Wavelet Transform and Time-Frequency Analysis

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Information

1. **The file (PowerPoint) of lesson can be download from following homepage:**

   http://is.pse.tut.ac.jp/
   User name: keisoku
   password: kougi

Outline of the lesson

1. Introduction and Digital and Sampling Proposition
2. Signal Type and Choosing Analysis Method
3. Continues Wavelet Transform
4. Discrete Wavelet Transform
5. Real signal Mother Wavelet and Instantaneous Correlation
6. Image processing using Wavelet Transform
7. Information Theory and Signal Processing

*Evaluations method: Report

1.1 Introduction

1.1.1 In the car production

Design on the strength and outside

(CAD and Design)

Mechanic foundation

Materials
Materials Engineering

Processing
Manufacturing Engineering

Management, Assembly, Testing
Production Systems Design, Control

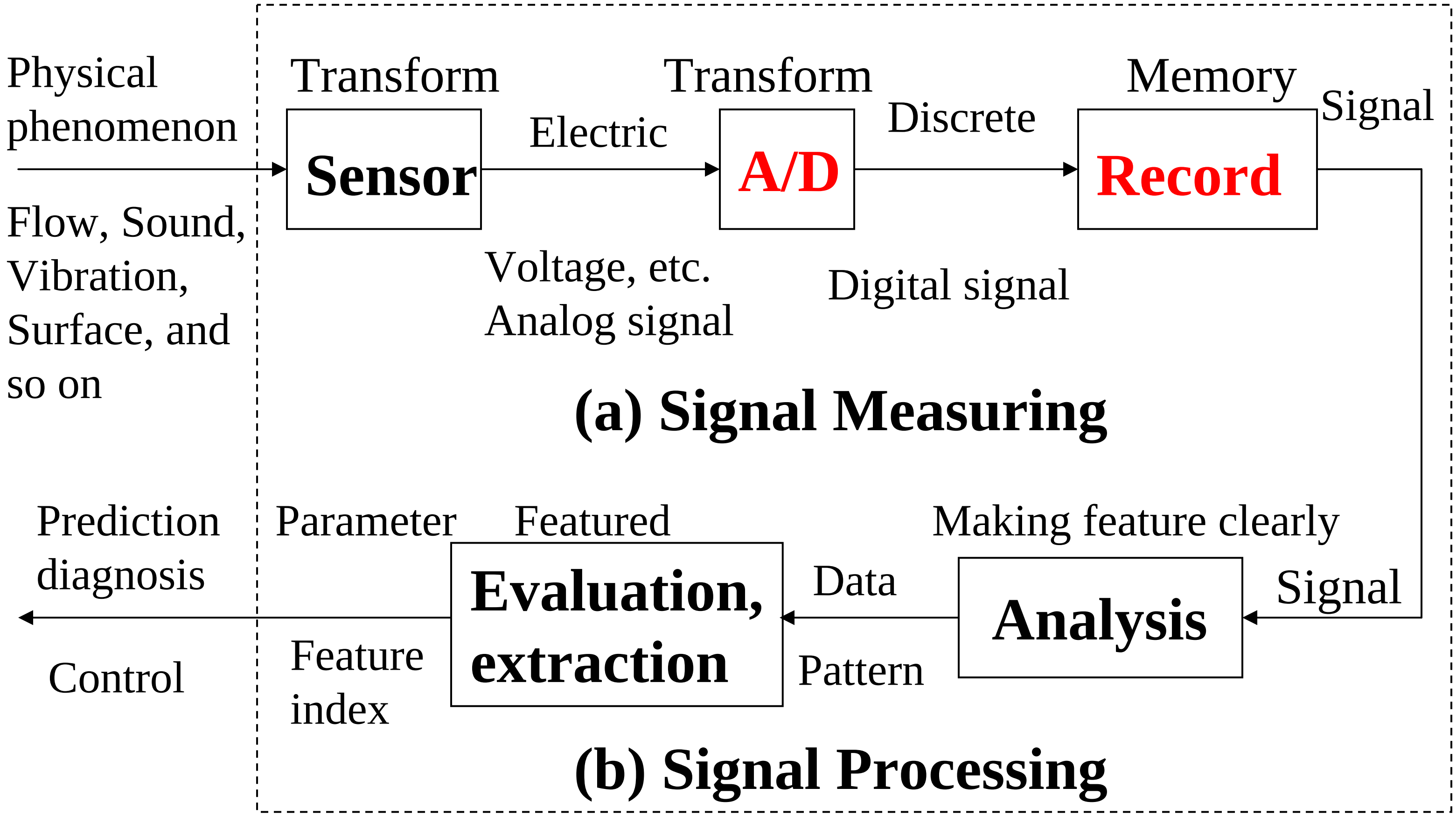
Product(Car)

