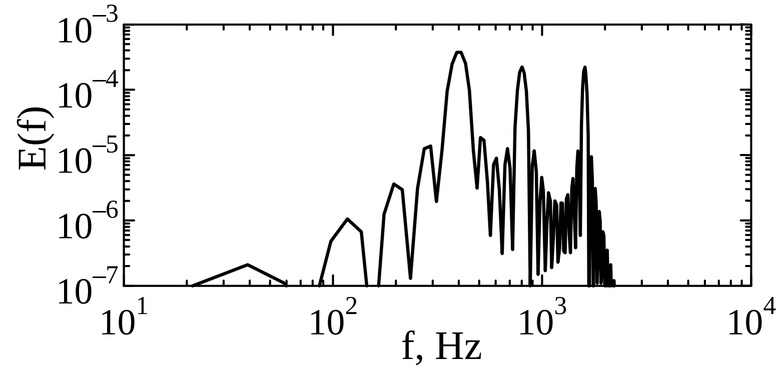
Spectrum of the RMW



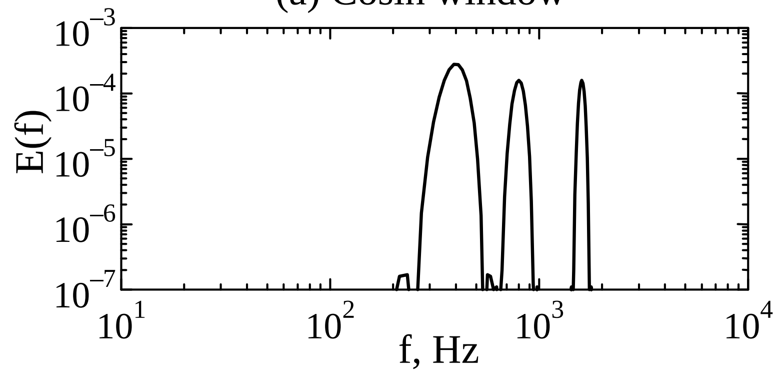
(a) Model Signal



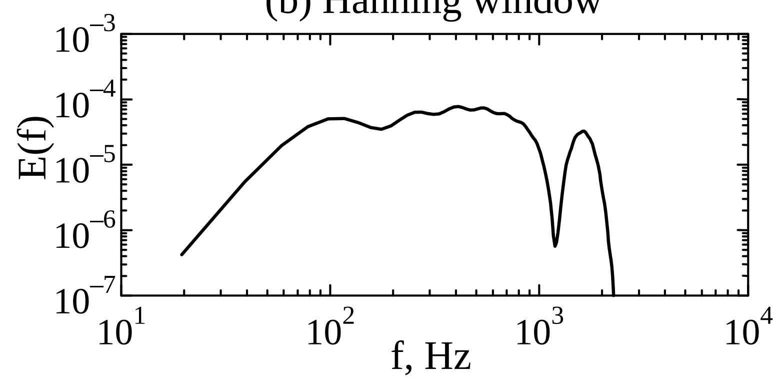
(b) Window functions



(a) Cosin window



(b) Hanning window



(c) Gaussina window

3) Construction of a complex RMW

1）Characteristic of Fourier transform：

R(t) —— FFT ——→ R(f) $f>0$, R(f)=0 $f<0$

I(t) ——————→ I(f)=0

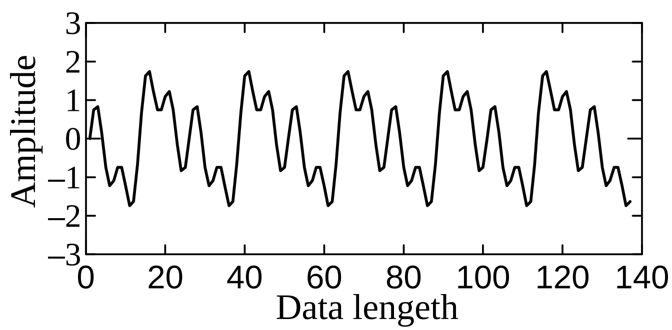
2)Constructing complex RMW using Hilbert transform：

R(t) ←—— Inverse FFT —— 2R(f) $f>0$, R(f)=0 $f<0$

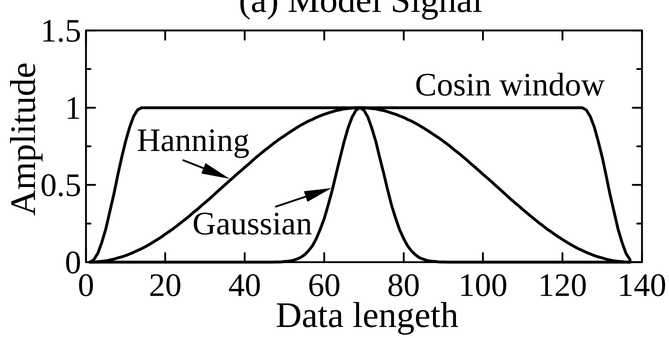
I(t) ←—————— I(f)=0

Example of complex RMW

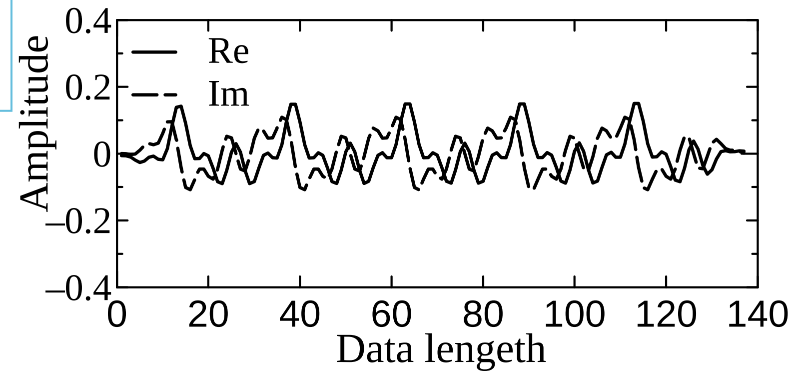
Both Complex and Real RMW have
Same frequency components



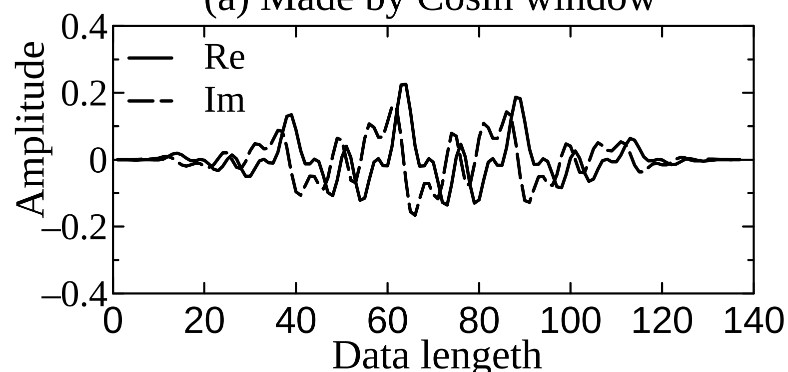
(a) Model Signal



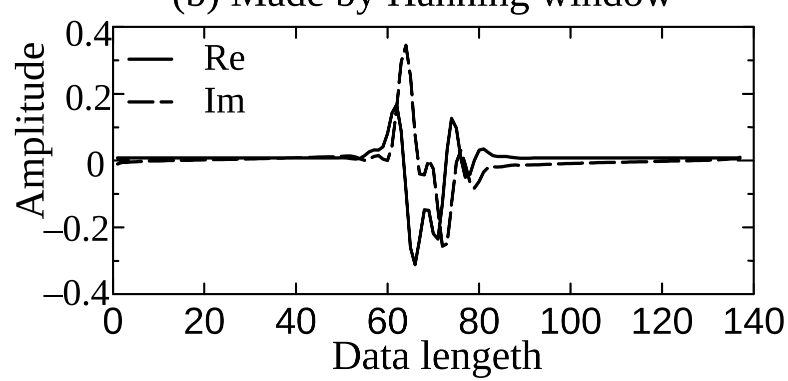
(b) Window functions



(a) Made by Cosin window

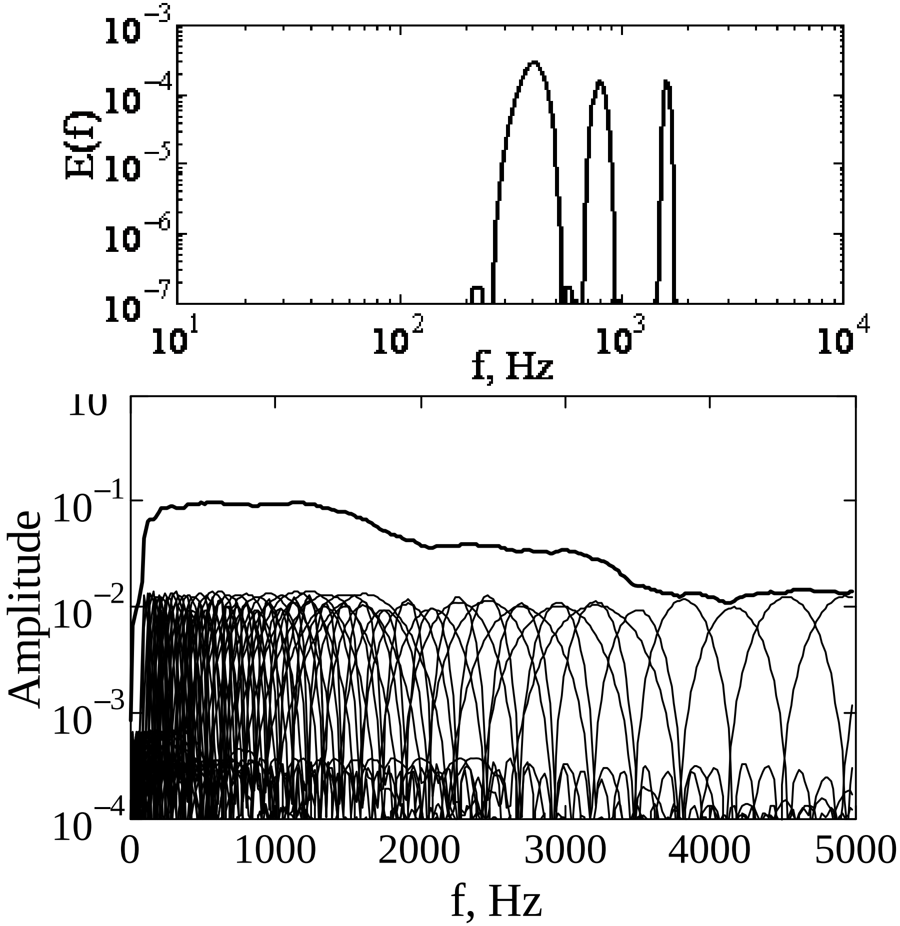


(b) Made by Hanning window

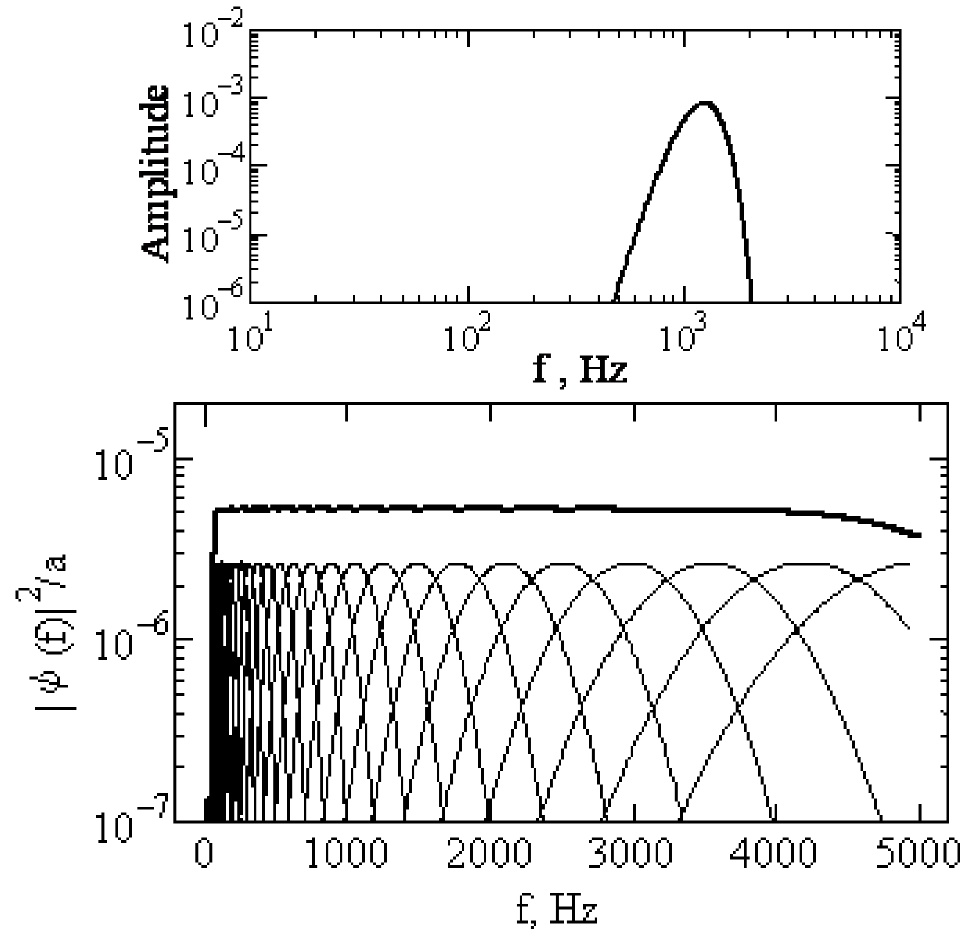


(c) Made by Gaussian function

Basis of complex RMW



(a) Basis of R-WT for 6 octave, 4 divided



(b) Basis of CWT for 6 octave, 4 divided

But, it isn't problem since our purpose is detection abnormal signal and isn't reconstruct signal.

