4. Measurement of the length and angle

4. 1 Measure Distance

Distance is length of line that connecting two distant points. It is often used as word “length”, but it should be considered as “a long length that ruler cannot measurement”. It is ambiguous use word “distance” in situation that a very short length likes interatomic distance. This section will descript a method that measure distance from observer to object by non-touch instead of ruler.

International common unit of length is meter [m]. Used of metric prototype during 90 years and be abolished in 1960s. A more accurate standard has been adopted. Which is “1650,763.73 times of light wavelength in vacuum, where the light is that transition between the energy levels $2p_{10}$ and $5d_5$ of the krypton 86 atoms”. And from 1982, General Conference on Weights and Measures use “the length of the path travelled by light in a vacuum in 1/299,792,458 second” as definition.

4. 1. 1 Basic measurement method

There are methods to follow the stereoscopic sense and echo:

1) Triangulation,
2) Pulse electromagnetic wave propagation time measurement,
3) Phase measurement of a tuned electromagnetic wave signal,
4) Light wave method.

Triangulation is a method that watching the object from two different watching points and calculates length from parallax, it is total as same as visual observation by two eyes.

Shoot a ballus-like electromagnetic wave to object, measure the time that reflection signal is needed, this method uses same principle of “echo”. In modern days, instead of electromagnetic wave or sound wave, light beam can arrive target correctly are used generally. Especially, laser is rapid develop and can be used as long-distance measurement.

1) Triangulation

Triangulation is a method that developed from 1960s. In Fig.4.1, two different watching points where distance is $d[m]$ when watch target, the shake angle (parallax angle) is θ, and distance $L[m]$ can be calculated by this equation.

$$L = \frac{d}{tan\theta}$$

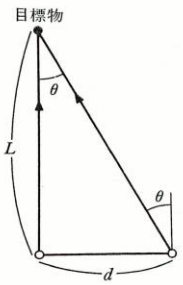

Fig.4.1 Principle of Triangulation

2) Optical pulse propagation time measurement

In Fig.4.2, irradiate target from distance $L(m)$ and receive the reflection signal.

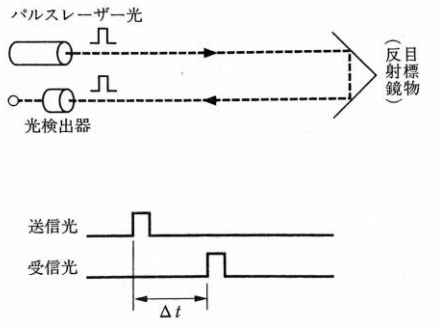

Fig.4.2 Measurement by Optical Pulse

Measure the time $\Delta t[s]$ of reflection signal against transmission signal. The distance can be calculated by next equation.

$$L = \frac{c\Delta t}{2},$$

where c is speed of light in measurement environment. The characteristic of above equation is, in the point where absolute distance is immediately known from the delay time. But, decomposed distance of $1[mm]$ must measure the decomposition of Δt by $6[ps]$, so in application, decomposition of few $[mm]$ is limit.

3) Phase measurement of intensity modulated light

In Fig.4.3, use intensity modulated light instead of optical pulse, and measure phase of reflection light. Measure phase by observe the

propagation of light waves. It is same principle as “2) optical pulse propagation time measurement”. Modulation wavelength $\Lambda(m)$ of modulated light signal by frequency f[Hz] is

$$\Lambda = \frac{c}{f}$$

There is a relation of get distance L from phase difference where the phase difference $\emptyset[rad]$ is between send a signal and receive the signal, which can be expressed next equation.

$$L = \frac{1}{2}\left(N + \frac{\emptyset}{2\pi}\right)\Lambda,$$

where N is times. But, if L is longer than $\Lambda/2$, and absolute length is become not understand. Therefore, for getting N use light source with $\Lambda' > 2L$ and get phase difference $\emptyset'$.

