

7. Information theory and Signal Processing Method

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6. Real Signal Wavelet Transform

6.1 Construction of Real-Signal MW

- 1) Selection of window function
- 3) Construction of a complex RMW

6.2 Detection of abnormal signal

- 1) Definition of Wavelet Instantaneous Correlation
- 2) Knocking extraction from block vibration

6.3 Construction of RMW using more than two signal

- 1) Flow chart of SCRMW construction
- 2) Construction Method of A-RMW
- 3) Rattle Noise Source Detection

6.4 New First WIC by using DWT

- 1) Parasitic-Discrete Wavelet Transform (P-DWT)
- 2) Design method of the parasitic filter
- 4) Example of Detection result in the MGR (z direction)

Report title:

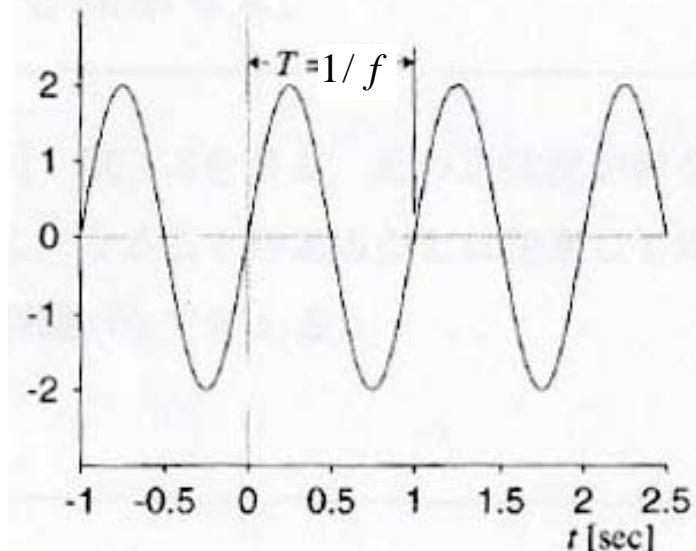
Wavelet transform from my viewpoint

Deadline: February 28, PM5:00, Office

7.1 Sampling Seen from Information

1) Signal Sampling

Sin signal :



$$x(t) = \underline{A} \sin(\underline{\omega}t + \underline{\theta})$$

Amplitude

Initial Phase

$$\omega = 2\pi f [\text{rad/sec}]$$
$$f = 1/T [\text{Hz}]$$

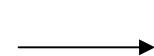
Period (cycle)

Frequency

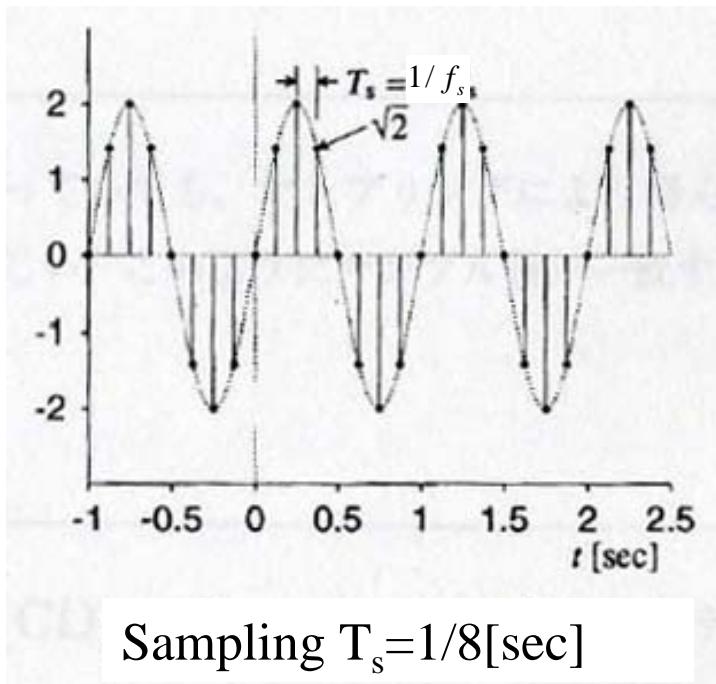
Sampling

The sampling is operation which extracts the value of the signal in discrete time.