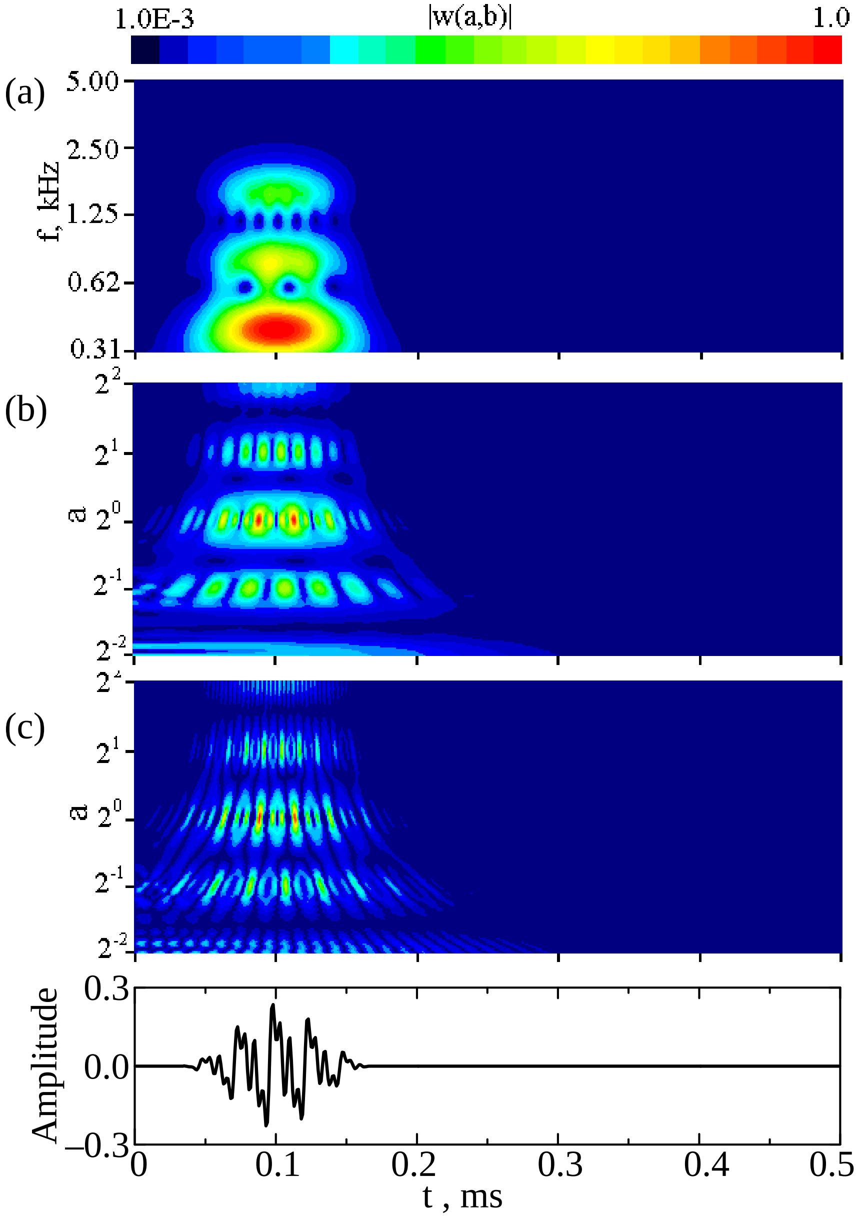
4) Example of analysis

Results:

1）It is clear that three patterns consisting mainly of 400Hz, 800Hz, and 1600Hz were obtained

2) Compared with (a), in (b) and (c), the pattern consisting mainly of scale a=1 and 1ms has appeared strongly.

3) In addition, the differences between (b) and (c) are the striped pattern obtained by the real RMW and continuation pattern obtained by the complex RMW.



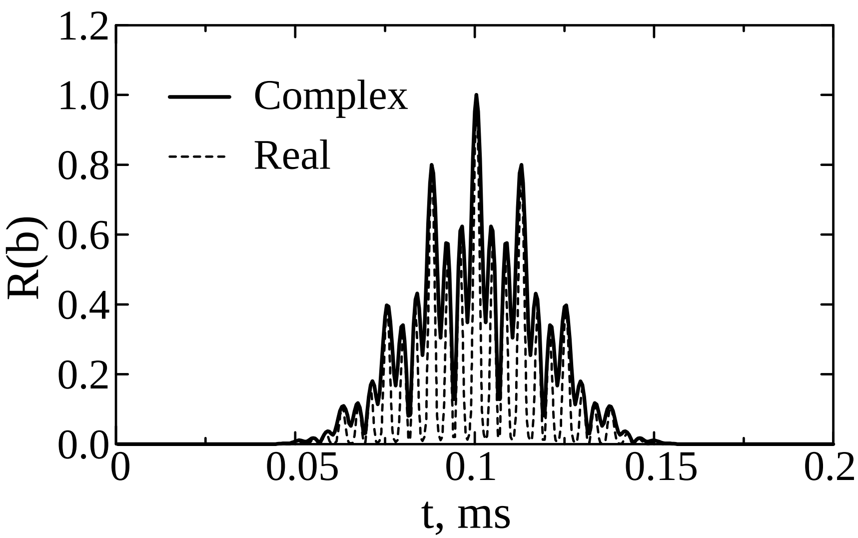
6.2 Detection of abnormal signal

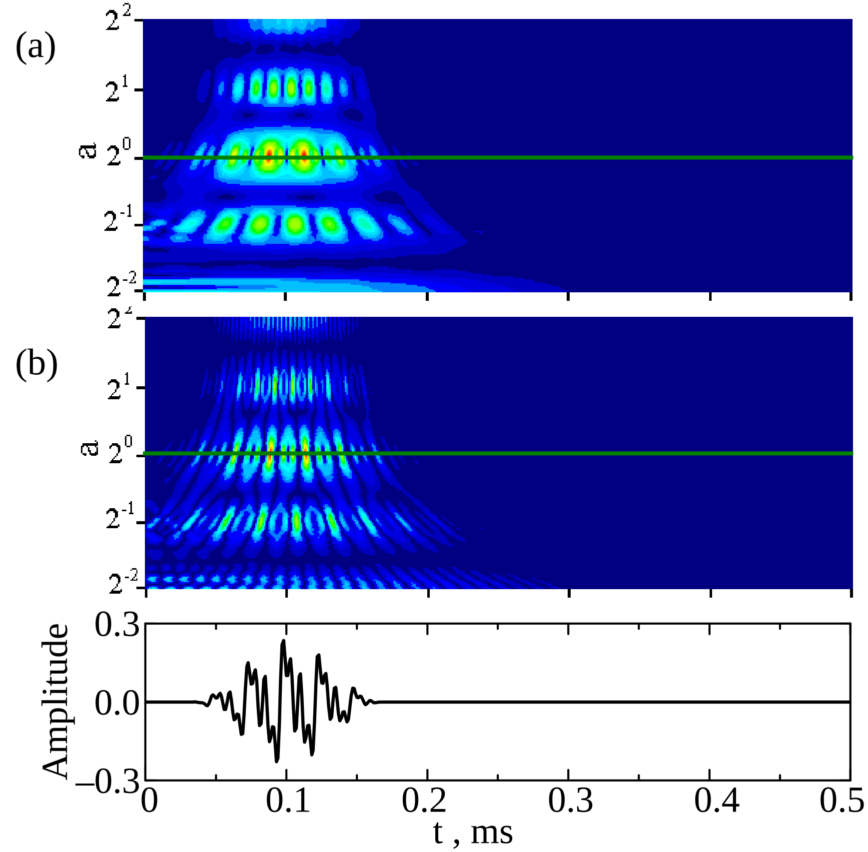
1) Definition of Wavelet Instantaneous Correlation

The value $|w(a, b)|$ obtained by the RMW in scale a=1 is defined by the Wavelet Instantaneous Correlation (WIC) value *R(b)* and is shown as follows:

$$R(b) = W(a = 1, b)$$

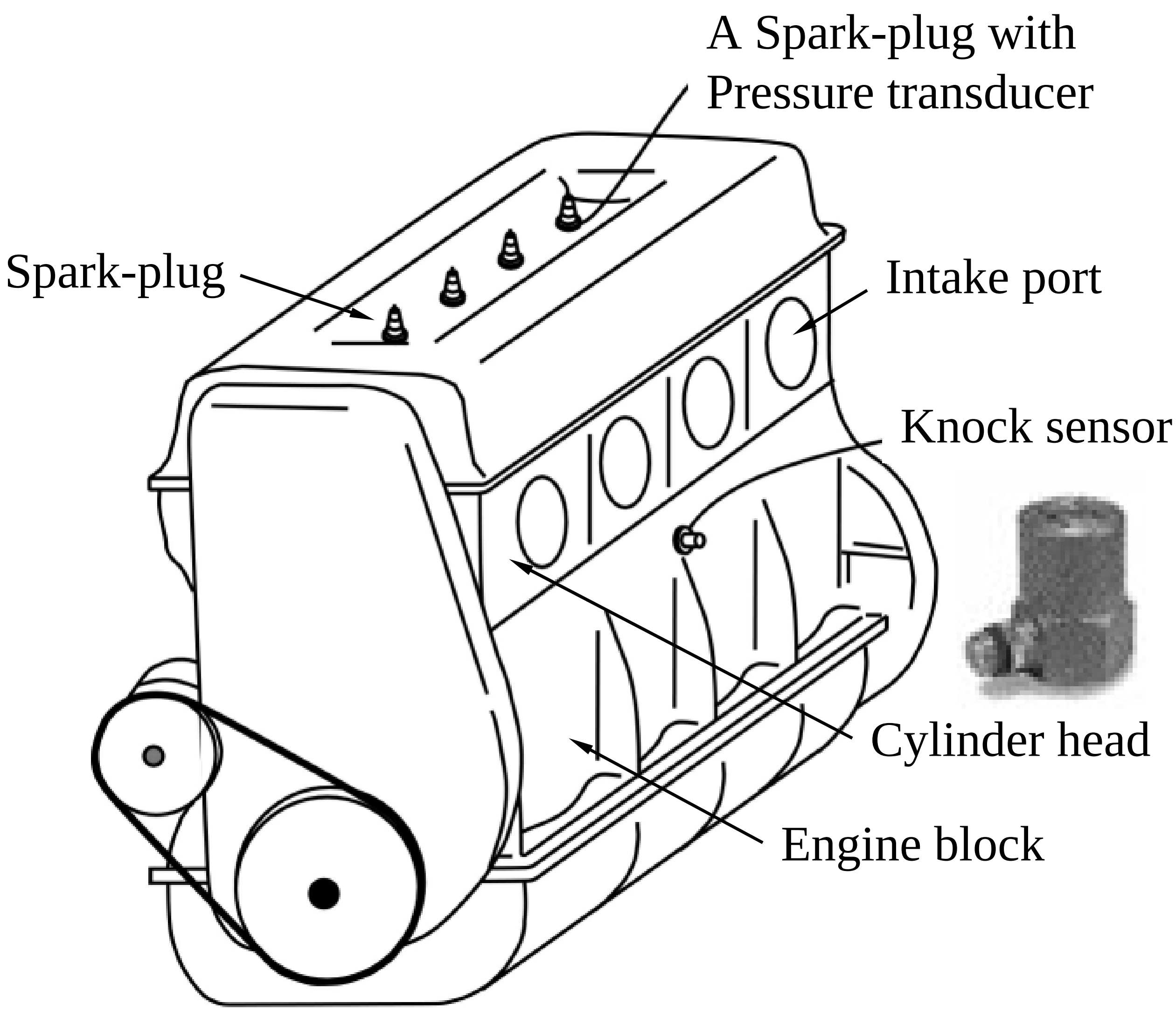
2) Example of the WIC

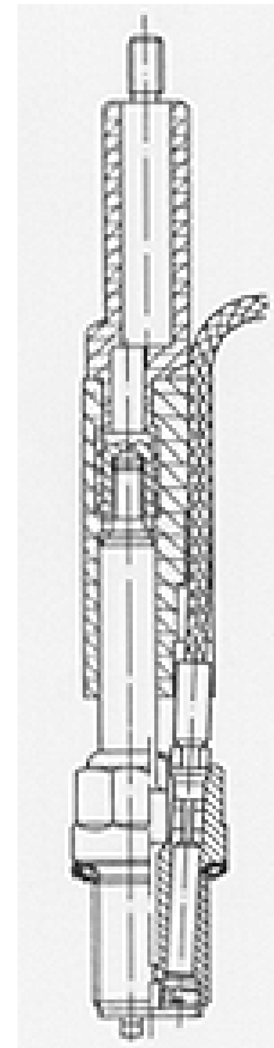




1) R(b) =1.0 can be obtained in 0.1ms since the RMW is completely the same as the components of the signal.
2) The R(b) using real RMW has an oscillating phenomenon, this phenomenon can be improved by using the complex RMW .

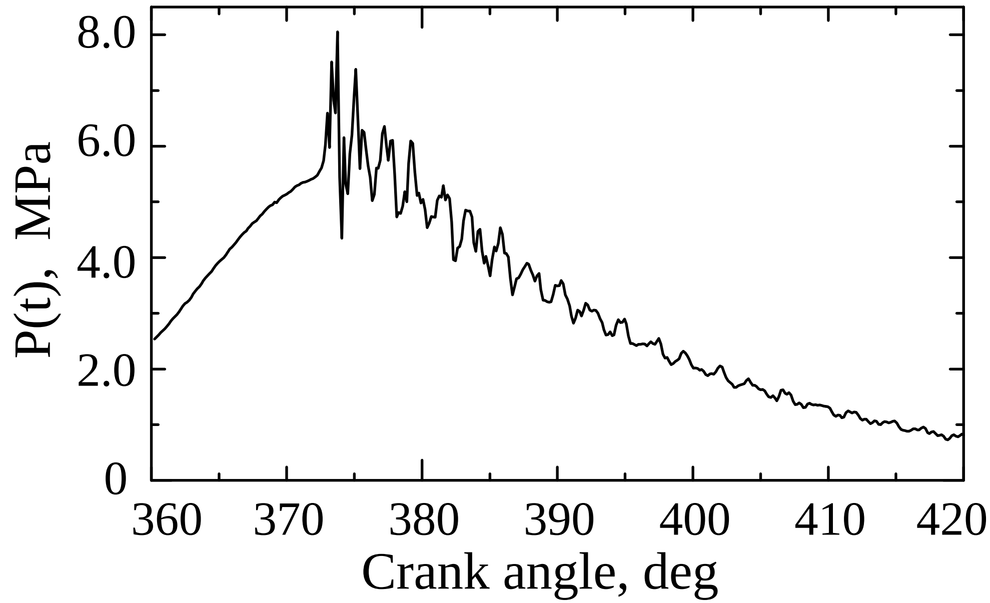
3) Knocking extraction from block vibration



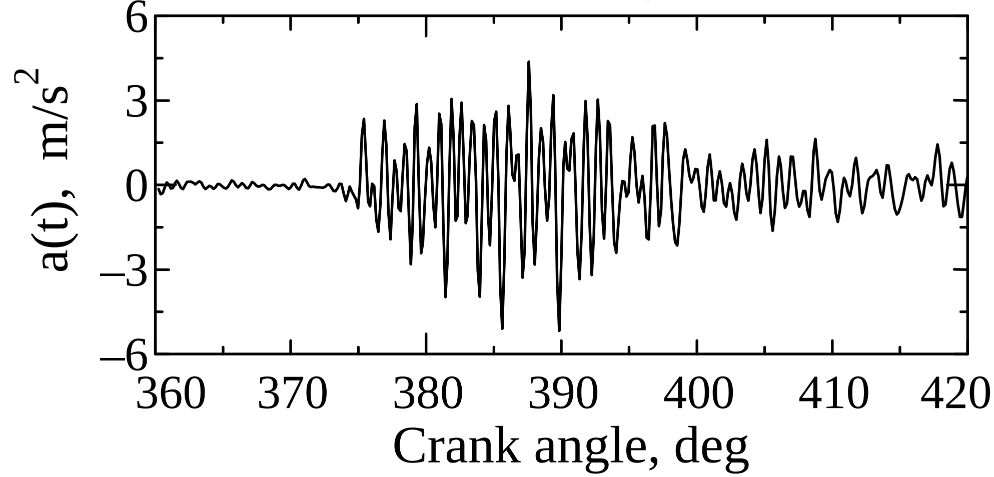


Example of signal

When a knock occurs, there is corresponding vibration of the pressure and vibration of the engine block.