Performance Test of a Car

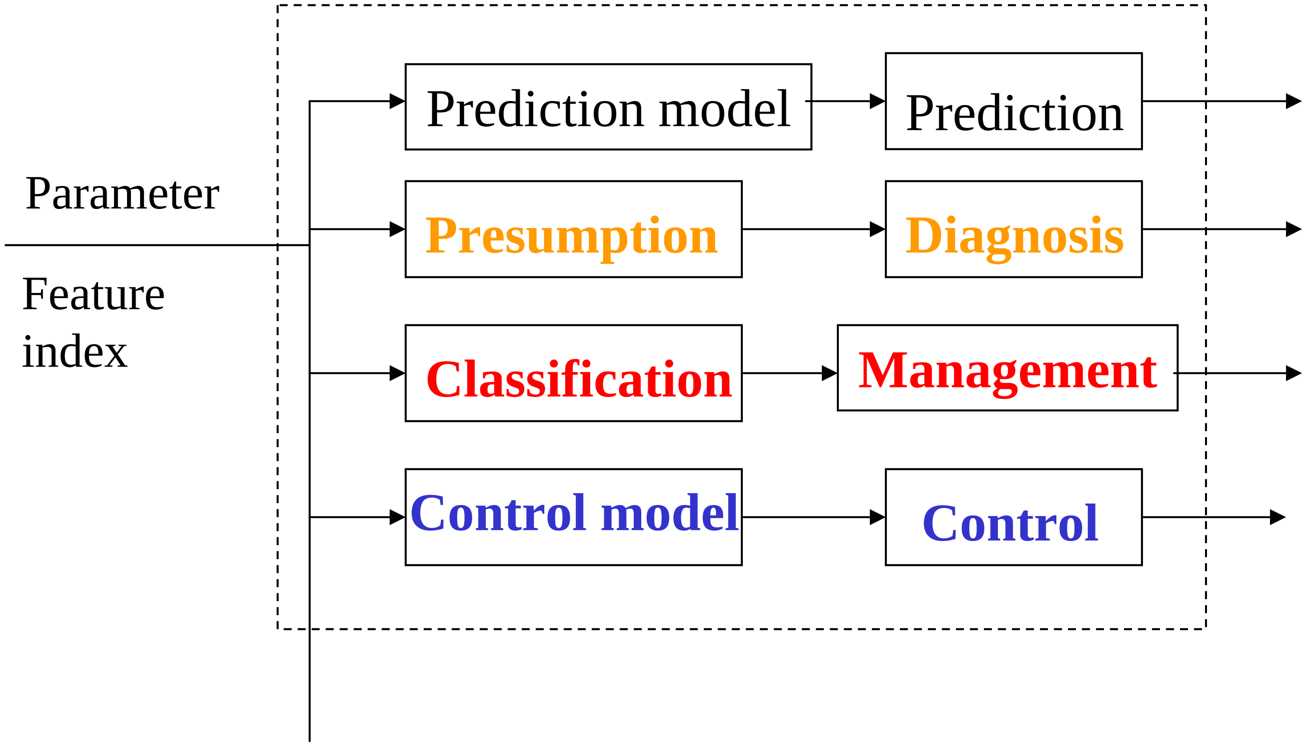
Distortion sensor

Acceleration sensor

1.1.2 Measuring System



Intelligent System

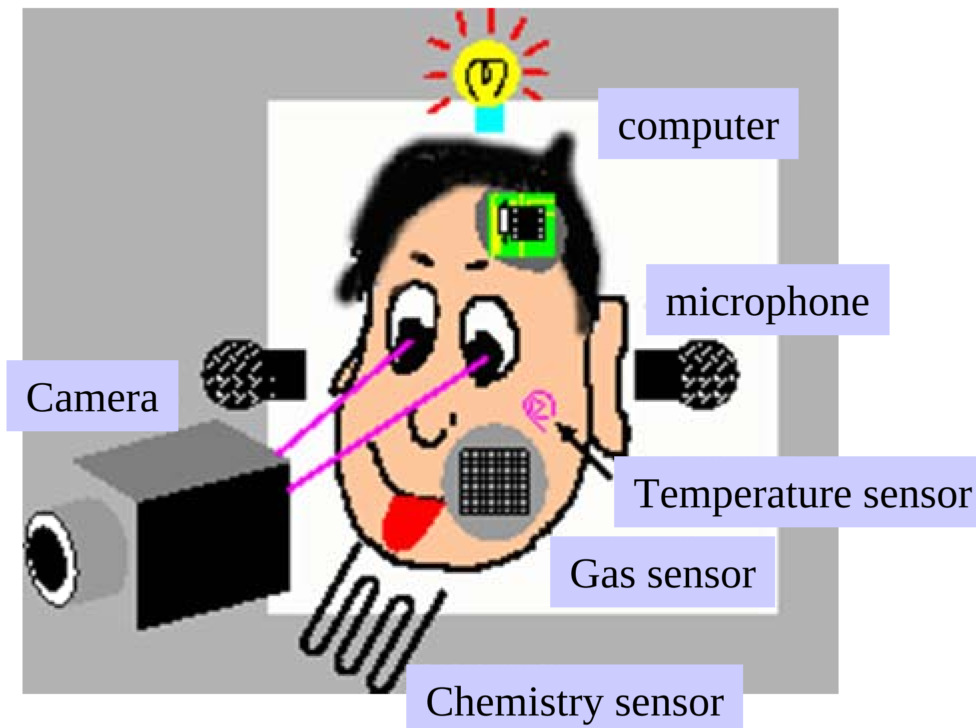


1.1.3 Sensor

(1) What's sensor?

The artificial sense organ for giving feeling to a machine

A sensor realizes human's senses, vision, hearing, and the sense of smell in engineering.



(2) The sensors in our life

Refrigerator, Washing machine, Air conditioner, etc.

Temperature sensor,

Water sensor, Flow sensor, etc.

Automatic door etc.：

Infrared sensor,

Ultrasonic sensor, etc.

In a car, there are dozens of kinds of sensors;

The robot for industry with a variety of sensors in a Manufacture line.

1.1.4 Signal Processing

(1) What's signal?

Signal: Informational physical bearer

A signal is mathematically expressed with the function of some independent variables, and, in most cases, it has only one independent variable called time (time series data).

Signal Type

time classification: Continuation, discontinuity

Freq. classification: Cyclic, Random

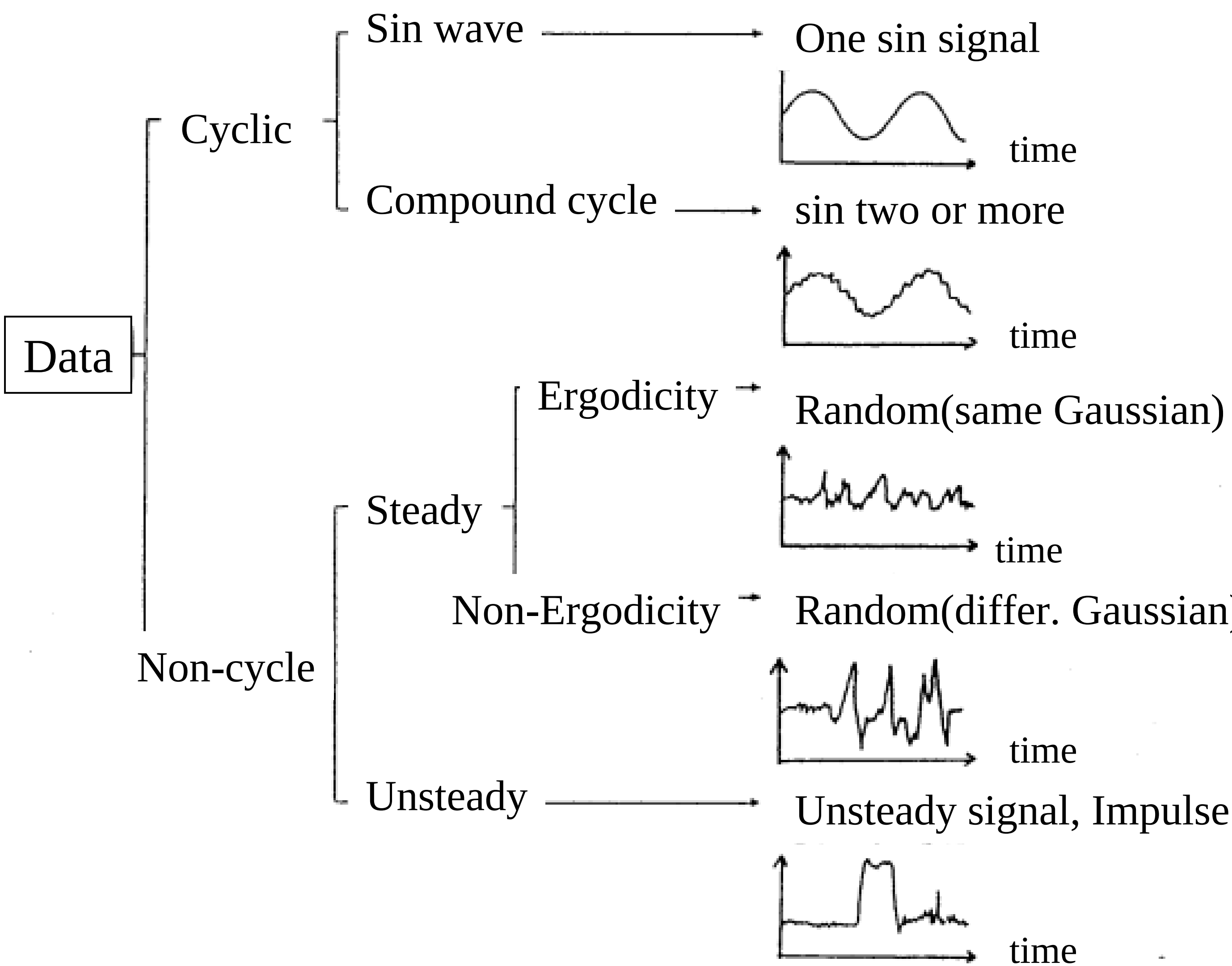
Non-steady signal:（Frequency components change with time

(1) Continuation-steady → Artificial sound which compounded one or more sin waves

(2) Continuation-unsteady → Noise of road, Music performance, etc

(3) Non-steady → Impulse response etc.

(2) Classification of the signal



Signal Analysis

Purpose: Feature extraction

- Sound analysis
- Equipment diagnosis
- State evaluation of a living body etc.

Steady signal

- Fourier Transform etc.

Unsteady signal

Time-Frequency Analysis

- Short Time Fourier transform
- Wigner distribution
- Wavelet transform etc.

Signal control

Purpose: Control of the signal in actual space

Noise problem

- Noise from equipment
- Inside of car, airplane

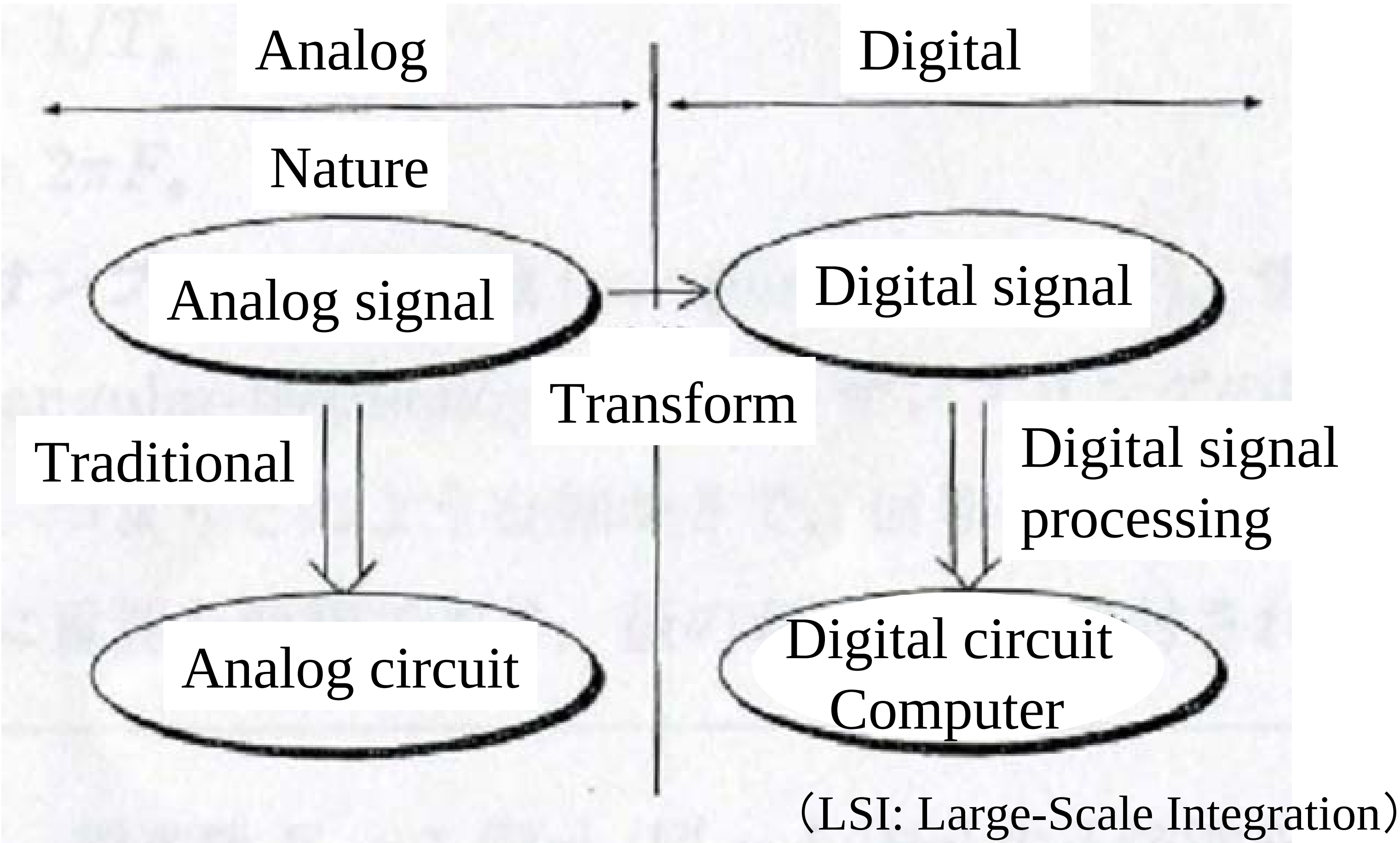
Noise reduction etc.