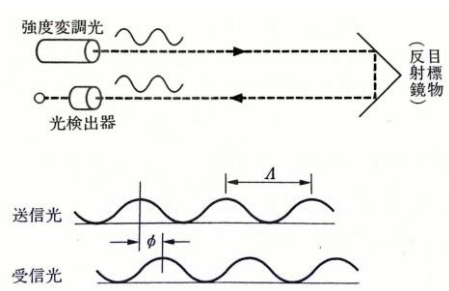

Fig.4.3 Measurement by Measure Phase

$$L = \frac{\emptyset'\Lambda'}{4\pi}$$

$$N = \frac{1}{2\pi}\left(\frac{\Lambda'}{\Lambda}\emptyset' - \emptyset\right)$$

But, if $2L$ is larger than Λ', because of two degrees and L ’s 3 degrees are become unknown, use matching method by successive calculation to calculate L.

4) Interferometry

Because light is transverse wave and can be seen as sine wave occurred by amplitude of monochromatic light, it can be used as same principle as “3) Phase Measurement of Intensity Modulated Light”. But frequency of light is 500[THz] (Terahertz: $10^{12}[Hz]$) , modern device to observe phase of wave is impossible. Therefore, get phase information as low-frequency light intensity information, use reflection signal from target (reflect mirror M2) to interference base light. This device is called Michelson interferometer. Fig.4.4 shows Michelson Interferometer, where (a) shows optical system, and (b) shows output of Detector.

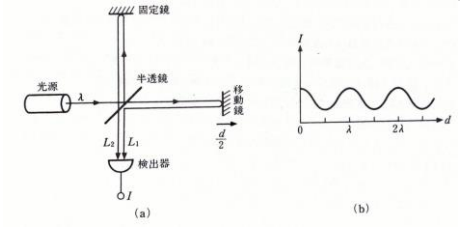

Fig.4.4 Michelson Interferometer

Wave length of light is $\lambda[m]$, basis optical path length is L_2 and measurement optical length is L_1, and output of detector is I.

$$I = A + Bcos\left(\frac{2\pi d}{\lambda}\right),$$

where, $A > B > 0$. Observe change of I and can get moving distance by decomposition of $1/10$ of wave length (around $50[nm]$.

4. 1. 2 Visual Measurement

Measurement by visual could be very correct if have experience. Measure distance of object in front of self by eye, line of sight match the position of object, get distance by two movement that adjust thickness of eye and focus.

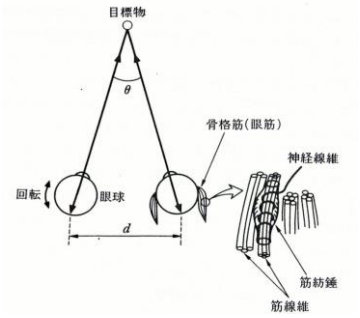

Fig.4.5 Measurement by Eye

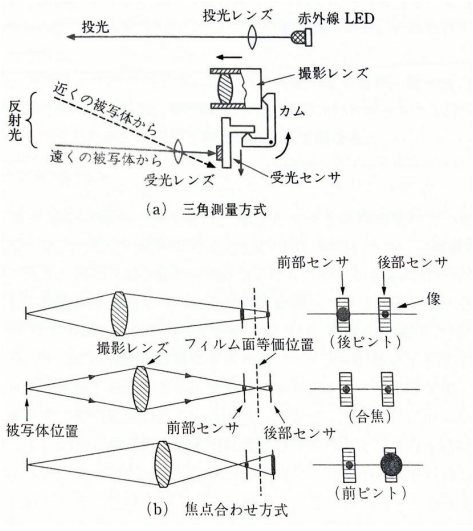

Fig.4.6 Auto-focus in Camera

In Fig.4.5, when sight is match with object, and images of two eyes by retina is same and sharp, the parallax angle θ can be detected by muscle spindle in ocular muscle. Auto-focus in camera, uses same principle as visual measurement, which is triangulation or match focus.