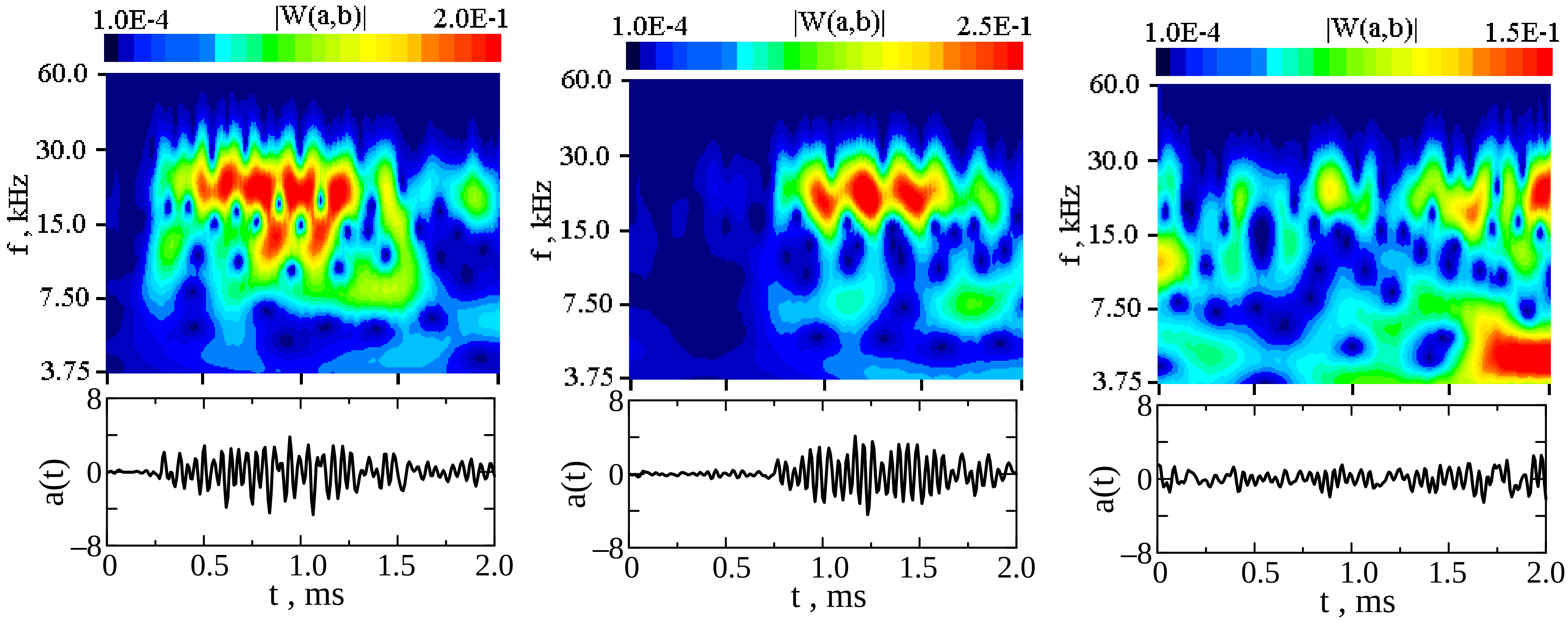


(a) Pressure signal



(b) Vibration signal

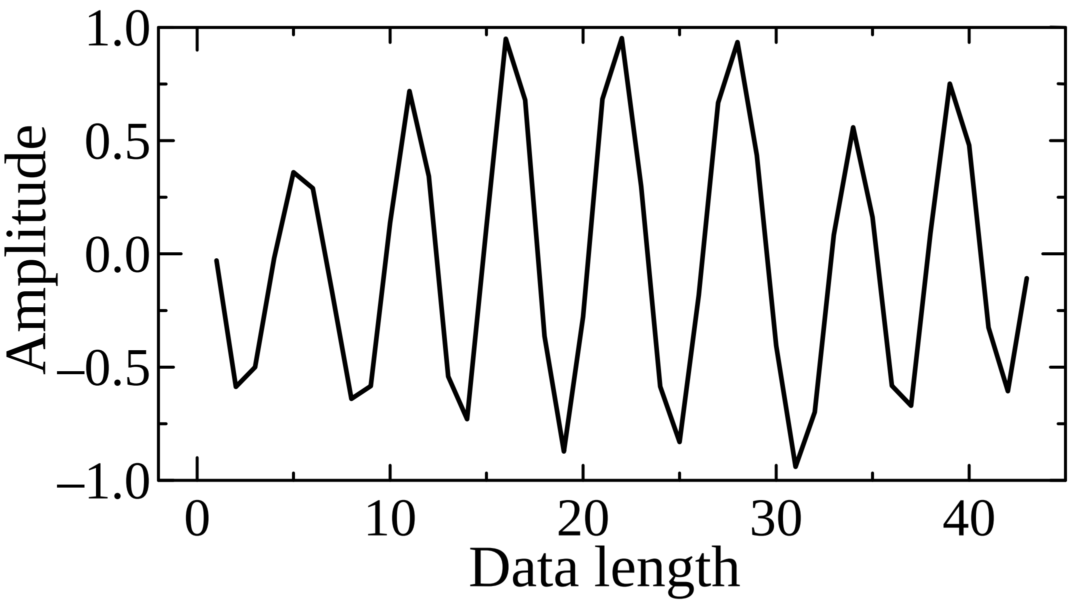
Wavelet Transform of the vibration signals



(a)Stronger Knocking **(b)Weaker Knocking** **(c)Normal Combustion**

Fig.4 Wavelet Transform of the vibration signals by knock sensor at 3000rpm

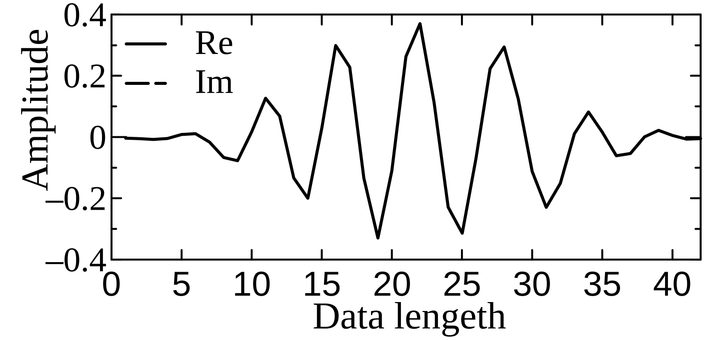
Portion of vibration



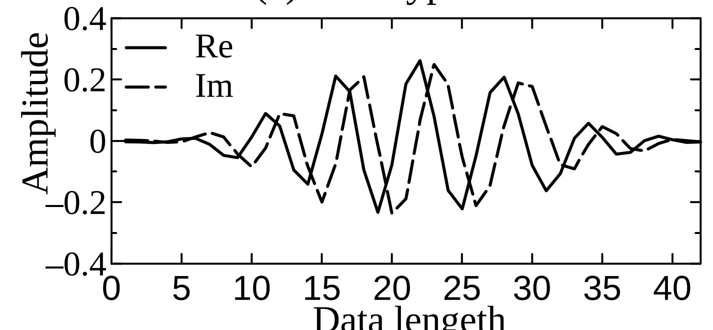
Extracting portion of vibration signal obtained in the condition of Weaker knocking

Constructing RMW

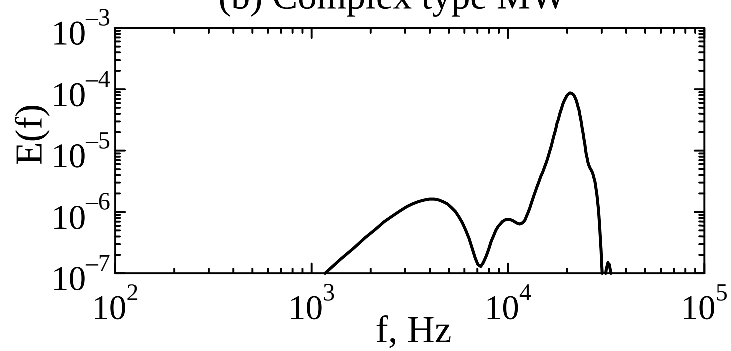
1) Multiplying the real signal with Hanning window function and removing the average value.
2) Carrying out Hilbert transform of real RMW and obtaining complex RMW