Noise control

- Active noise control etc.

1.2 Digital and Sampling Proposition

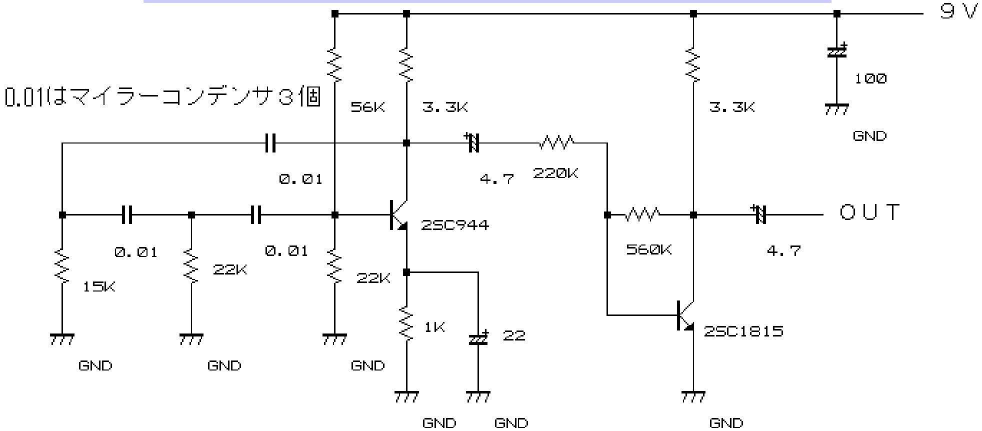
1.2.1 Analog and Digital Signal



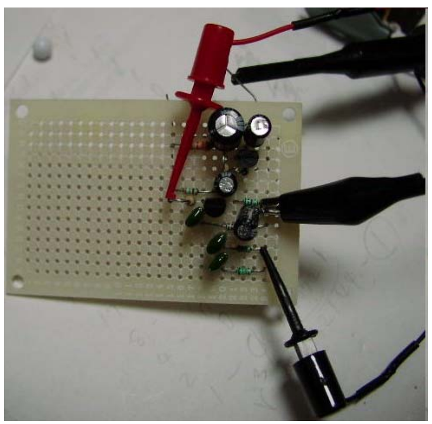
What’s Digital signal Processing

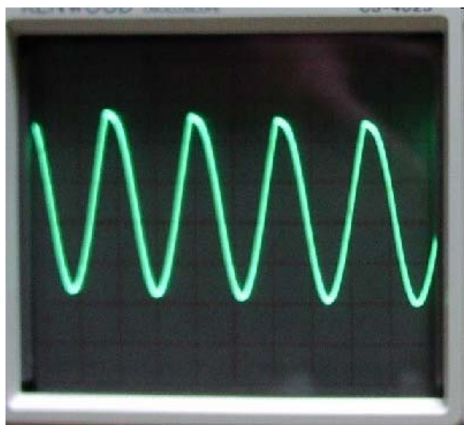
Example

Analog oscillation circuit



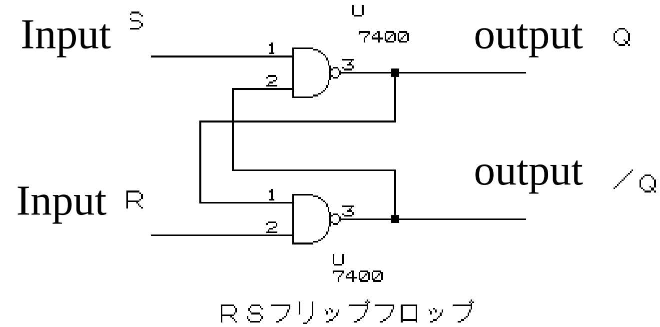
Oscillation at 443Hz

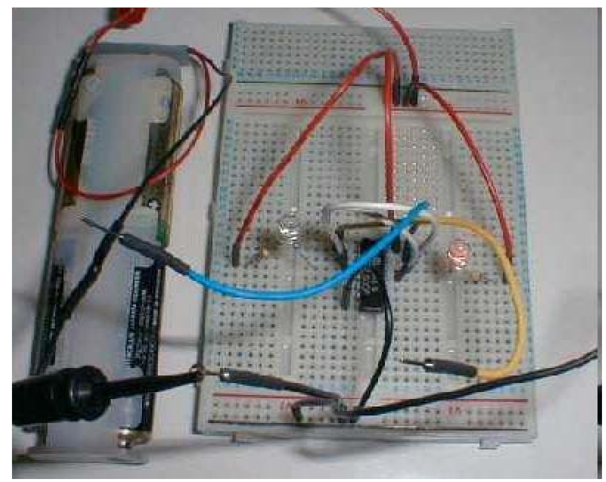




Oscillation circuit with the transistor and its output

Digital logic circuit





S	R	Q	／Q
0	0	1※	1※
0	1	1	0
1	0	0	1
1	1	No change	No change

Logic circuit and its output

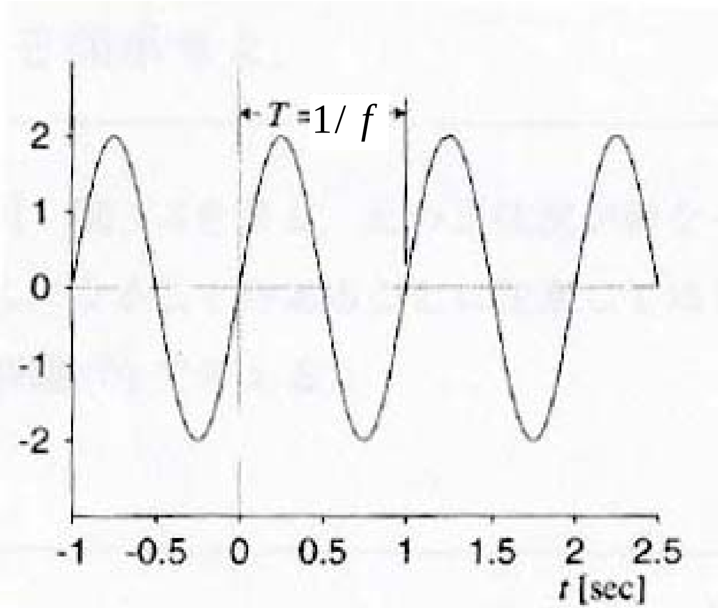
1.2.2 Signal Sampling

Sine signal:

Amplitude

$$x(t) = A\sin(\omega t + \theta)$$

Initial Phase



(a) $x(t) = 2\sin(2\pi t)$

$$\omega = 2\pi f [rad/\sec]$$

$$f = 1/T [Hz]$$

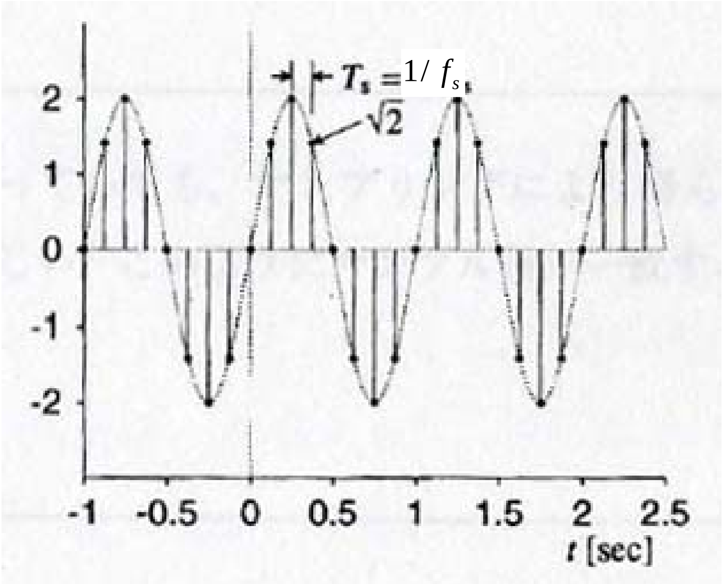
Period (cycle)

Frequency

Sampling

The sampling is a operation that extracts the value of the signal in discrete time.