In Fig.4.6, figure (a) is triangulation and figure (b) is match focus. In figure (a), use infrared laser to irradiate object, and receive reflection signal by sensor interlocked with lens extension and cam. Reflection signal from object in short-distance, where reflection angle is big and sensor move down, lens focus on the object at a short-distance. But object in long-distance, the reflection signal is parallel light, and lens back off. This method is used by compact camera with lens shutter. In figure (b), set sensor at film surface in front and back. Adjust lens to make the images of sensors are same. This method is used by single lens reflex camera which can exchange lens mostly.

4. 1. 3 Difference in measurement method by distance

1) Measure Long-distance

The definition of long-distance is uncertain, but above 100[km] height airspace is outer space, so 100[km] can be considered as long-distance. Set lights year away start as target, use triangulation to measurement by annual period difference. In this situation, receive light or radio wave from start. In modern days, there is a plan that measure distance of 10-thousand light years away from earth.

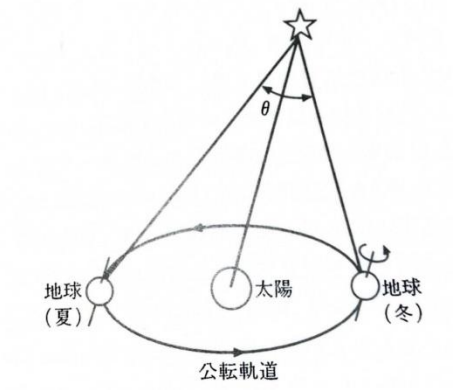

Fig.4.7 Triangulation by Annual Period

GPS (Global Positioning system) is a method that get position at earth by receive signal from 20-thousand kilometers away artificial satellite, and is applicated by car navigation. GPS is a military technology that developed by United States Department of Defense, and used by private. It is not only applicated by car navigation and also by ship. There are 24 artificial satellites around earth and 2 rounds in 1 day. Time information is in the radio wave, so the delay is known, and uses same principle as optical pulse propagation time measurement, the distance of receiver and artificial satellite will be calculated. For make sure orbit of satellite, in Fig.4.8, distances between 3 satellite is known and vertex of triangle cone is known, but in practical there are at least 4 satellites are used, the another one is for fix time. Car navigation get correct position by in addition, receiver settled in main road, or calculate mileage by wheel revolutions.

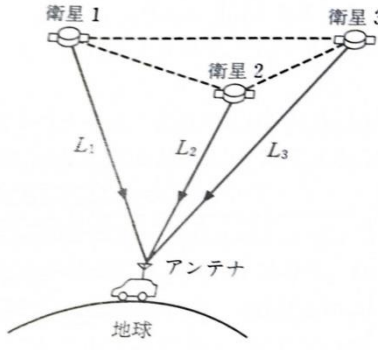

Fig.4.8 Principle of GPS

2) Measure Mid-distance

Mid-distance is from tens of kilometers to 100[m], topographic surveying on earth is mid-distance, and use triangulation and phase measurement of intensity modulated light. Triangulation is basic measurement of distance, which advantage is not depending on complex device. But main measurement is light wave distance measurement. He-Ne laser can measure distance until 50[km], but measure longer distance will need high voltage power, which make movable become difficult. Therefore, a portable type, which is battery used light emitting diodes or laser diodes as light source, receiver is photodiode, can measure longer distance.

Measurement under water use ultrasound pulses. In this situation, if send ultrasonic in air, most of ultrasonic will be reflected by water. So, it is necessary that transmitter is under water. Ultrasound pulses can be used as search for sinks, or fish detection. Use 50~200[kHz] ultrasound can detective fish under

200~300[m], and accuracy is tens centimeters. And can explore terrain under 1000[m] with accuracy of tens centimeters.