(a) Real type MW

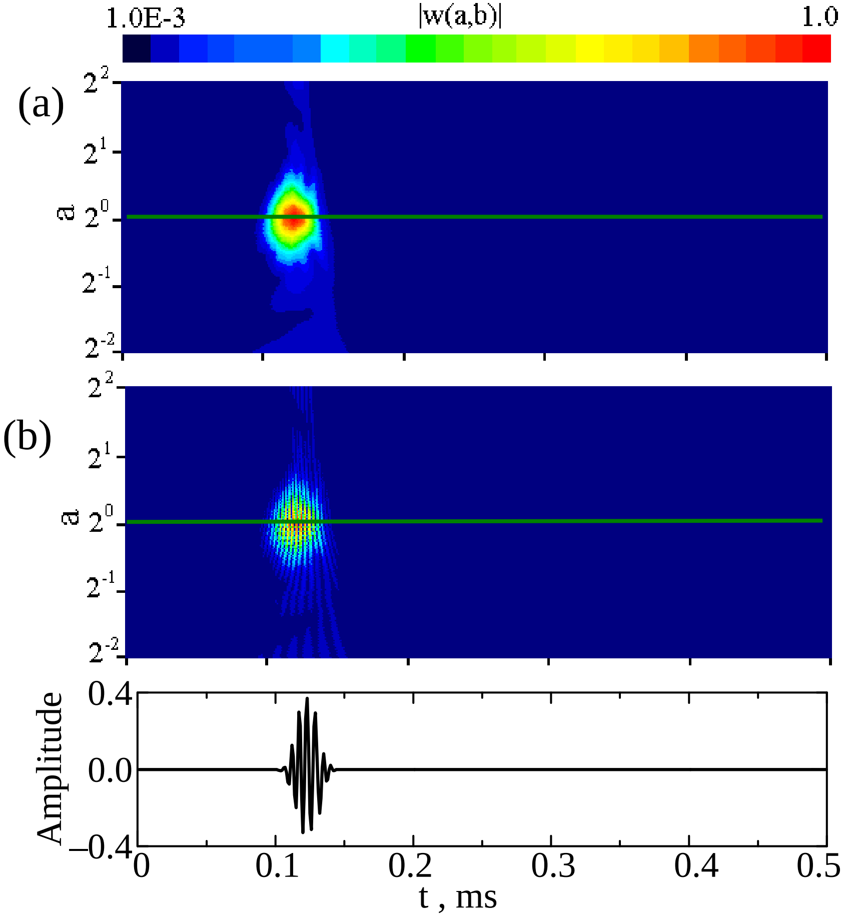


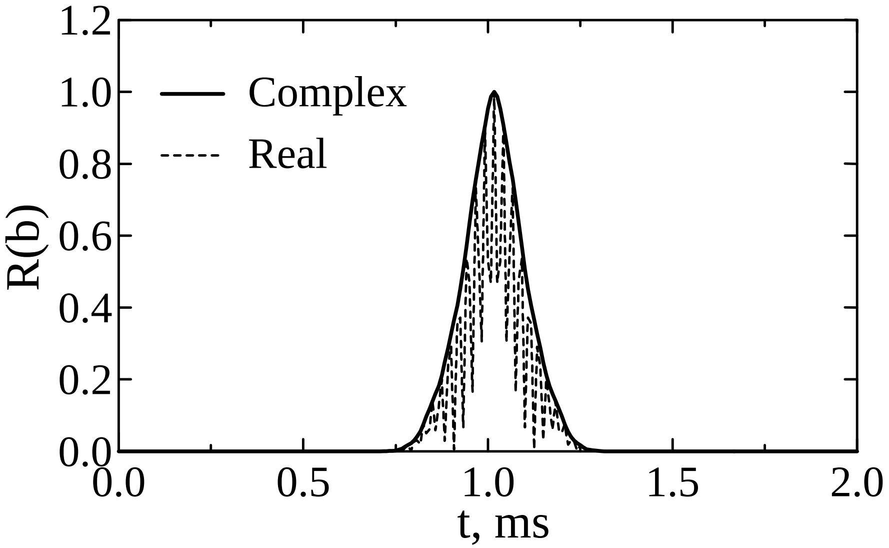
(b) Complex type MW



(c) Power spectrum of MW

Example of the WIC

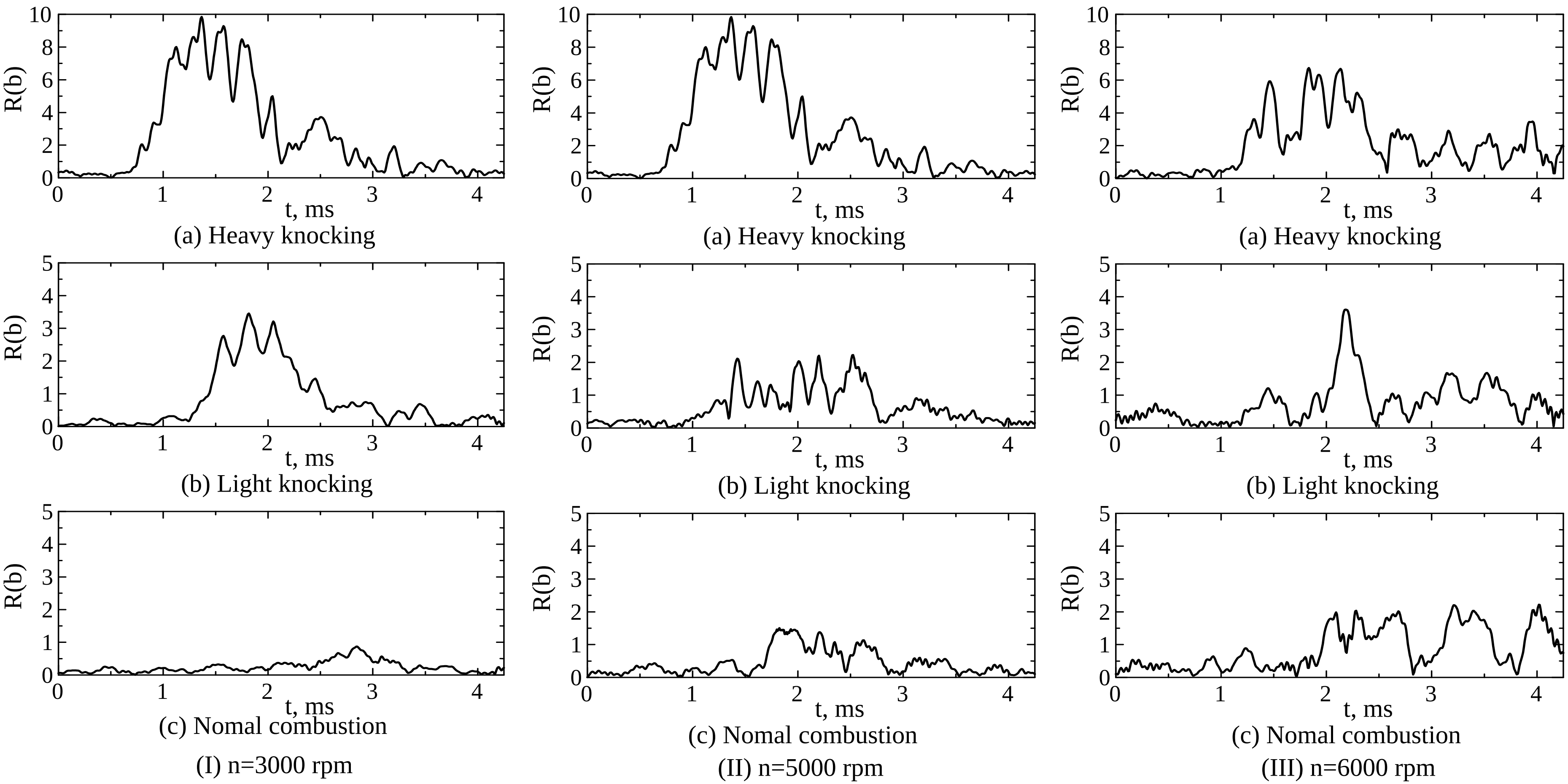




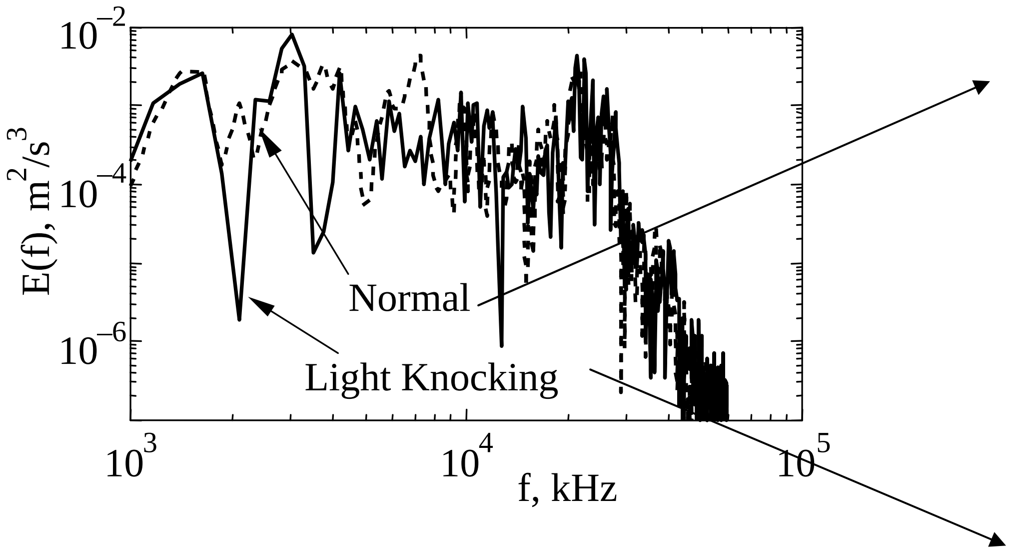
1) R(b) =1.0 can be obtained in 1.2ms since the RMW is completely the same as the components of the signal.

2) the R(b) using real RMW has an oscillating phenomenon, this phenomenon can be improved by using the complex RMW .

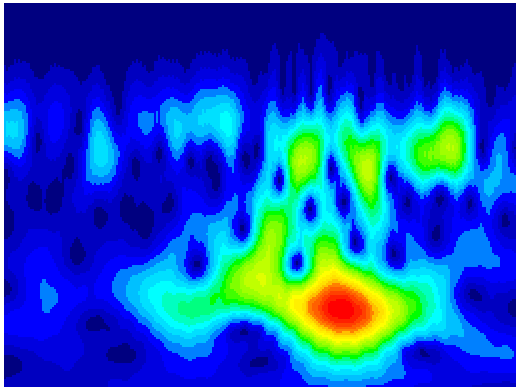
Example of Knocking Extraction

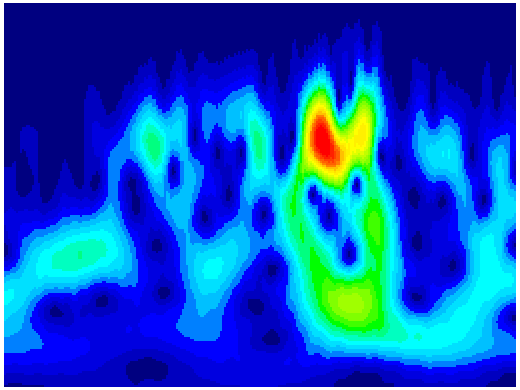


Comparing Spectrum



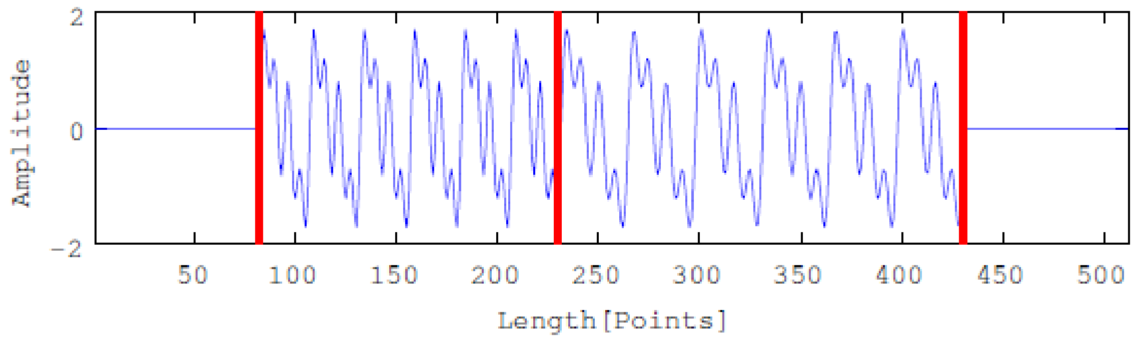
Fourier transform



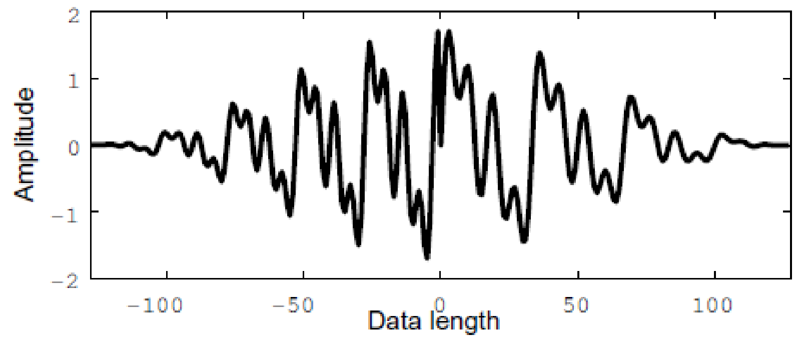


Wavelet Transform

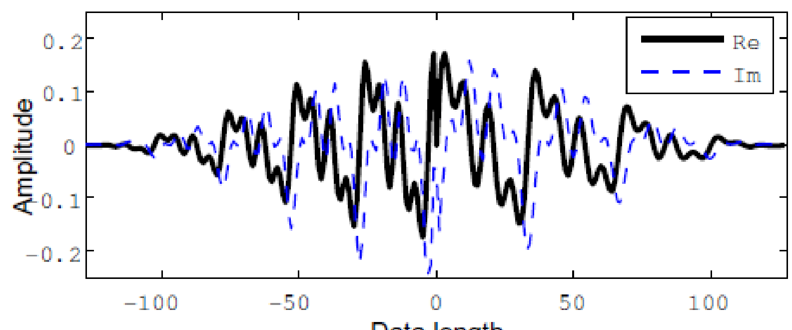
6.3 Construction of RMW using more than two signal



The average characteristic of the two signals can not be obtained from the average CMRW since using the construction method of CRMW, the phase information of the two signals were held and a discontinuity arose in the connection portion.



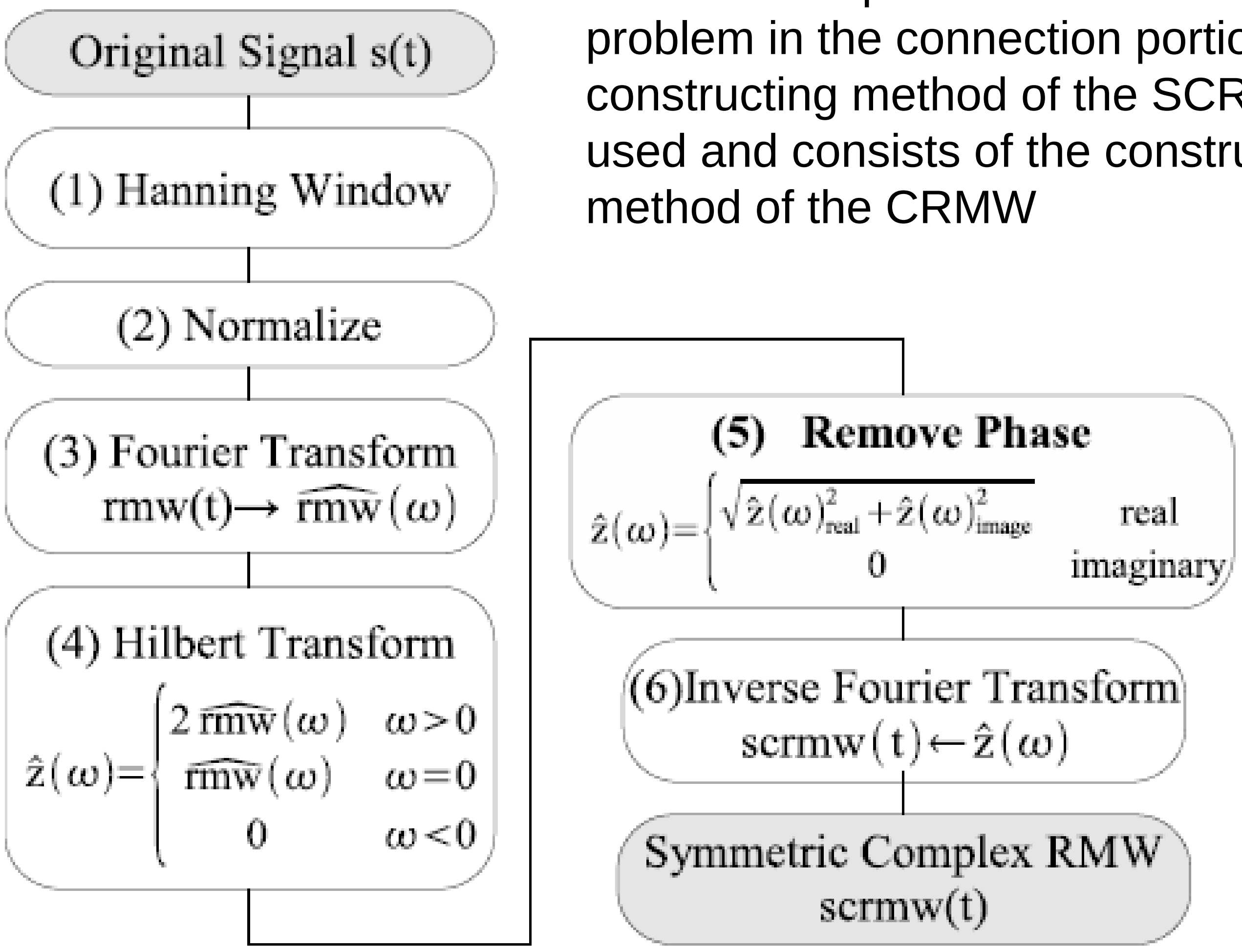
(a) RMW



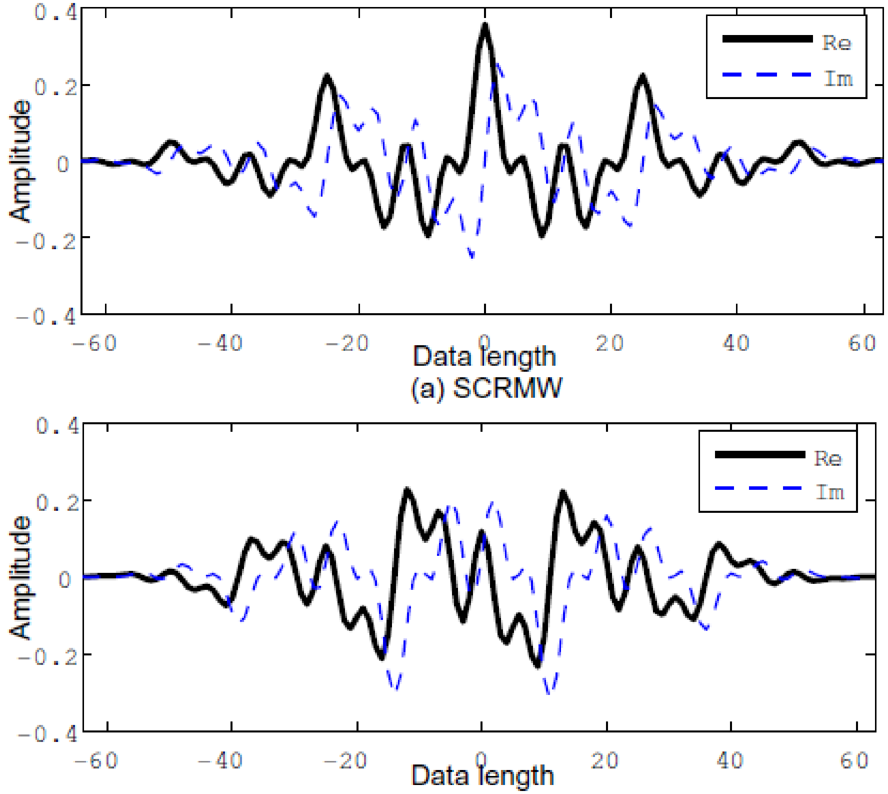
(b) CRMW

1) Flow chart of SCRMW construction

In order to improve the discontinuity problem in the connection portion, the constructing method of the SCRMW was used and consists of the construction method of the CRMW



Example of the SCRMW

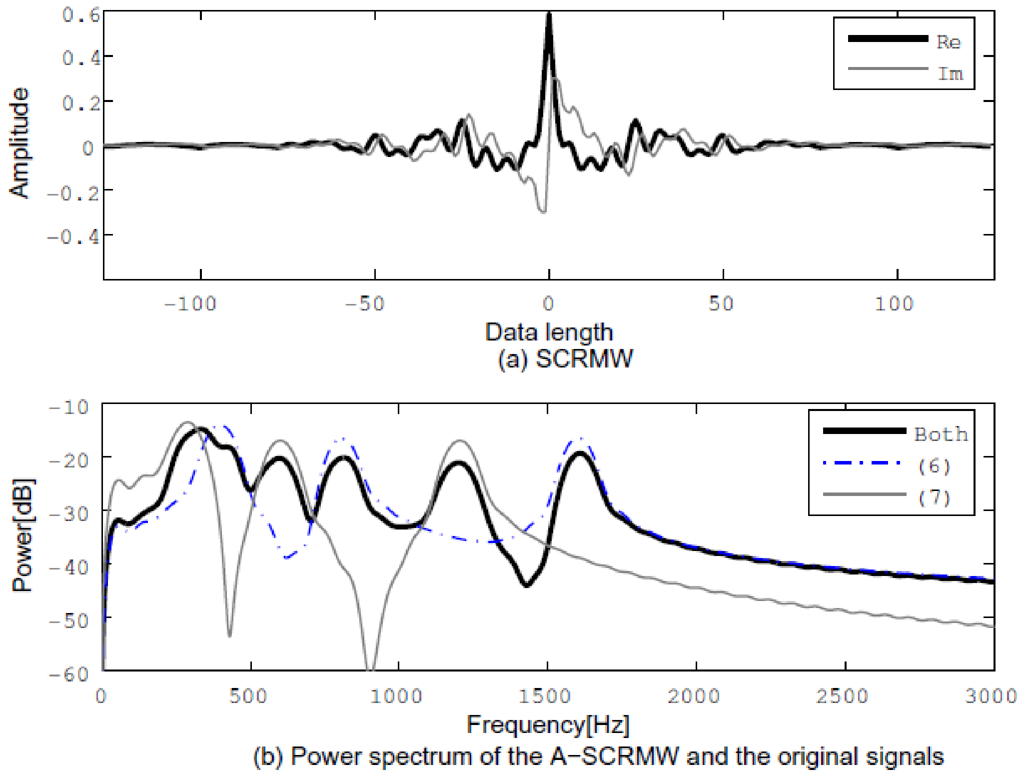


(a) SCRMW

(b) CRMW

the energy of the SCRMW is concentrated around the center of the SCRMW. In addition, the real component of the SCRMW has symmetry and the imaginary component has anti-symmetry.