Extracted value ⟶ Sampling value



Sampling T_s=1/8[sec]

Ts: Sampling period / Sampling interval

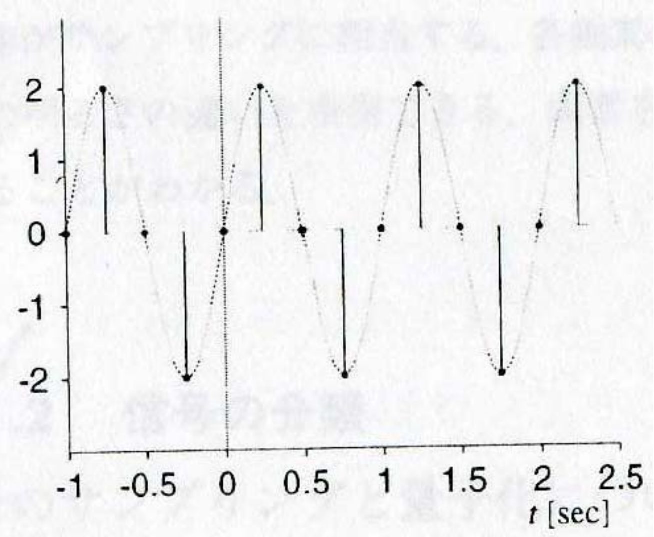
$f_s = 1/T_s$ Sampling frequency

$\omega_s = 2\pi f_s$ Sampling angle frequency

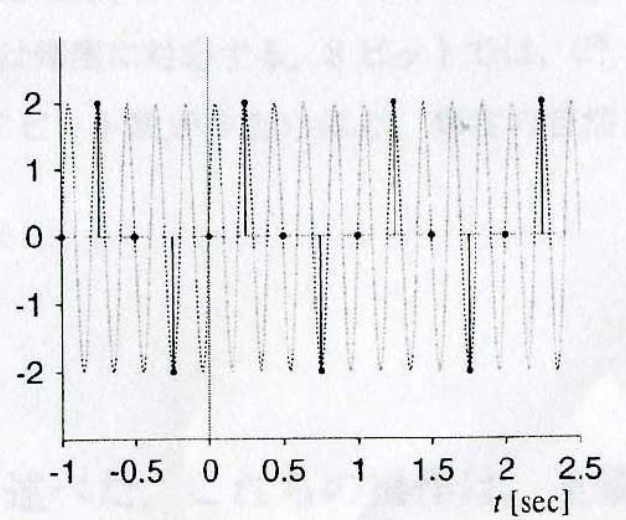
【Example 1】

Q: The signals that have frequency f=1Hz and f=5Hz are sampled by using the sampling frequency 4Hz. Please show the results obtained

- A: First sampling period $T_s = 1/f_s$ then $T_s = 1/4$ [sec]



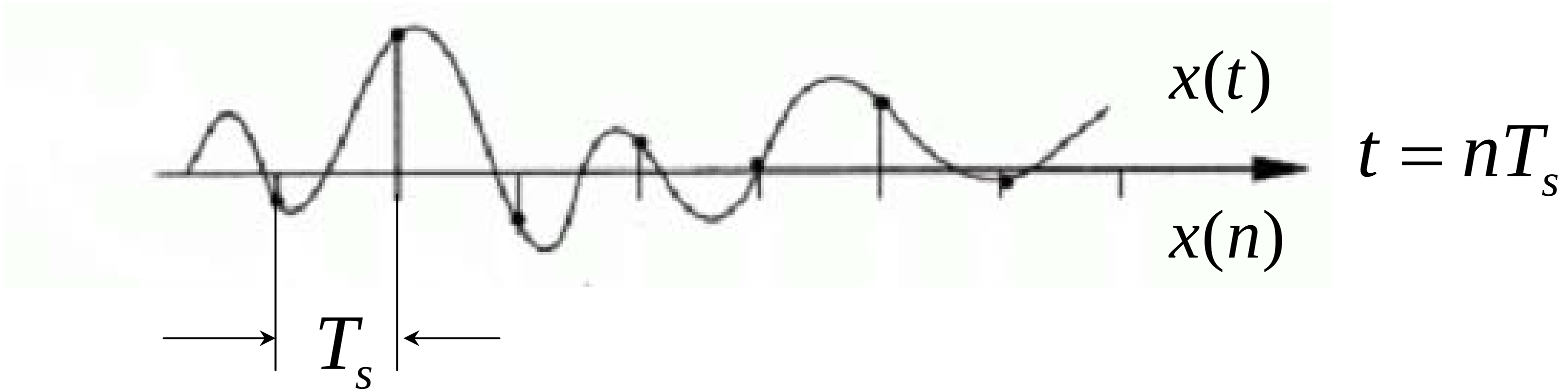
(a) $F = 1$[Hz], $F_s = 4$[Hz]



(b) $F' = 5$[Hz], $F_s = 4$[Hz]

Notices: Signals obtained are same even the original signals are different

Shannon' sampling theorem



Shannon' sampling theorem:

In the case of $f_s = 1/T_s$, the maximum frequency that can be analysis becomes:

$$f_N = f_s/2 = 1/2T_s$$

we call this frequency Nyquist frequency

Example: $f = 5Hz > f_N = f_s/2 = 2Hz$

It is impossible for catching information of f=5Hz, so the 5Hz was mistaken to the1Hz.

【Example 2】

Q:In the case of a compact disk (CD), audio signal is changed into digital signal using sampling frequency f_s=44.1[kHz]. Ask for a sampling period.

Answer:

$$f_s = 1/Ts$$

$$Ts = 1/f_s$$

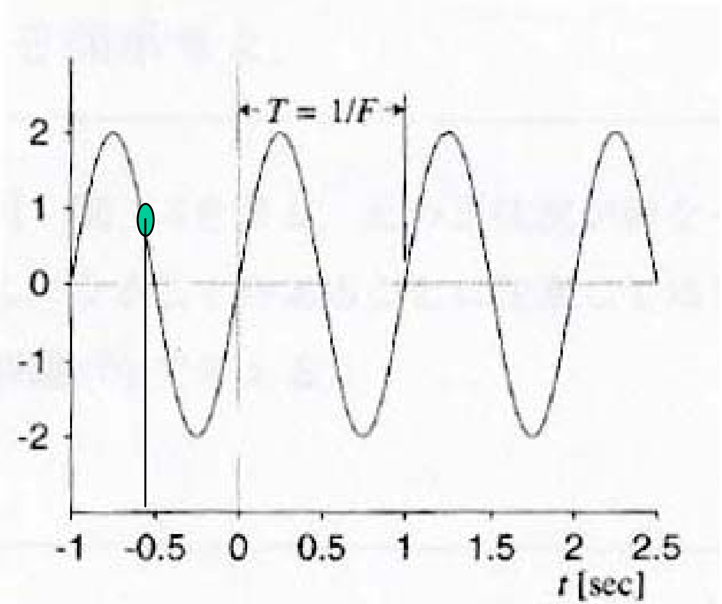
$$= 1/(44.1 \times 10^3)$$

$$= 22.7 \times 10^{-6} \quad \text{[sec]}$$

1.2.3 Signal quantification

Quantification is a operation for expressing each sample value with the binary number (2 bits or 8 bits) of the limited number of beams.