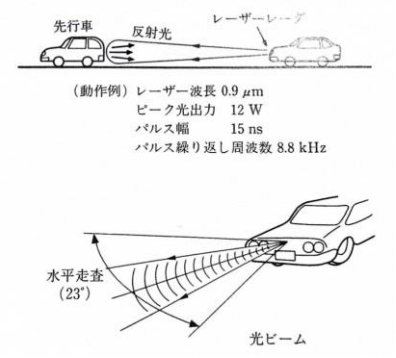

Fig.4.9 Laser radar

3) Measure Short-distance

Example of measure distance 10~100[m] is laser radar used by car (Fig.4.9). This device is aim for keep inter-vehicular distance and prevents hit human or obstacle. Principle is same as optical pulse propagation time measurement. For miniaturization and vibration resistance, semiconductor pulse laser is used, and for radar operation, beam travel mechanism is used. It is necessary that detective tens meters obstacle to prevent hit, high distance resolution needed tens centimeters. But since there is no reflection mirror, light beam shoot to object surface directly, reception signal is affected by object surface, and marginal distance is different. Here is an example, to car in sunny day can detective 150m, but can detective 140m in rainy day. To human only can detective 40~50m because of cloth.

4. 2 Measure Length

4. 2. 1 Directive Measurement and Indirective Measurement

Measure length or size is used in daily life and production site. Use length meter or length measuring machine as measurement device. Especially, device for measure length under 10[mm] called micrometer.

Limit of read from ruler is 0.1mm, But if expand smallest scale, and can read more and correctly. Consider method like, Vernier scale, Mechanical expansion (screw, gear, lever), optical expansion (lens, light lever, moire fringes), electrical expansion (differential transformer). Performance of these methods are shown by Tab.4.1.

Tab.4.1 Performance of length meter

種類	最小目盛り(mm)	確度(±μm)	測定範囲(mm)	目盛り拡大方法
直尺	1または0.5	最良で50	0～1,000（各種）	
ノギス	0.05	20	0～1,000（各種）	バーニア機構
マイクロメータ	0.01	2	0～25	ねじ送り
ダイヤルゲージ	0.01	10	0～10	歯車
ミニメータ	0.001	0.5	±0.03	てこ
オプチメータ	0.001	0.5	±0.1	光てこ
電気マイクロメータ	0.0005～0.005	0.25～2.5	±0.015～±0.15	
空気マイクロメータ	0.001	1	±0.015	空気圧
測長機	0.001	1	0～1,000（6mのものまである）	読み取り顕微鏡

4. 2. 2 Expand scale

1) Vernier Scale

Divide basic length of ruler scale by n equally. And make Vernier with $(n-1)$ scale and divide it by n equally. And align ruling. In Fig.4.10, basic length is $1[cm]$, and divide it by 10 equally, and Vernier is divide $9mm$ by 10 parts. Read $71.4[mm]$ from Vernier.

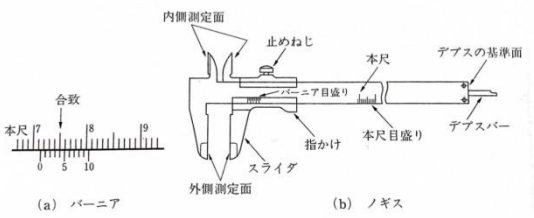

Fig.4.10 Vernier Scale and Calipers

2) Mechanical Expansion

Mechanical expansion is most simple method. Use this principle, there is minimeter which can expand 1000 times and can measure during $\pm 30[\mu m]$ with smallest scale $1[\mu m]$. Dial gauge is being shown in Fig.4.11.

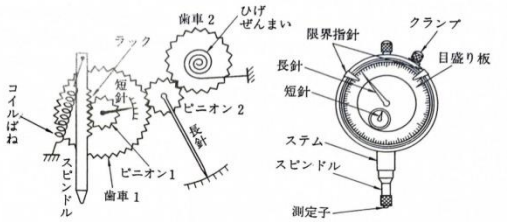

Fig.4.11 Dial Gauge

Straight movement of spindle make rack and pinion 1 turn around. Then expand rotation angle of gear 1 and pinion 2. It is straight relation in movement of spindle and angle of pointer rotation. The aim of gear 2 is for initialization. And because of there is error in gear, this method has error around $10[\mu m]$.

Micrometer shown in Fig.4.12 uses spring to expand. Screw with pitch p rotate θ, and

forward a. This relation can be expressed by next equation.

$$a = \frac{\theta p}{2\pi}$$

From above equation can know that, there is a straight relation between a and θ. The smaller p the bigger expansion rate.