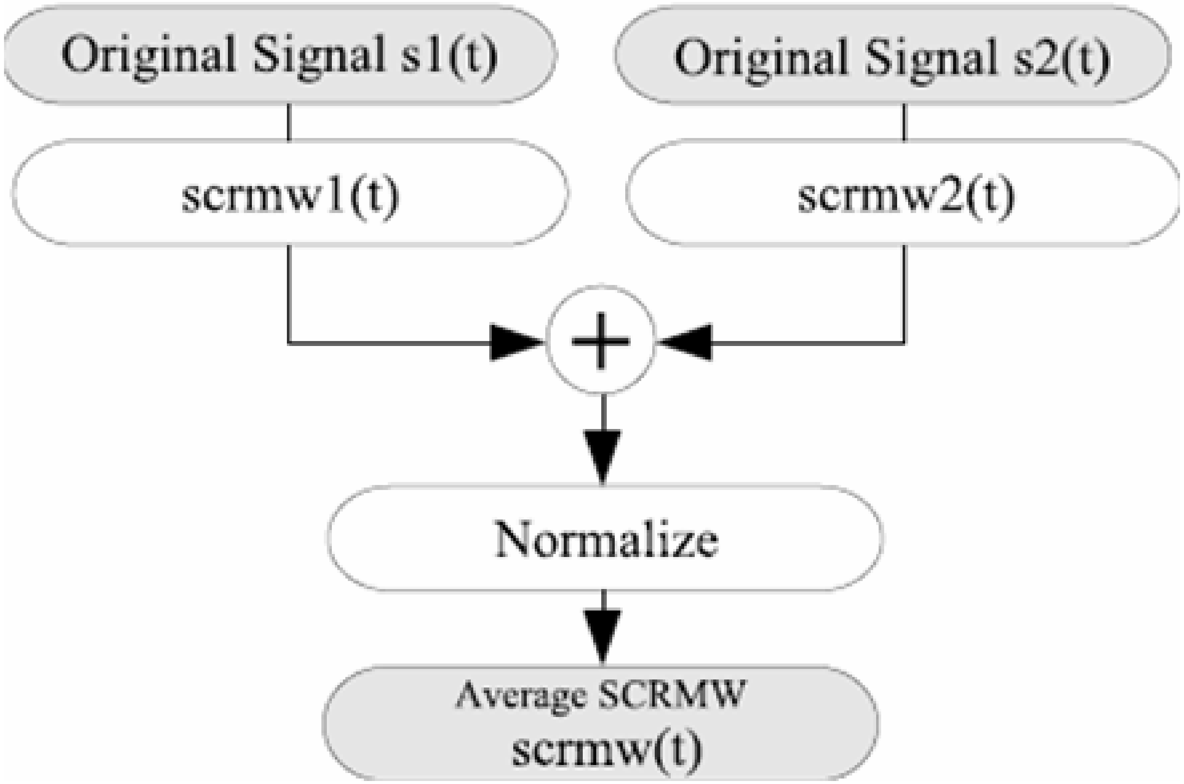
Example of the average SRMW



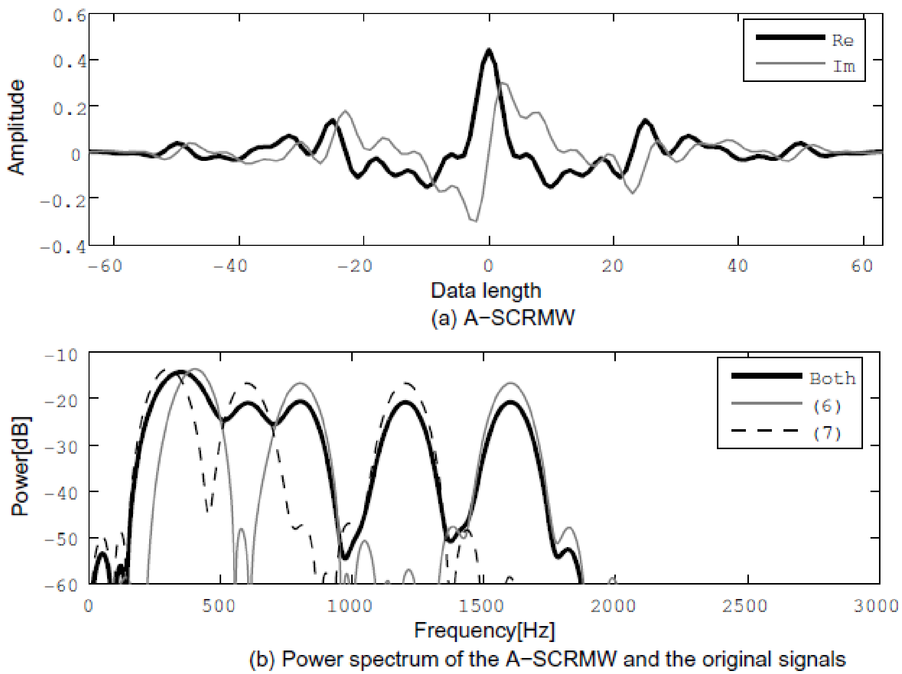
Result: the average SCRMW has the SCRMW's property.

Problem: the frequency property which high frequency components have been increased is not good because the average RMW has a discontinuous part in the connection portion.

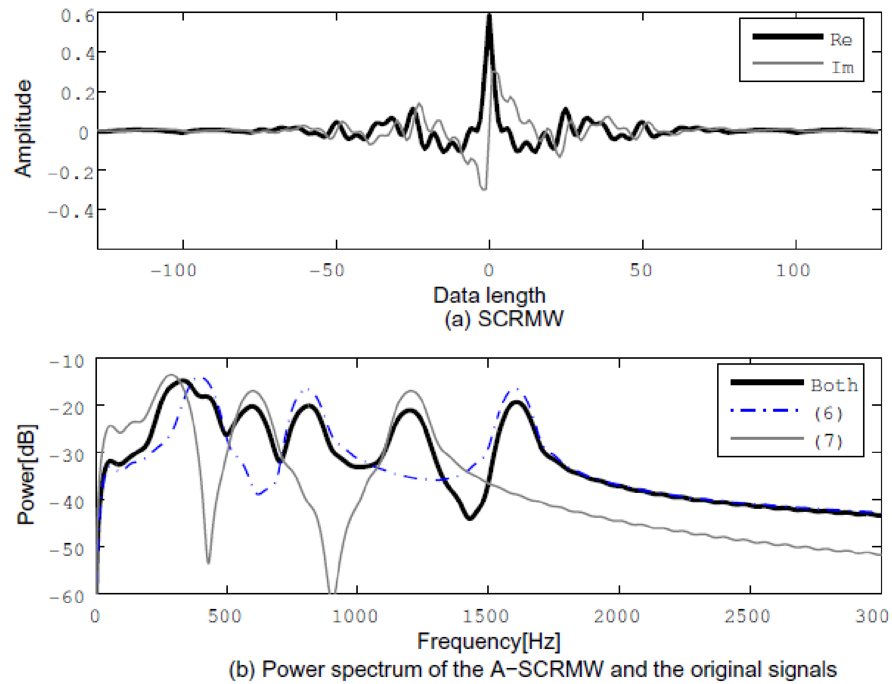
2. Construction Method of A-RMW



Example of the A-RMW



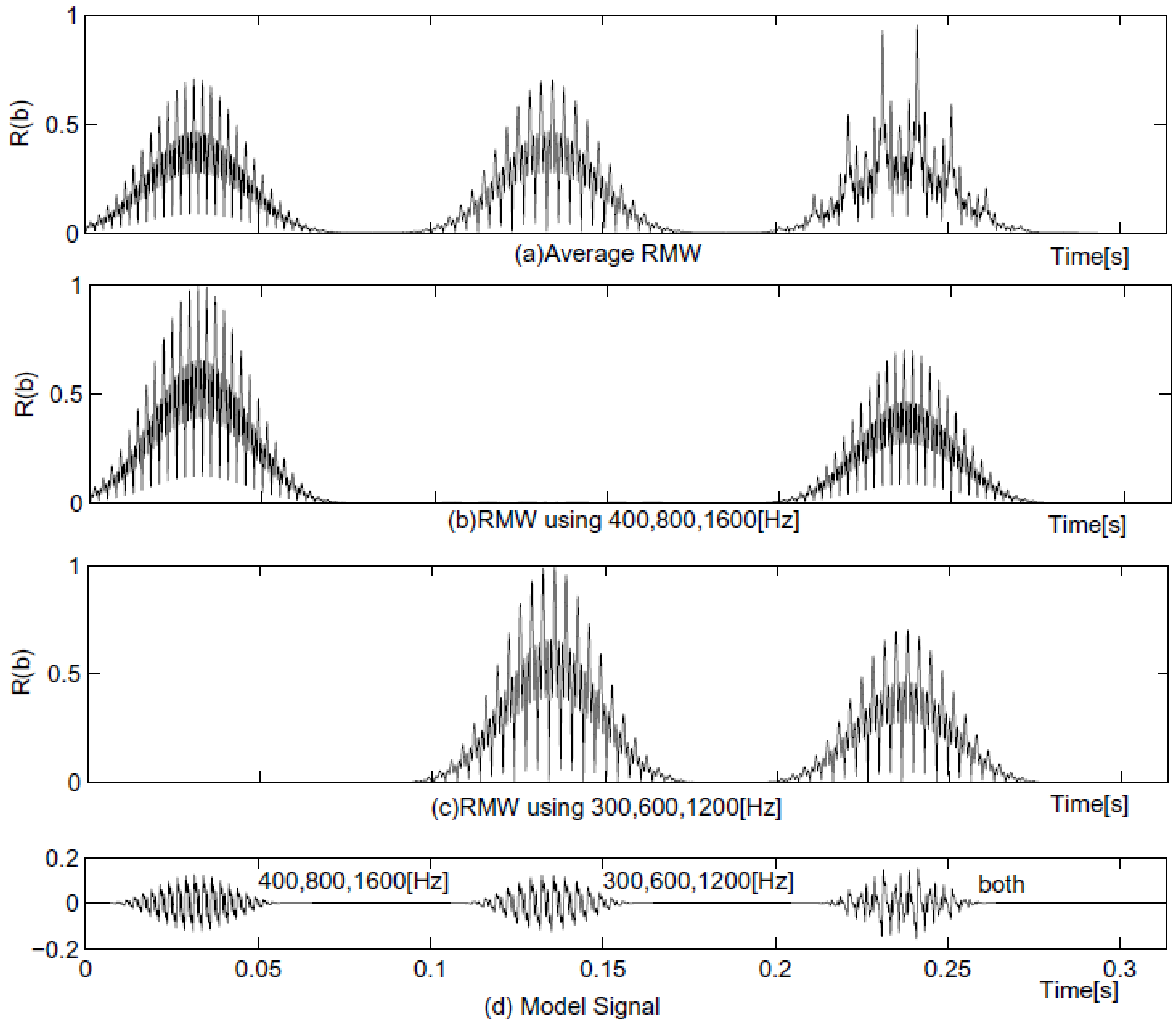
(a) Example of A-RMW



(b) Example of average CRMW

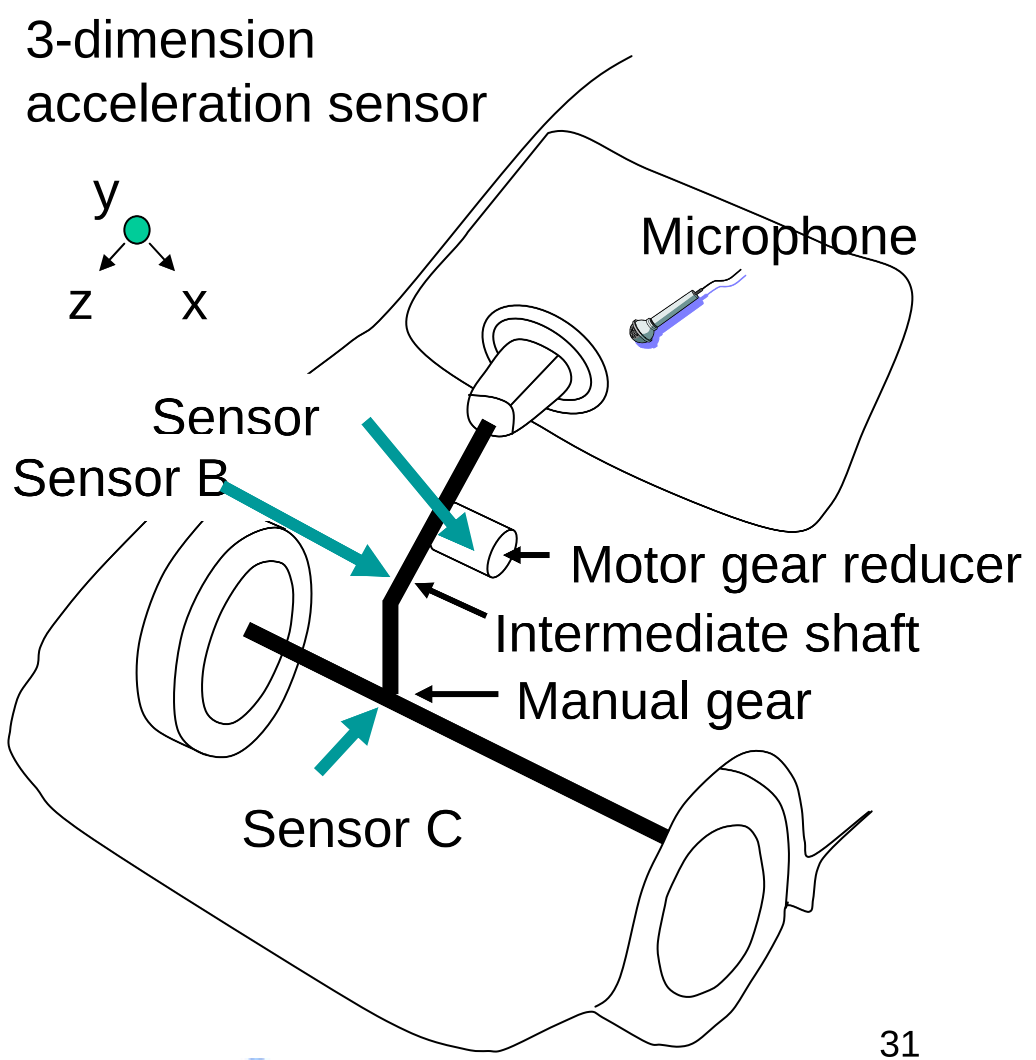
Result: the average SCRMW shown in (b), it has some frequency components which the original model signal does not have. However, in the case of the A-RMW, this phenomenon has been improved.