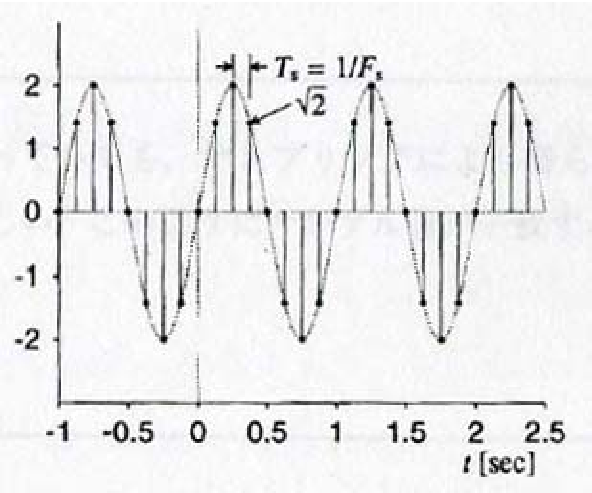
(The operation of quantization is needed for the next of sampling operation when an analog signal is changed into a digital signal)



(a) $x(t) = 2\sin(2\pi t)$

When the maximum and minimum values were decided as 2 and –2, a value of the signal for each time is infinity; it cannot be expressed with the binary number of the limited number of beams

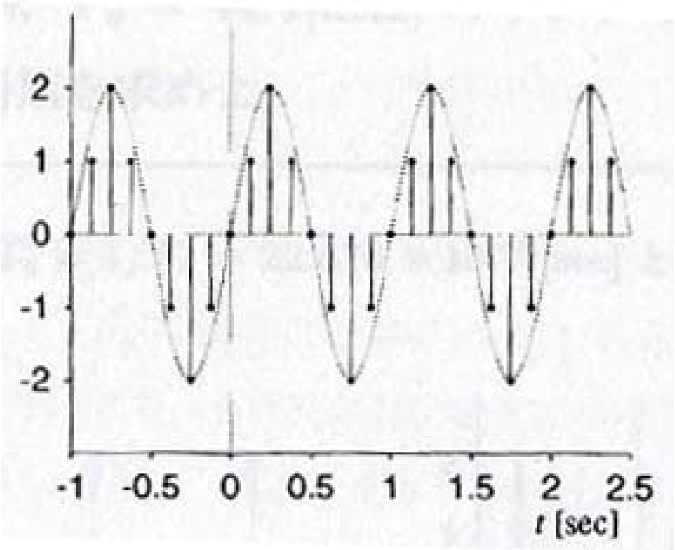
Structure of quantization



(a) Sampling (Ts=1/8[sec])

If each sample value is expressed using five values (2, -2, 0 and 1, -1), which can be expressed with five 3 bits binary numbers (010,110,000,001,101). However each sample value are not necessarily in agreement with the five values.

In order to express a sample value with five values, the value nearest to each sample value is chosen from five values, and a sample value is replaced with the value.



(b) Quantization

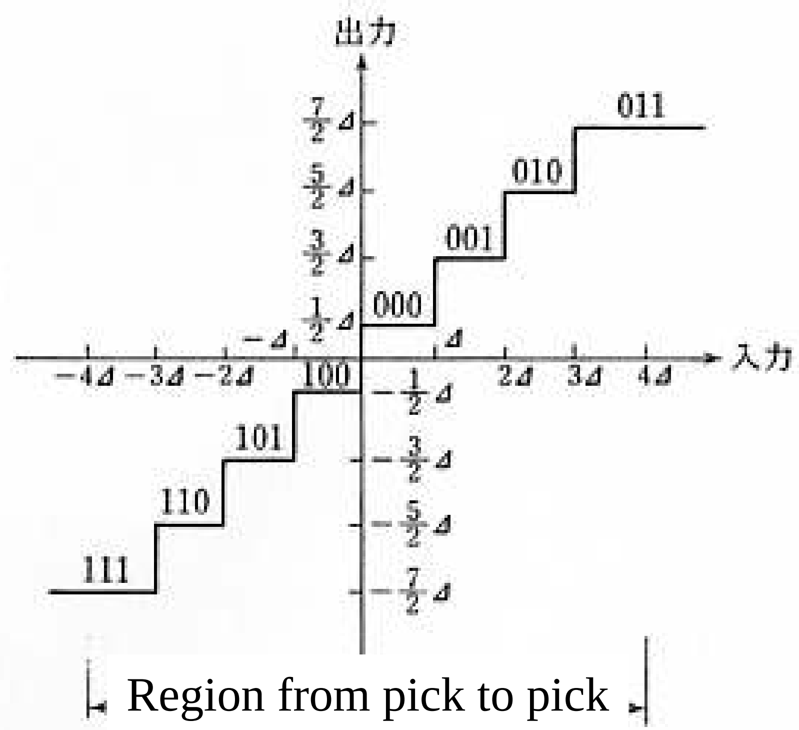
It is cut off by the 1st place below a decimal point.

・By the operation of quantization, a sample value will have a different value from the original value.

・The difference between value after quantization and original sample value is called quantization error.

Quantization step

The quantization characteristic is decided by number of levels, and quantization step △. When premised on coding of B (bit), the number of levels is common to make it 2^B piece.



Example: 8 levels（3bits）of the input-and-output characteristic of a quantization machine

Signal region: $|x_i| \le x_{\max}$

Level number: $2x_{\max} = \Delta 2^B$

Quantization error: $-\dfrac{\Delta}{2} \le e_i \le \dfrac{\Delta}{2}$

8 bit: Levels $2^8 = 256$
16 bit: Levels $2^{16} = 65536$

Calculation of quantization

Sampling value $x(n) = x(nT_s)$ (n:Integer)

$$s(n) = round\left[\frac{x(n)}{\Delta}\right]$$

Δ : Quantization step

▪ Round[y] is a operation which rounds off a value y by the 1st place below a decimal point .

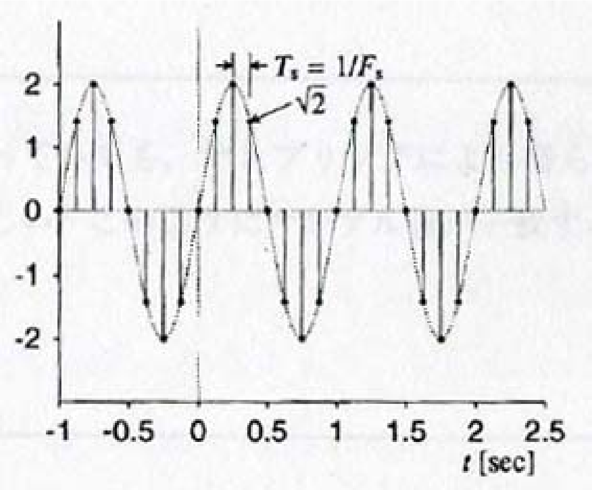
▪ s(n) is expressed with the binary number of the limited beam of the value of x(n)

▪Quantization error can be reduced when the s(n) showing a sample value increases as choosing a small value Δ

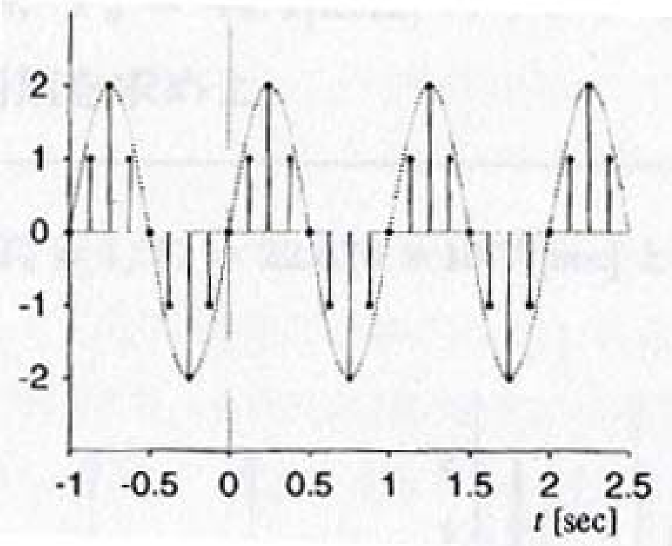
▪When Δ is chosen as a small value, then many numbers of bits are so required for expression of the sample

▪The original signal including quantization error will obtained when calculating s(n)XΔ

Example of quantification



(a) Sampling(Ts=1/8[sec])



(b) Quantification

Table: The quantization step Δ and digital signal

n	x(n)	Δ=1		Δ=0.5	
		s(n)	binary	s(n)	binary
0	0	0	000	0	0000
1	$\sqrt{2}$	1	001	3	0011
2	2	2	010	4	0100
3	$\sqrt{2}$	1	001	3	0011
4	0	0	000	0	0000
5	$-\sqrt{2}$	−1	101	−3	1011
6	−2	−2	110	−4	1100
7	$-\sqrt{2}$	−1	101	−3	1011