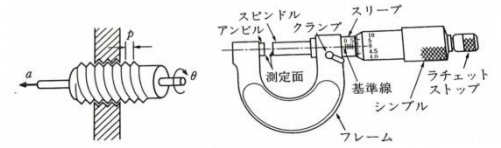

Fig.4.12 Micrometer

3) Optical Expansion

Principle of optical lever is been shown in Fig.4.13. When movement of spindle x is small, and angle of reflection mirror is

$$\theta = \frac{x}{a}$$

Shoot light beam in reflection mirror, observe reflect light at distance L, movement d by shake of beam can be expressed by next equation.

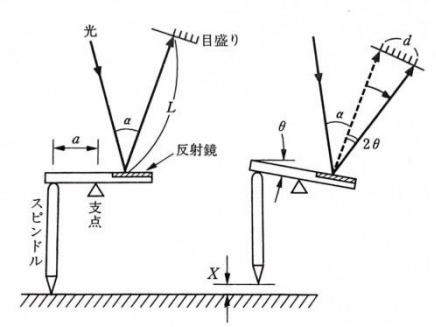

Fig.4.13 Optical Lever

$$d = 2\theta L = \frac{2xL}{a}$$

Expand movement of spindle, optimeter is one of application. Smallest scale $1[\mu m]$ within arrange $\pm 100[\mu m]$.

4) Light Wave Drought Expansion

Put glass board on reflection mirror with some angle, can observe drought. In Fig.4.14, monochromatic light follow arrow and arrive AB, part of it reflect at point B and follow BC, rest of if pass point B and follow $BDEF$. Path of latter one is longer than former one at $BDE \cong 2d$. If phase of B, E is same and light will be bright, if not light will be dark. Because phase will be flipped at reflection point D.

$2d = n\lambda, (d = n \cdot \lambda/2\ dark)$

$2d = (2n+1)\lambda/2, (d = (n+1/2)\lambda/2\ light),$

where, λ is wave length.

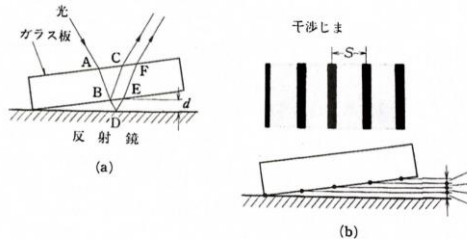

Fig.4.14 Light Wave Drought

Therefore, get drought of figure (b). If d is known, and magnification rate E can be calculated by next equation.

$$E = 1/\varphi = 2S/\lambda$$

For example, $\lambda/2 = 0.32[\mu m]$ (Red Interference Shield), if $S = 2[mm]$, and $E = 6250$.

5) Electrical Expansion

a) Inductance conversion

Turn movement of spindle into electrical value, it is not only expanding any times by amplifier, but also can record data. Differential transformer is most be used, and principle is been shown by Fig.4.15.

It is 1st coil in middle, and 2nd coil differential connected. Add AC voltage at 1st coil, and AC magnetic field transfer to 2nd coil by iron core, and produce induced voltage (e_1, e_2), and output e_0 is

$$e_0 = e_1 - e_2$$

In middle of iron core, cause of $e_1 = e_2$, output is zero, and in upper of iron core, cause of less voltage transfer to down coil, which means $e_1 > e_2$, output is sine wave.

Around middle of iron core, there is straight relation between movement d, and e_0. So measure voltage and movement can be calculated. Move down of iron core, in this situation, phase will flip. So measure phase and voltage will get movement.

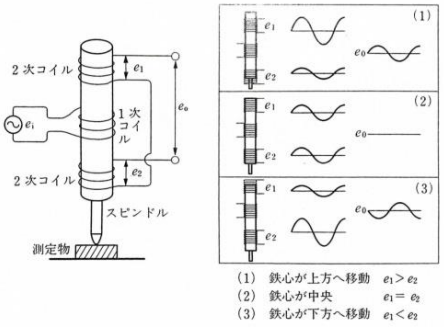

Fig.4.15 Micrometer with Differential

b) Electrostatic Capacity

In Fig.4.16, electrostatic capacity C between parallel electrode plate can be expressed by next equation.

$$C = \frac{\varepsilon S}{d}$$

Where, ε is dielectric constant, $S(m^2)$ is effective area of electrode plate, d is distance between electrode plate.