Detection result obtained by WIC

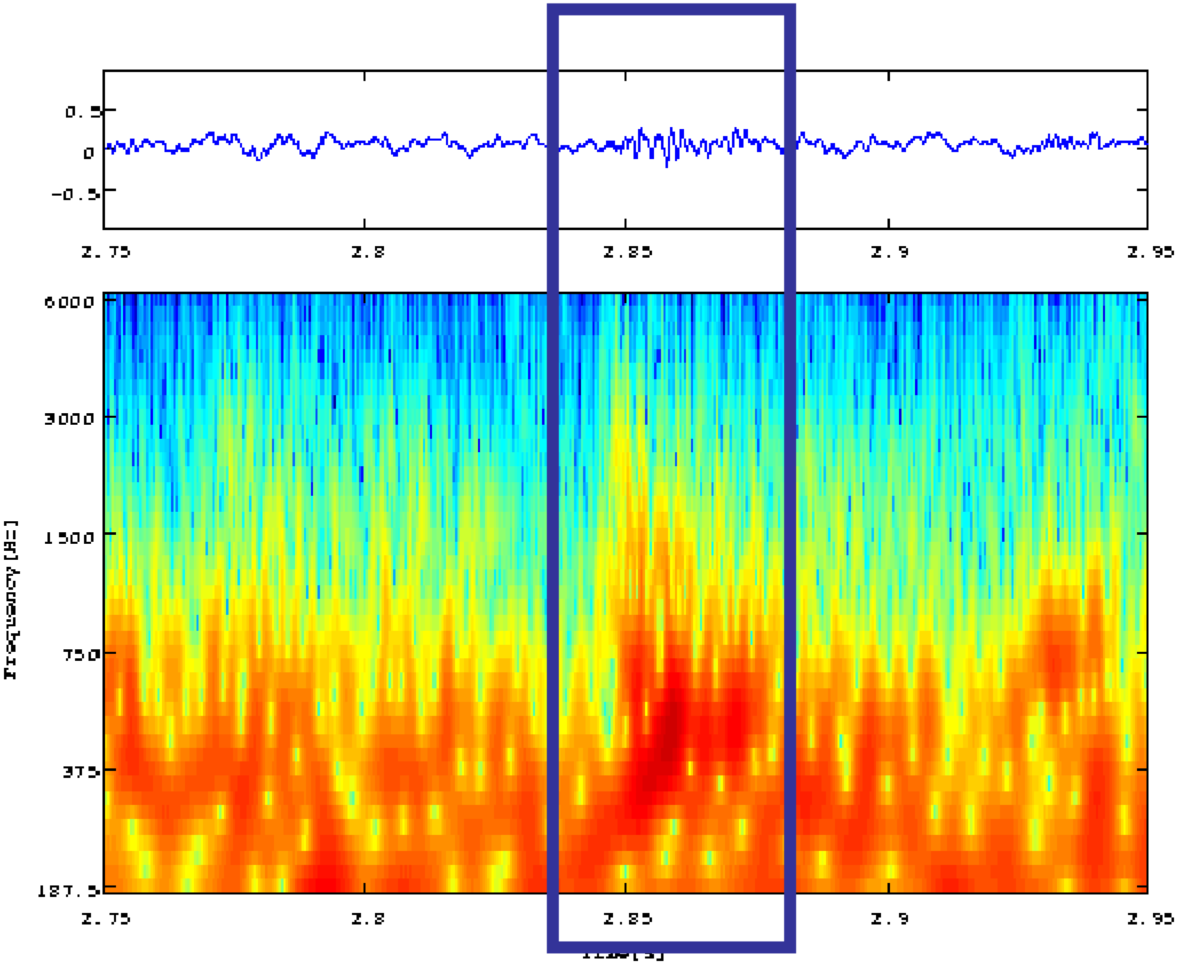


3) Rattle Noise Source Detection

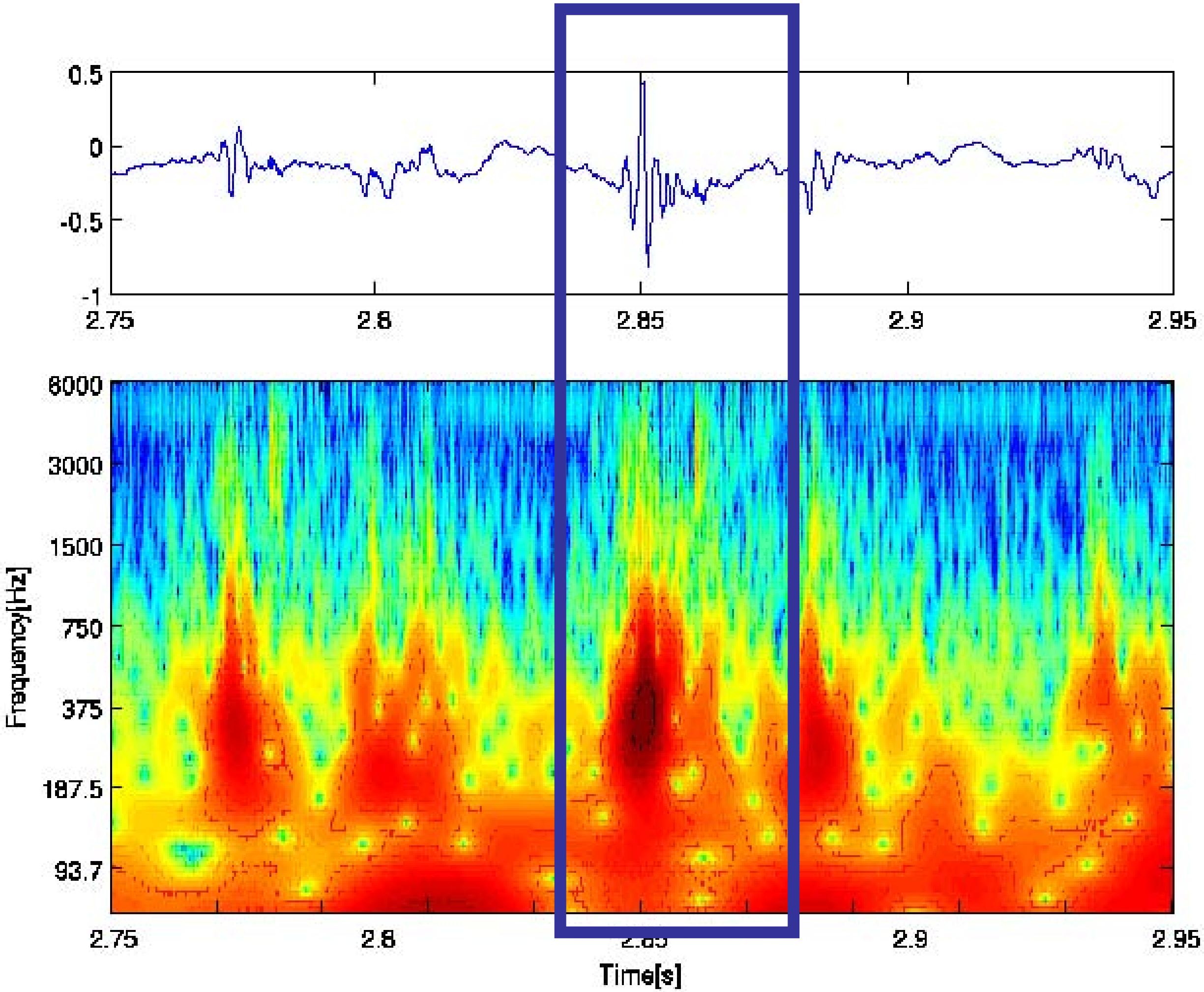
- Electric Power Steering (EPS)
- A rattle noise and a joint noise are occurred from EPS system.



Example of CWT (Rattle noise)

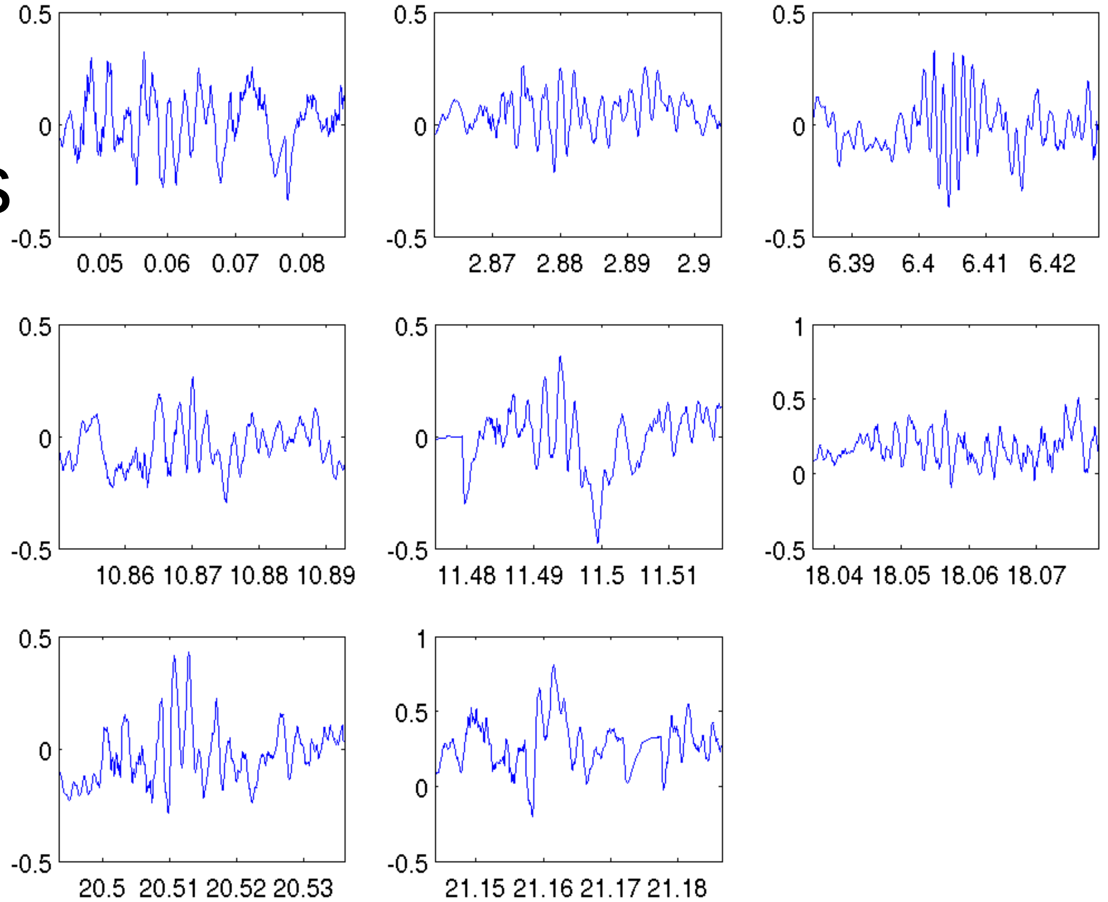


Example of CWT (Acceleration sensor signal)



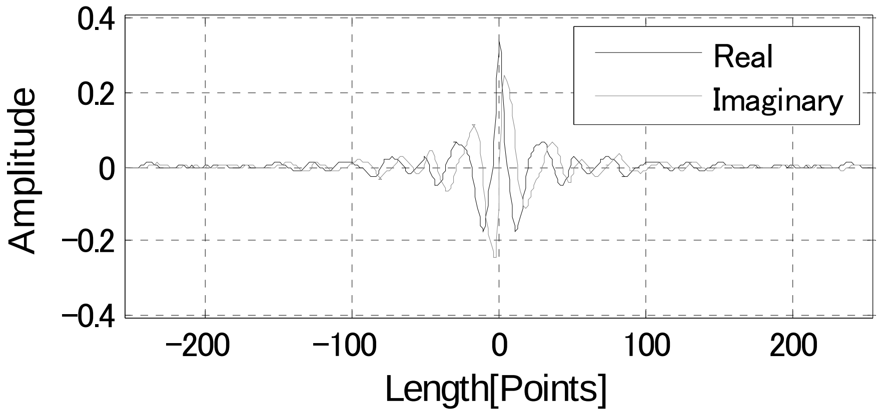
Constructing A-RMW

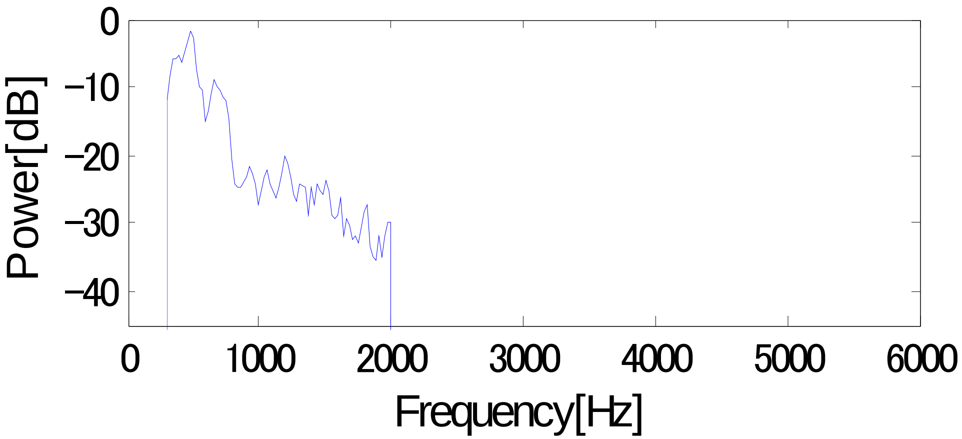
- The rattle sound signals picked up by using CWT.
- Construct A-RMW from eight signals.