Example of quantization of an Image signal

8 bits（Level $2^8 = 256$）

4 bits（$2^4 = 16$）

2 bits（$2^2=4$）

1 bit（$2^1=2$）

1.2.4 Signal's expressing method

1. Digital time signal expressing

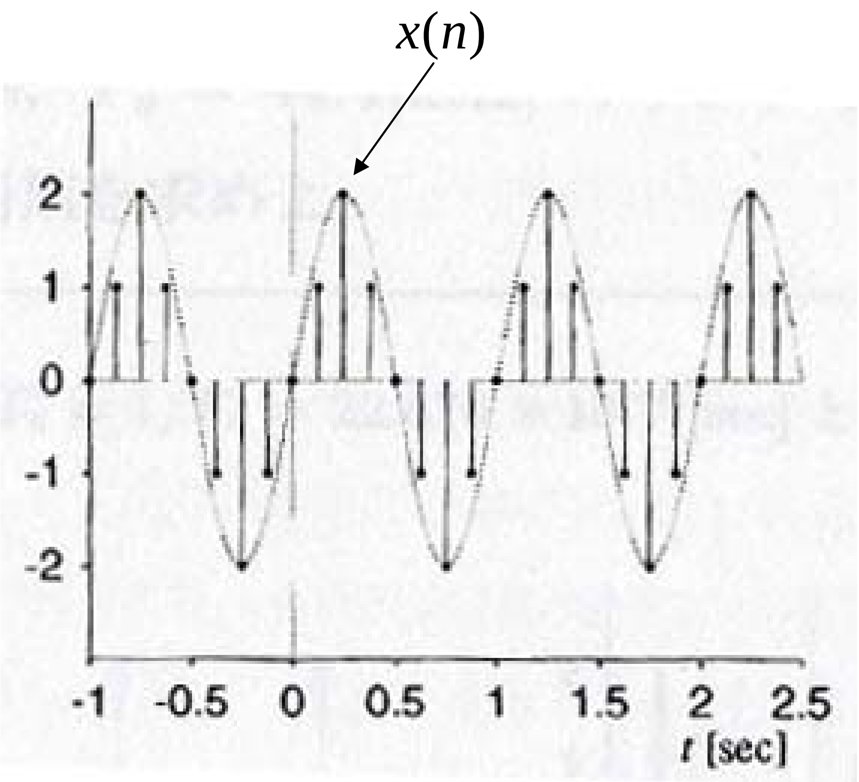
Analog signal：

$$\boxed{x(t) = A\sin(\omega t)}$$

If this signal is sampled with the sampling period T_s, then the discrete time nT_s is substituted for the time t

Discrete time signal can show as follows:

$$\boxed{x(n) = A\sin(\omega n T_s)}$$



2.Normalization expression

Simple expressing:

$$x(n) = A\sin(\Omega n)$$

Normalization angle frequency was shown as: Ω

Normalization angle frequency :

$$\Omega = \omega T_s = \omega / f_s [rad]$$

Normalization frequency :

$$F = \Omega / (2\pi) = f / f_s [-]$$

Example: express the Nyquist frequency on normalization frequency and normalization angle frequency.

Normalization frequency :

$$F_N = f_N / f_s = 1/2 = 0.5$$

Normalization angle frequency :

$$\Omega_s = \omega_s T_s = 2\pi f_N / f_s = \pi$$

Digitization's advantage

Digital signal processing ⟷ Analog signal processing

Improvement in economical and reliability

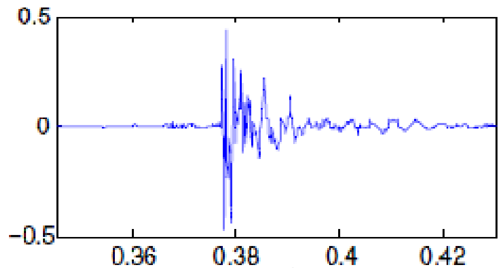
- LSI technology: Low-cost of the product in the mass production
- LSI technology: Product miniaturization; Up reliance
- Combined software: Change of specification and shortening of development period are possible

（LSI: Large-Scale Integration）

Diversification of signal processing

- By using computer, complicated and high flexibility processing are possible.
- Data compression , security etc. are possible.
- Parallel processing, nonlinear processing, etc. are possible.

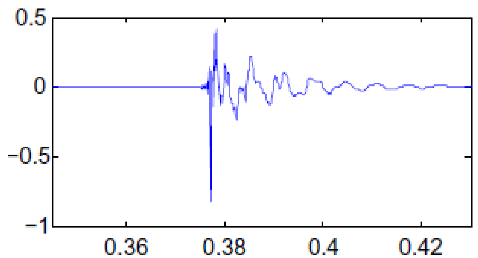
1.3 Example of Abnormal extraction



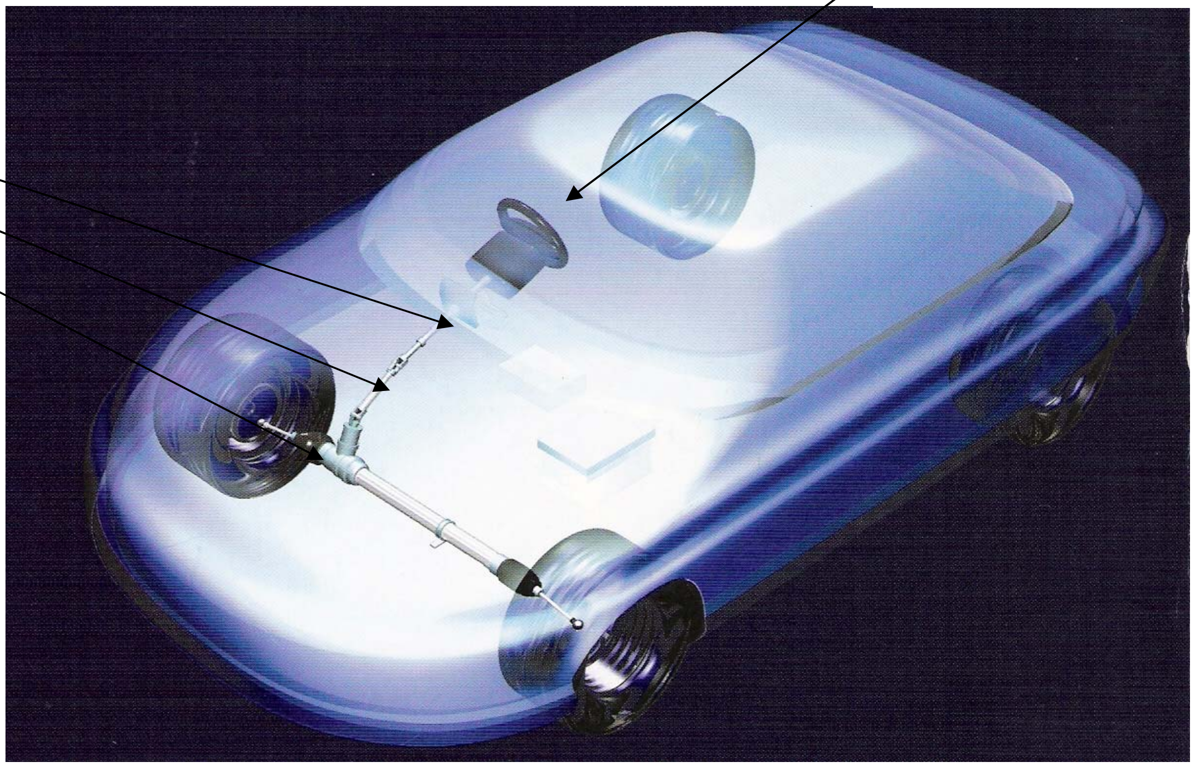
Example of sound

Micropho

Acceleration sensor



Example of vibration



Continuance wavelet transform

$$w(a,b)=\int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

$$\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother wavelet(MW)

a : Scale ($1/a$ Frequency)