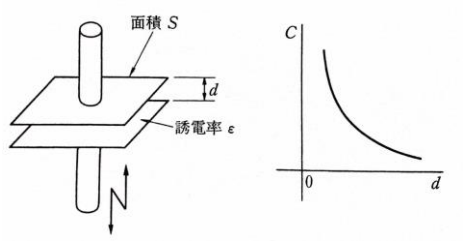

Fig.4.16 Electrostatic Capacity

From above equation, if there is a little change of d, which is Δd, and a little change of C, which is ΔC.

$$\frac{\Delta C}{C} = -\frac{\Delta d}{d}$$

Which means, variation of capacity reflects change of distance, and variation of voltage reflect change of capacity.

Generally, use differential method in Fig.4.17.

$C_1 = \frac{\varepsilon D}{d+\Delta d}, C_2 = \frac{\varepsilon D}{d-\Delta d}$

And little difference Δd, ΔV can be expressed by next equation.

$$\Delta V = V_1 - V_2 = \frac{V}{d}\Delta d$$

Advantage of this sensor is sensitivity $\Delta C/\Delta d$ is high. Although range is $1mm$, but can measure $1nm$ variation.

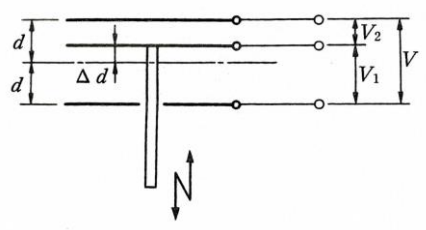

Fig.4.17 Differential system

6) Digitization

For automated in measurement or processing, in order to instead of human, output of measurement device is digital. Make straight movement to digital is called linear encoder. Optical pulse scale, magnetic scale is been used.

Fig.4.18 shows method that digital straight movement, where (a) is a scale with arrange elements whose magnetization directions are alternately reversed, and (b) is an optical Pulse Scale. Measure movement of scale by magnetically sensor, and get pulse number of percent of movement.

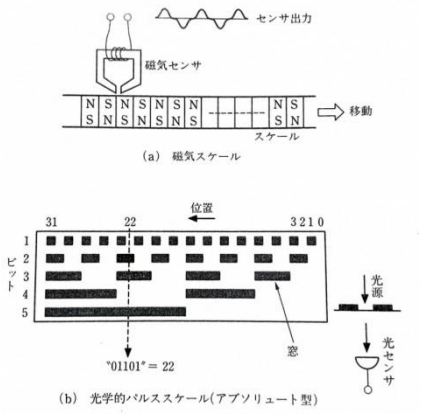

Fig.4.18 method that digital straight movement

4. 2. 3 Length Measuring Machine

Use length measuring machine with standard ruler to measure object. It is a precision and experience device in measurement. The construction of length measuring machine is been shown Fig.4.19.

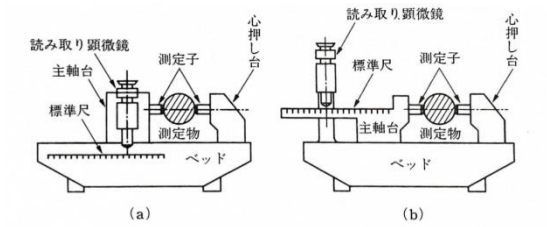

Fig.4.19 Length Measuring Machine

Read scale when two sides of probes are been contacted. And then read scale when objected are interposed. The difference is length of object. Smallest scale is $1[mm]$, but use microscope can read $1/1000$ of smallest scale. There are two types of length measuring machine shown Fig.4.19, where (a) is ruler in bed, and (b) is ruler can be moved at same line with object. Therefore, (a) is bigger, but (b) is more accuracy.