Constructing A-RMW

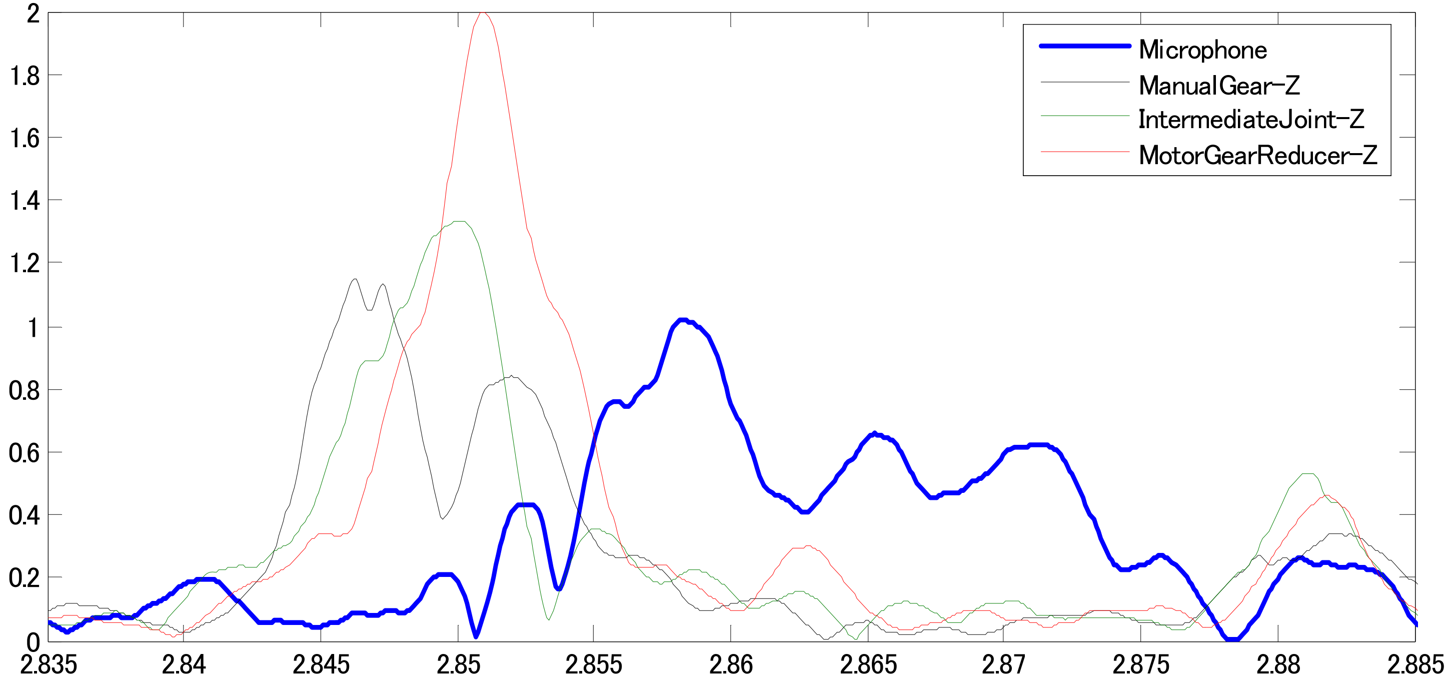
- We used band-pass filter.
- Because low frequency sound was very big and inaudibility sound.





Result of WICV

- Typical result (Z-axis).



4) Patient diagnosis by the WIC

- Patient's EEG of Alzheimer's disease (Wavelet Analysis)
- Choose Stable Wave

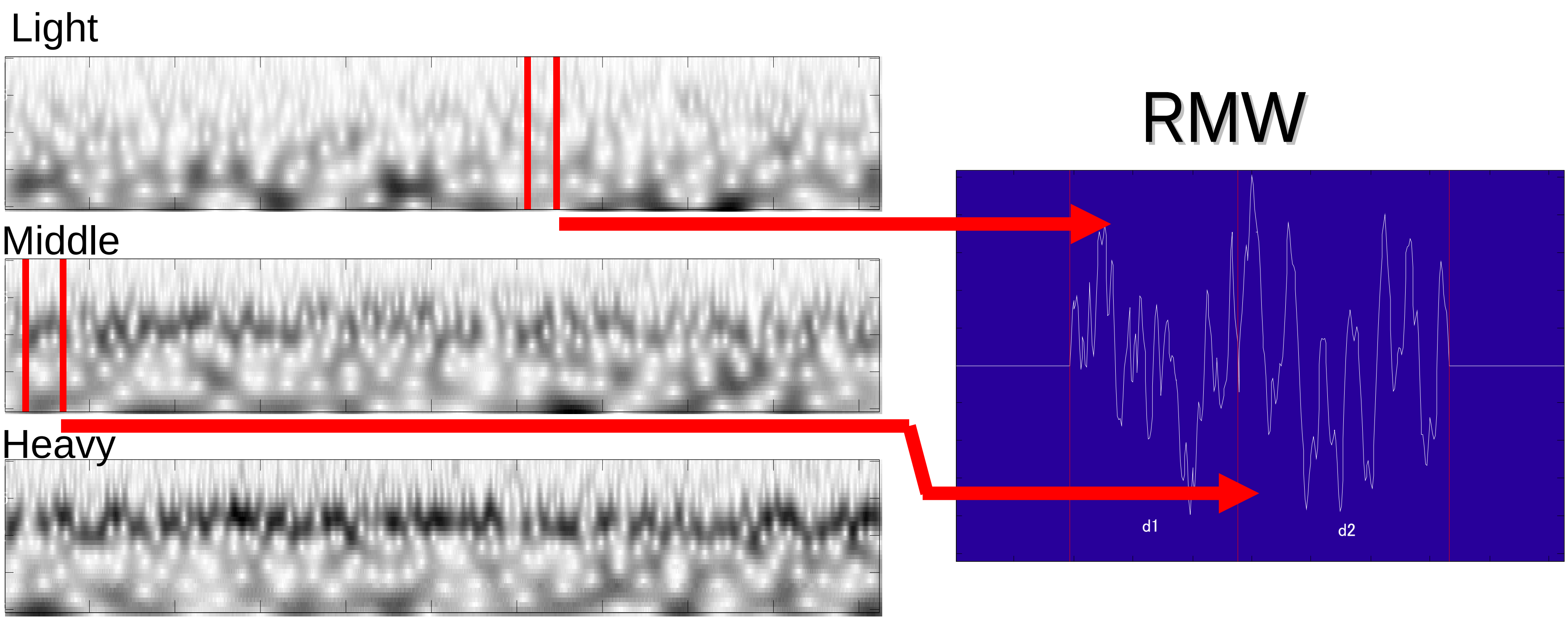


Fig.6 A-RMW constructed from EEG

Analytical Result

- Only Wavelet Coefficient at Scale 1.
- We set threshold to red line.

Healthy People

The Patients

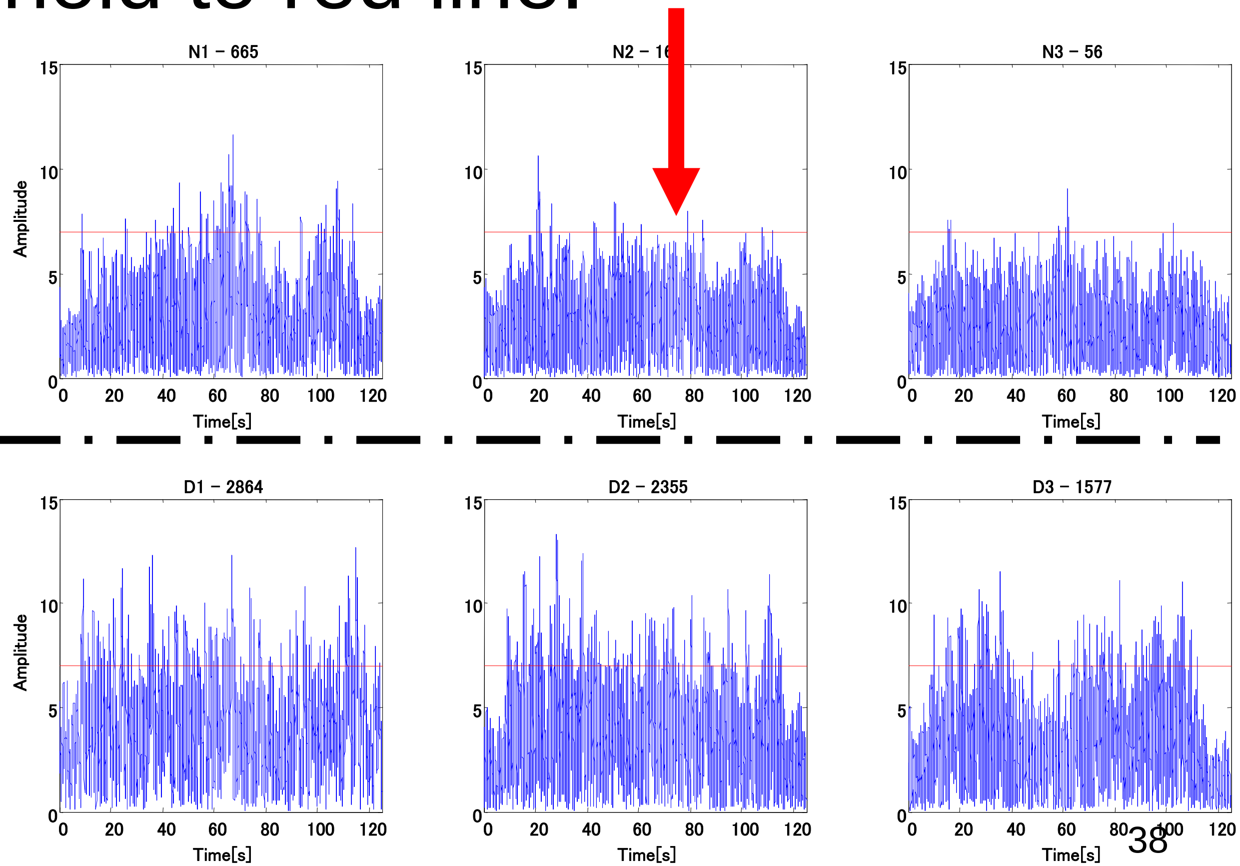


Fig.5 Wavelet Coefficient (Scale 1) using SCRMW

Analytical Result

- Statistic method
  - Threshold
  - Calc Average
- We could identify senile dementia.

Healthy People

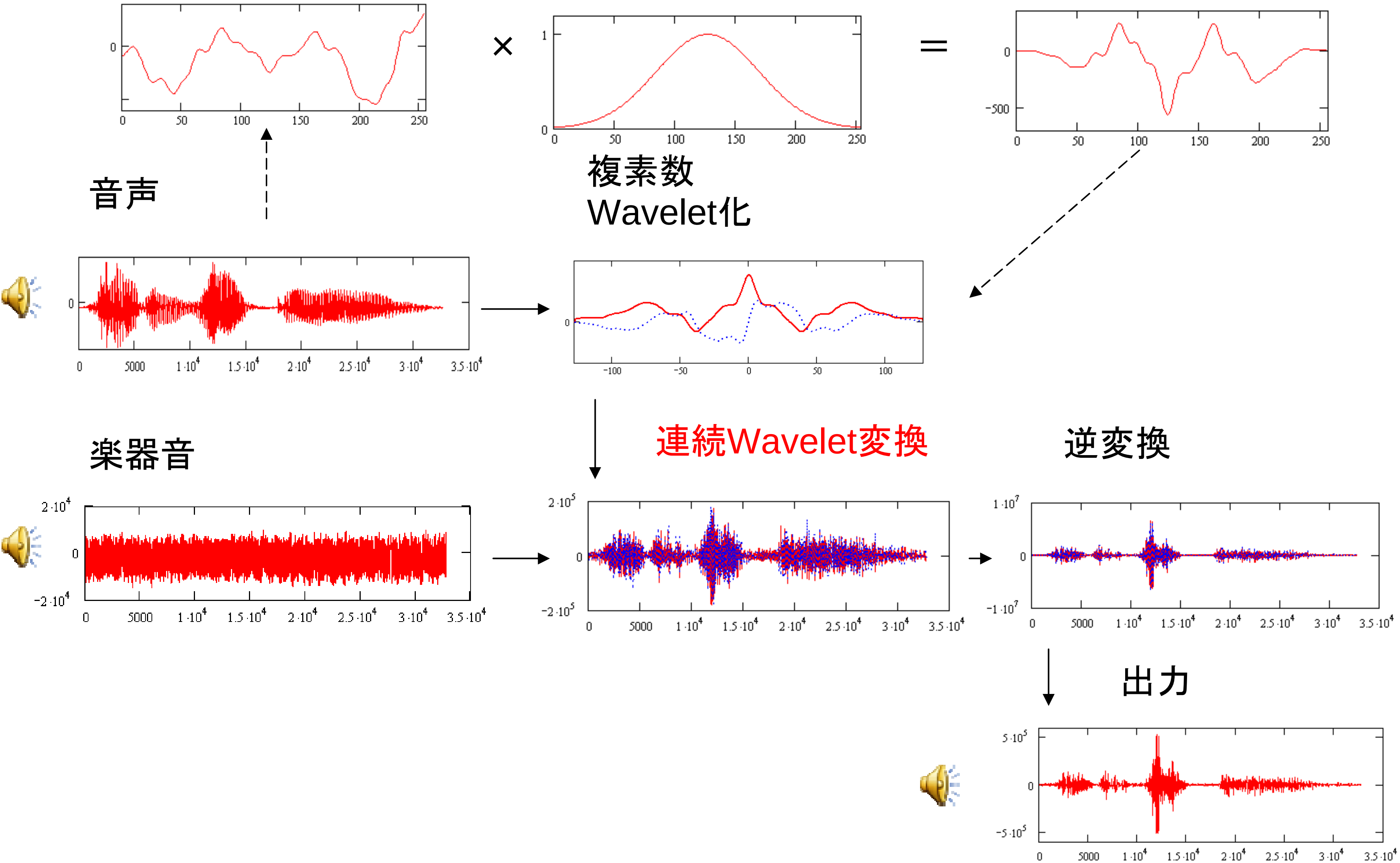
The Patients

	Over the threshold line	Average line
N1	665	3.04
N2	163	2.91
N3	56	2.62
D1	2864	4.27
D2	2355	3.98
D3	1577	3.63