b : Time

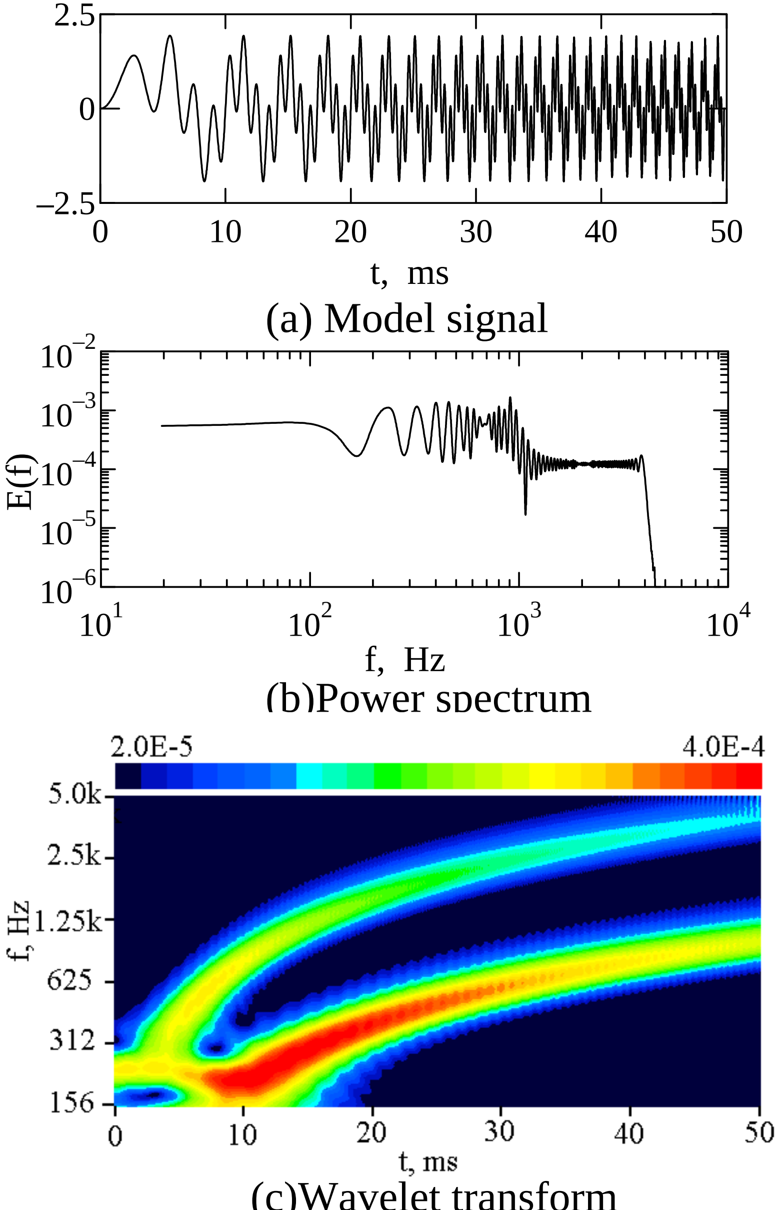
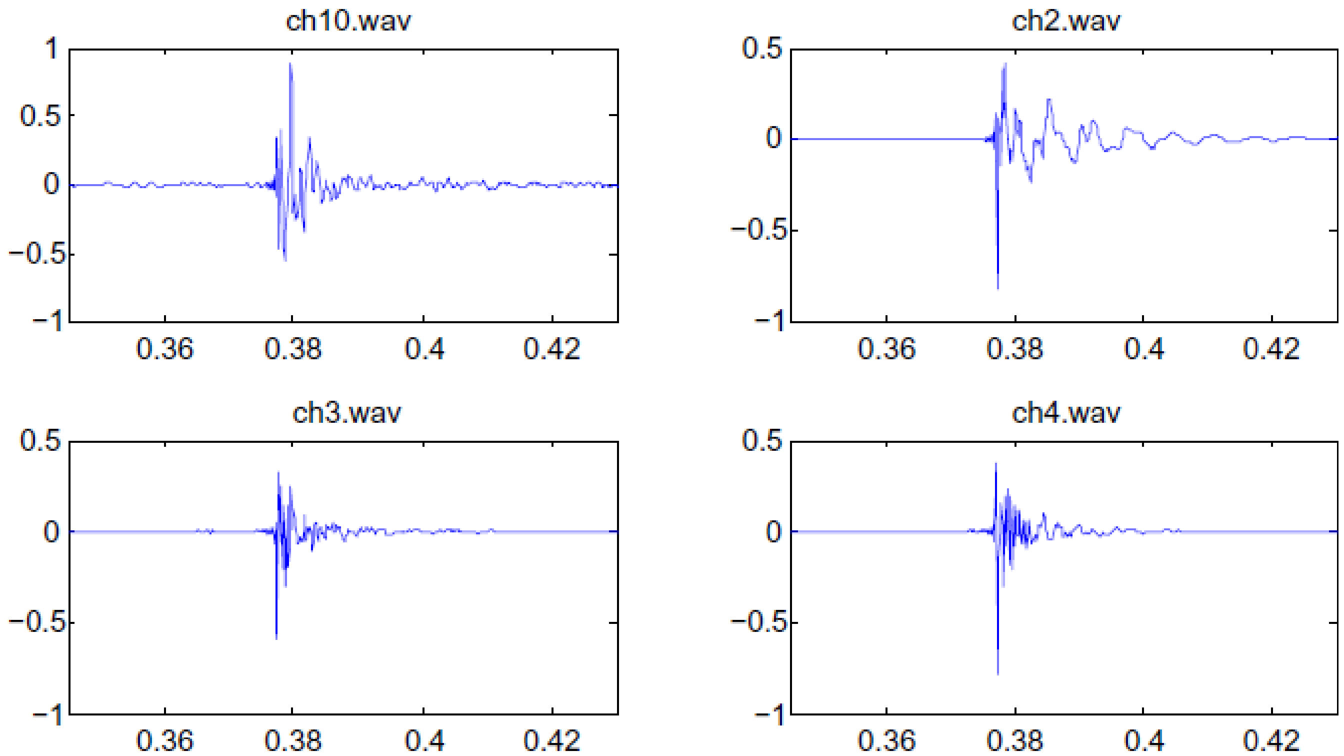
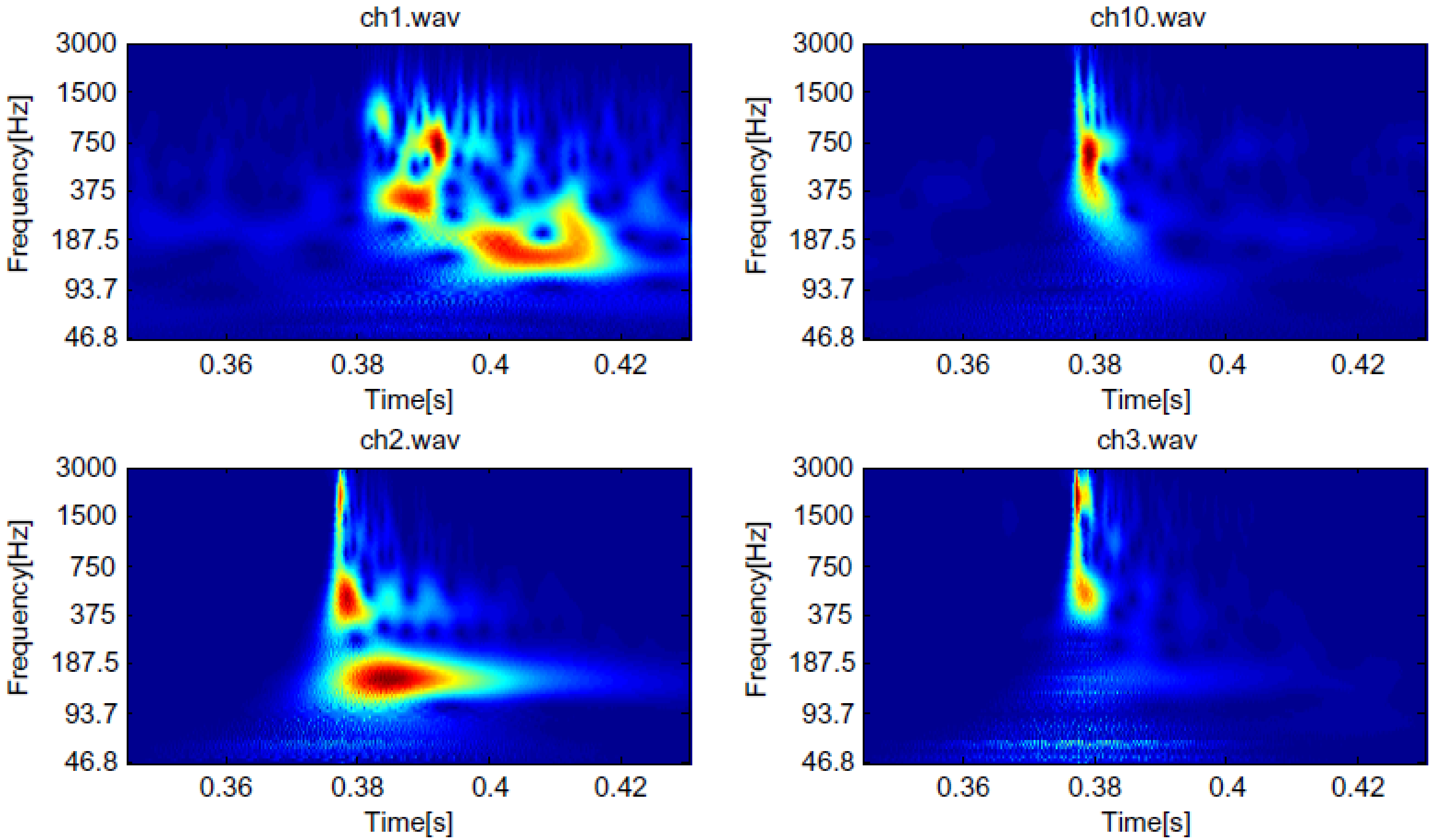