5)Sound composition

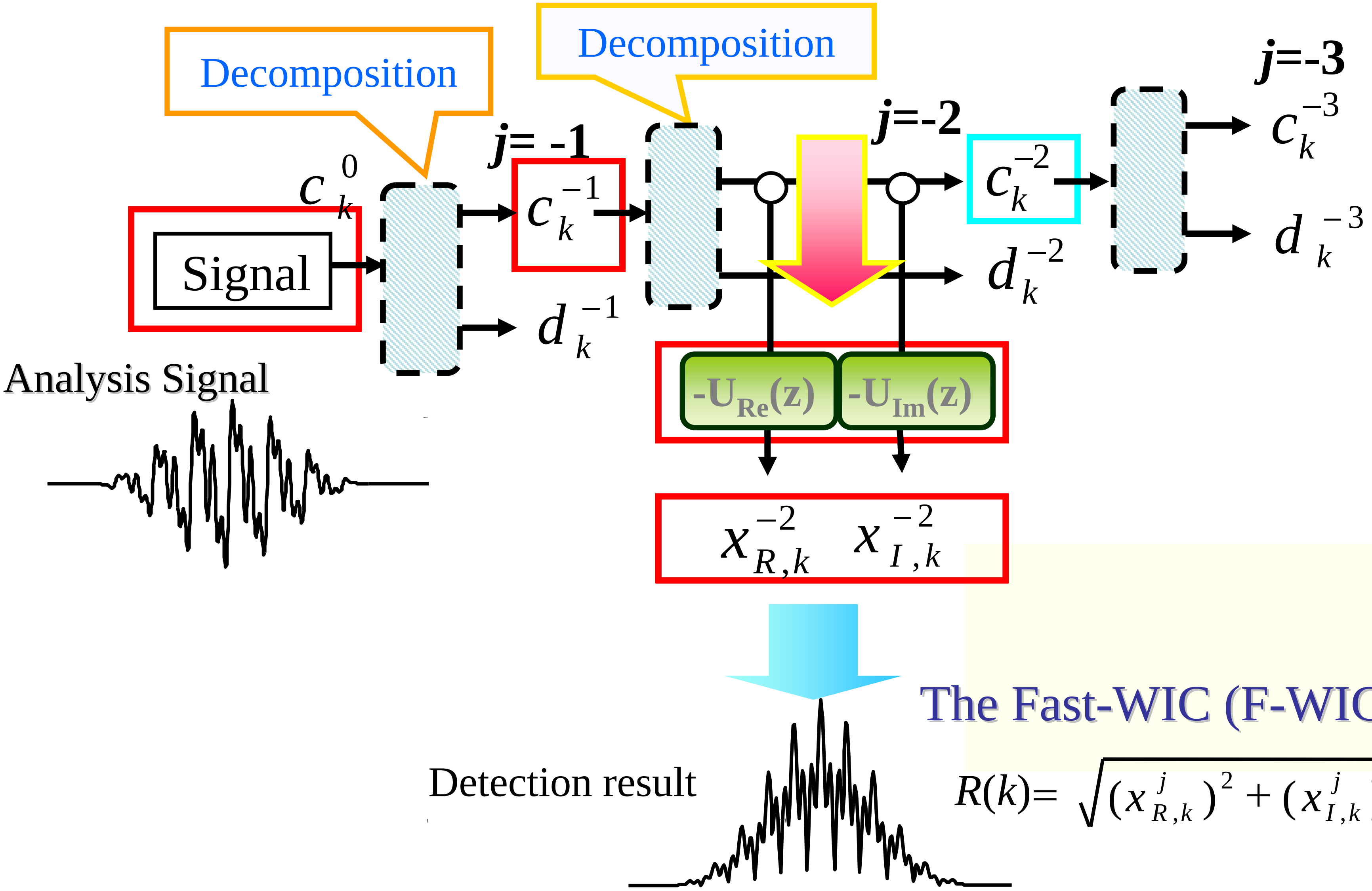
Conventional method

New method



6.4 New First WIC by using DWT

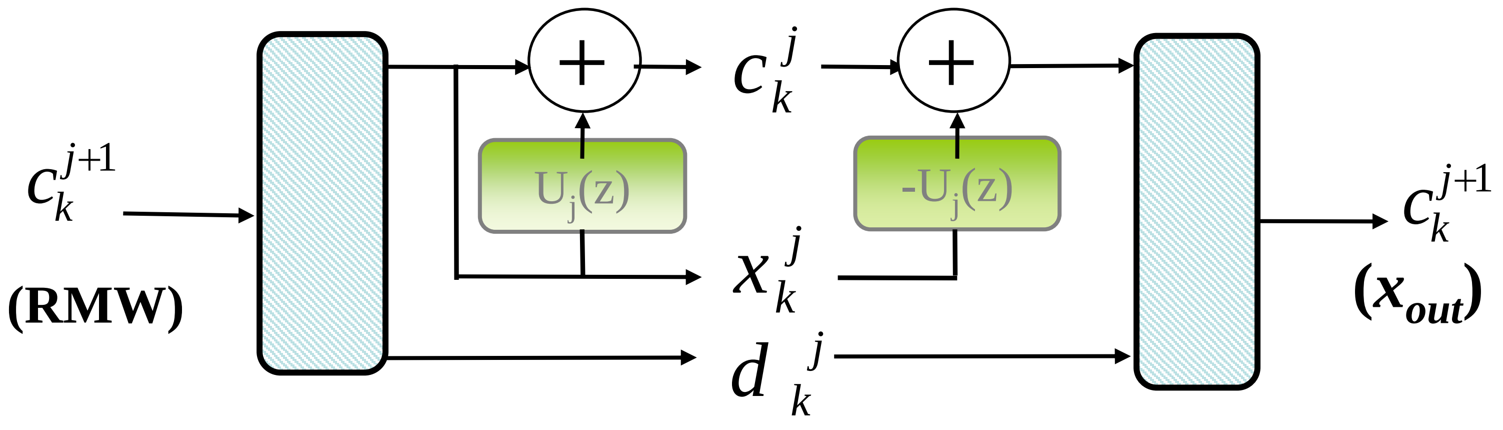
1) Parasitic-Discrete Wavelet Transform (P-DWT)



The Fast-WIC (F-WIC)

$$R(k)=\sqrt{(x_{R,k}^{j})^2+(x_{I,k}^{j})^2}$$

2) Design method of the parasitic filter



The Structure for design

1. Use the traditional DWT to decompose the constituted RMW to the parasitic level
2. Assume the coefficients $\{x_k^j\}$ are the initial parasitic filter sequences $\{u_k\}$
3. Set $c_k^j = 0$, $d_k^j = 0$, $x_k^j = \delta$ and calculate by using the inverse transform
4. Optimize $\{u_k\}$ with the following evaluation function.

$$\arg\min \left\| x_{out} - RMW \right\|$$

In the case of U_{Re} U_{Im}, they need to design each other

3) Verification experiment

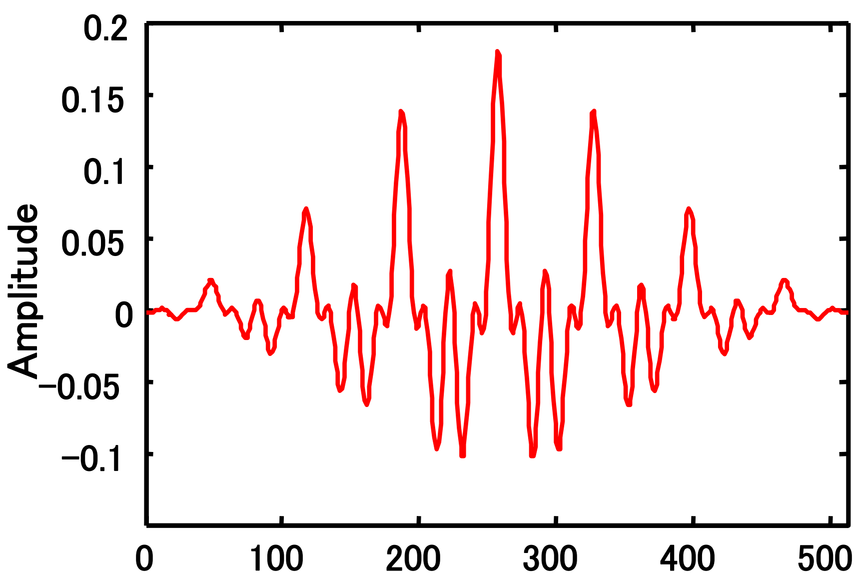
Design of the parasitic filter

- Model signal

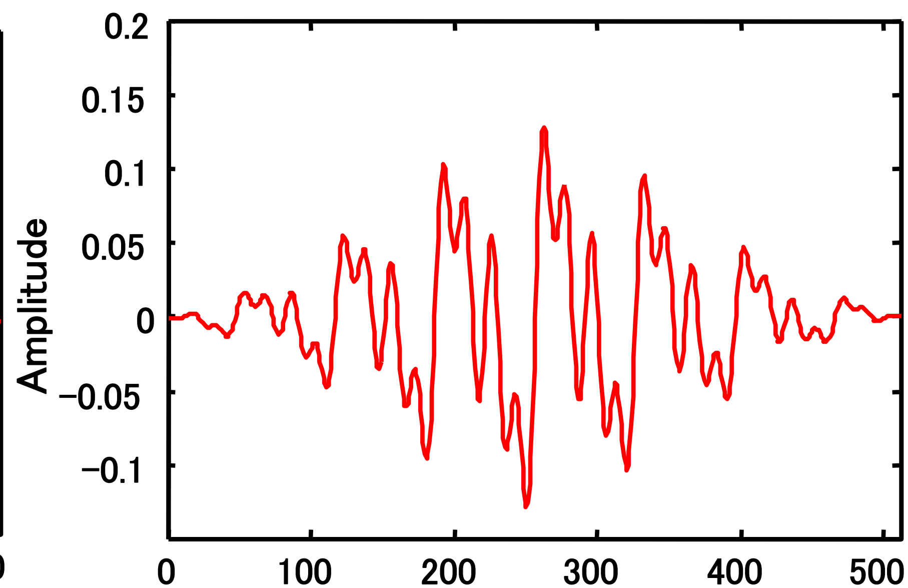
$$f(t) = \cos(2\pi 50t) + 0.7\cos(2\pi 100t) + 0.7\cos(2\pi 200t)$$

- **RMW width:512** ◆**Sampling frequency :3500Hz**

RMW



Real part



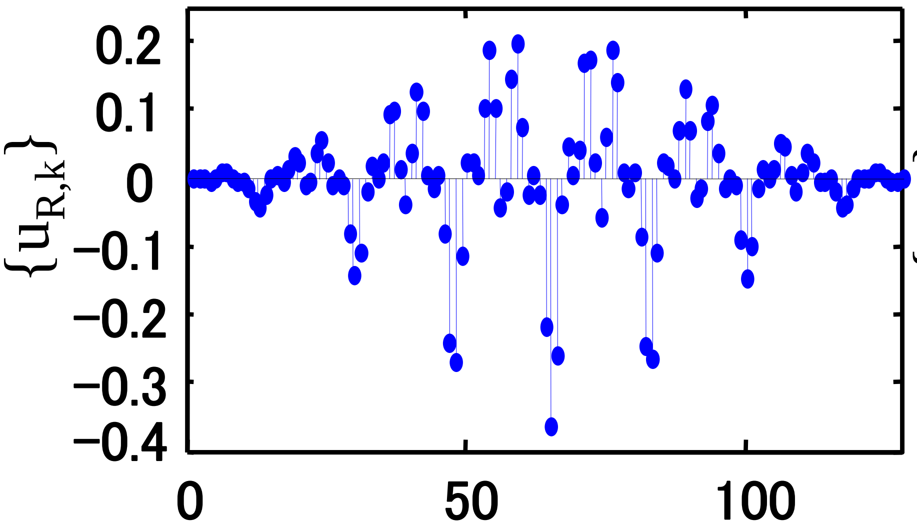
Imaginary part

Verification experiment

Design of the parasitic filter

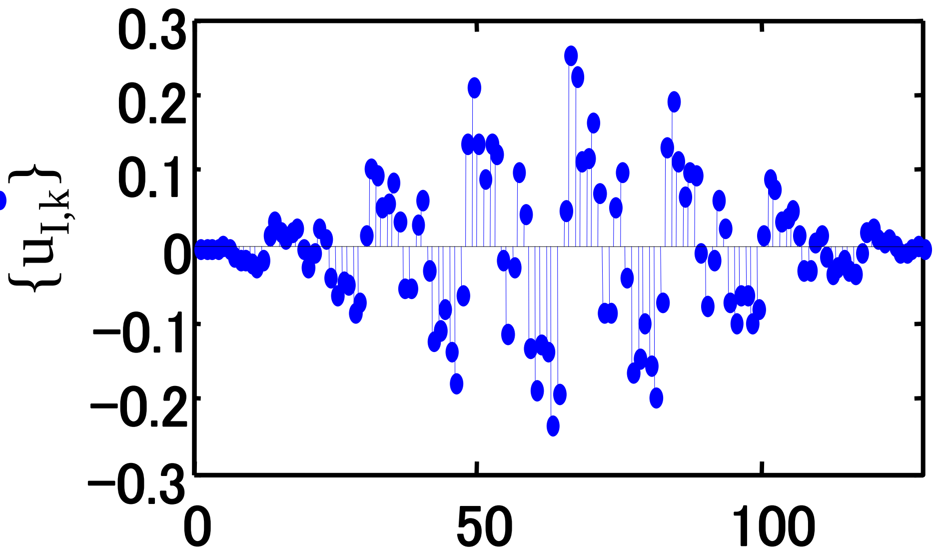
- **Parasitic level :-2**

Parasitic Filter



U_{Re}

Real part

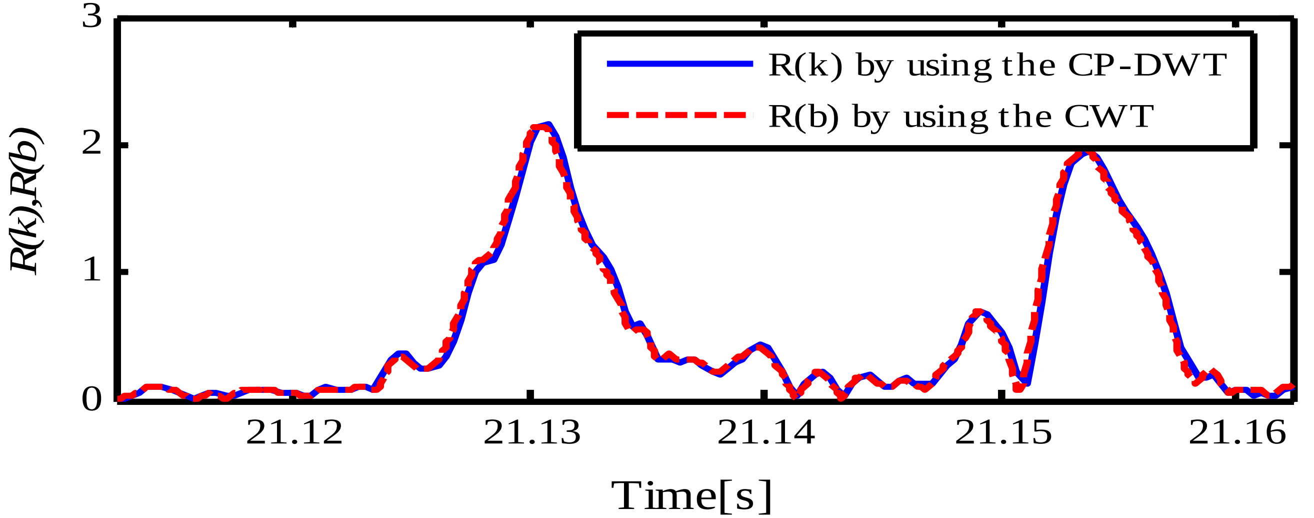


U_{Im}

Imaginaly part

4) Example of Detection result in the MGR (z direction)

- The F-WIC *R*(*k*) by using *the P-DWT* of the vibration signal in the *z* direction at the positions of the MGR.
- The WIC *R*(*b*) calculated by using the CWT also is shown in this figure
- we also define the mean square error (MSE) in the case of the MGR. MGR is -37.0 [dB].



6. Real Signal Wavelet Transform

6.1 Construction of Real-Signal MW

1)Selection of window function

3) Construction of a complex RMW

6.2 Detection of abnormal signal

1) Definition of Wavelet Instantaneous Correlation

2) Knocking extraction from block vibration

6.3 Construction of RMW using more than two signal

1) Flow chart of SCRMW construction

2) Construction Method of A-RMW

3) Rattle Noise Source Detection

6.4 New First WIC by using DWT

1) Parasitic-Discrete Wavelet Transform (P-DWT)

2) Design method of the parasitic filter

4) Example of Detection result in the MGR (z direction)

Report title:

Wavelet transform from my viewpoint

Deadline: February 28, PM5:00, Office