Fig.1 Wavelet transform of model signal

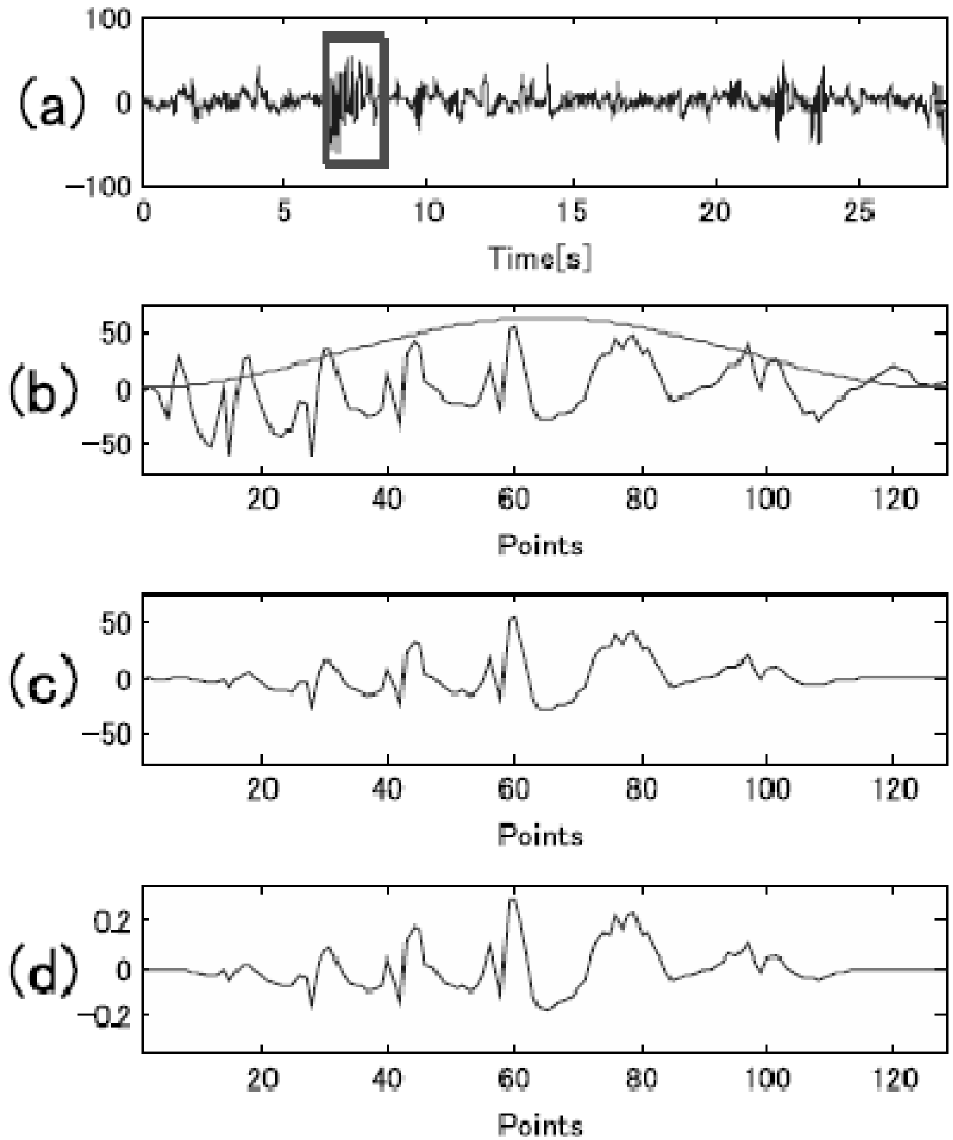
Example of vibration measured



Example of wavelet transform



Choosing Correlation Components



Create RMW using real signal

1) Selecting parts of signal Fig.(a)
2) Multiplying the real signal with a window function and removing average Fig.(b) and (c)

Then, we got the RMW shown In the Figure (d)

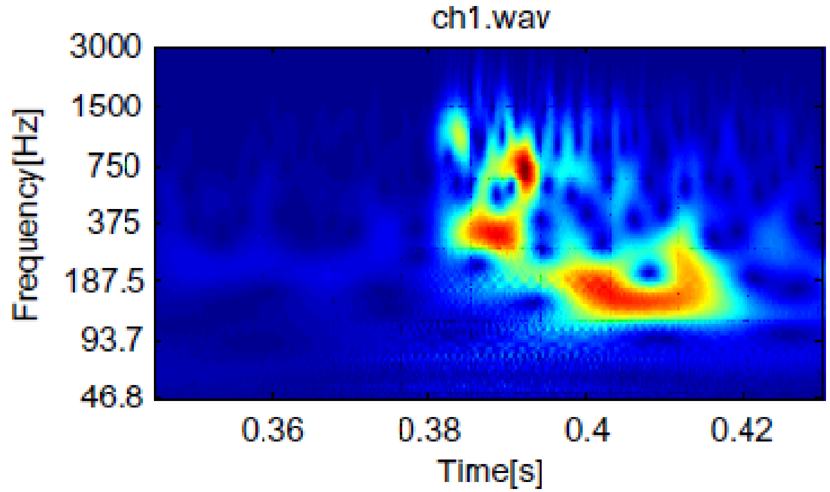
Result

Signal RMW

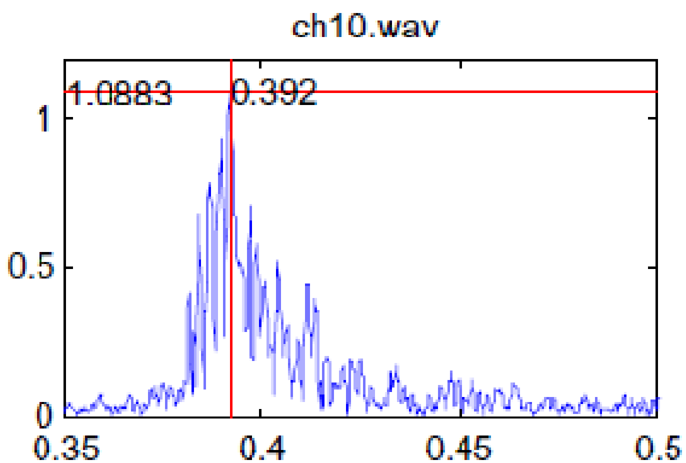
$$w(a,b) = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

Correlation of wavelet：

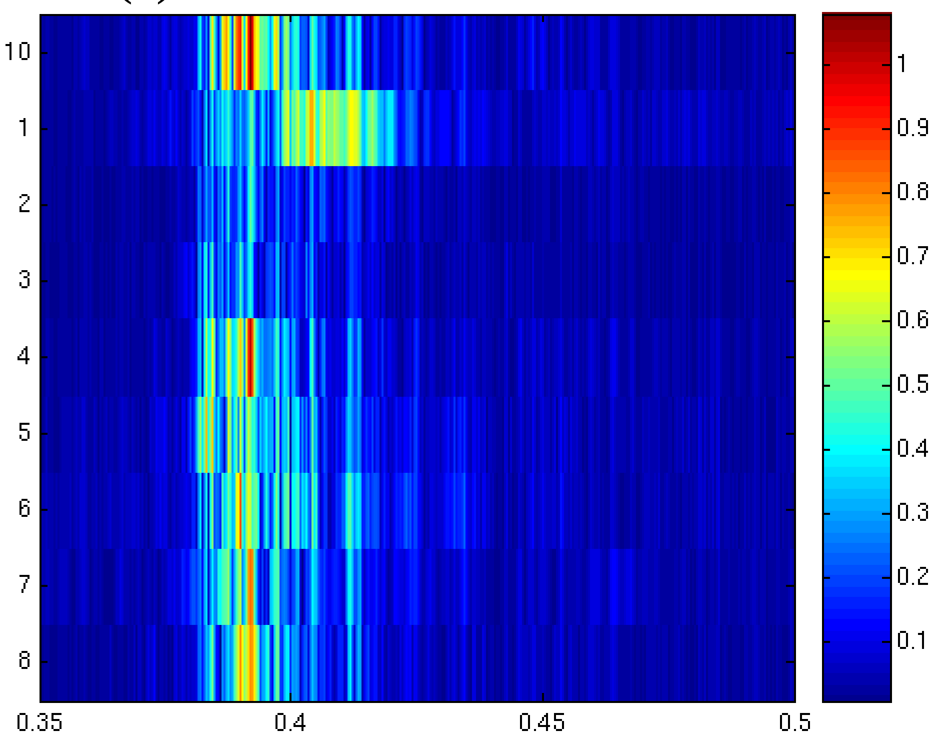
$$C(b) = |W(1,b)|$$



(a) Wavelet transform of the sound



Example of C(b)



(b) Distribution of C(b)