Extracted value



Sampling value



Sampling period
 T_s : Sampling interval

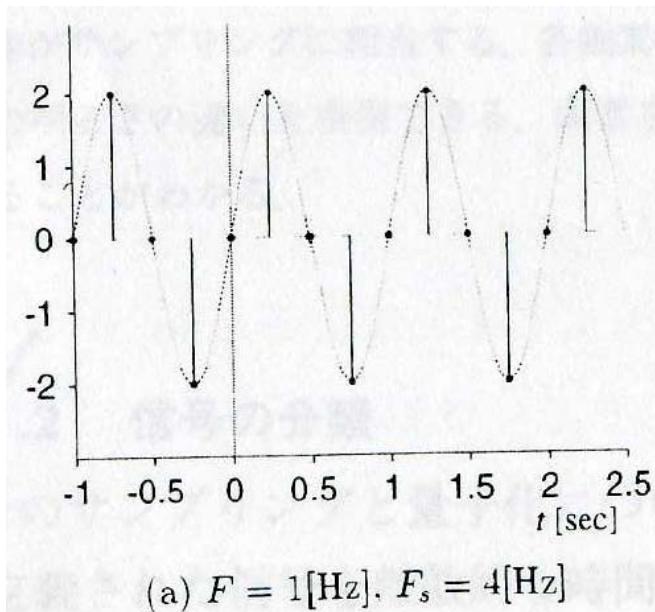
$$f_s = 1/T_s \quad \text{Sampling frequency}$$

$$\omega_s = 2\pi f_s \quad \text{Sampling angle frequency}$$

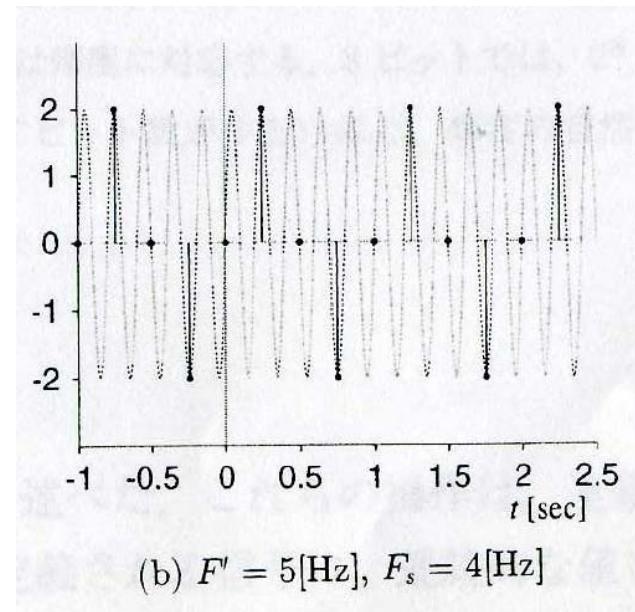
【Example 1】

The signals that have frequency $f=1\text{Hz}$ and $f=5\text{Hz}$ are sampled by using the sampling frequency 4Hz . Please show the results obtained

- Answer: First sampling period $T_s = 1/f_s$ then $T_s = 1/4$ [sec]



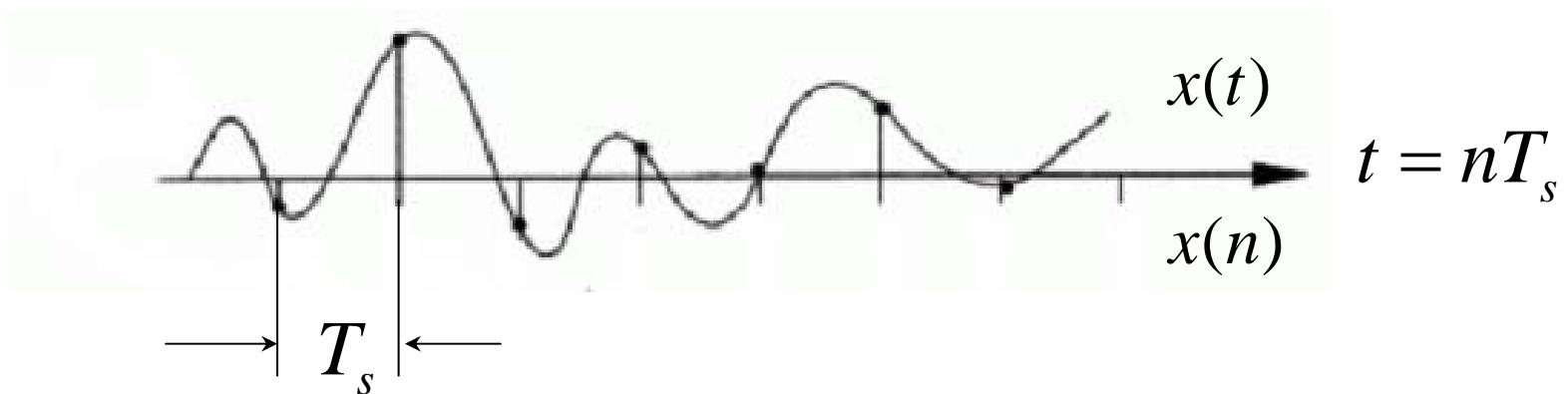
(a) $F = 1[\text{Hz}]$, $F_s = 4[\text{Hz}]$



(b) $F' = 5[\text{Hz}]$, $F_s = 4[\text{Hz}]$

Notices : Signals obtained are same even the original signals are different

2)Shannon' sampling theorem



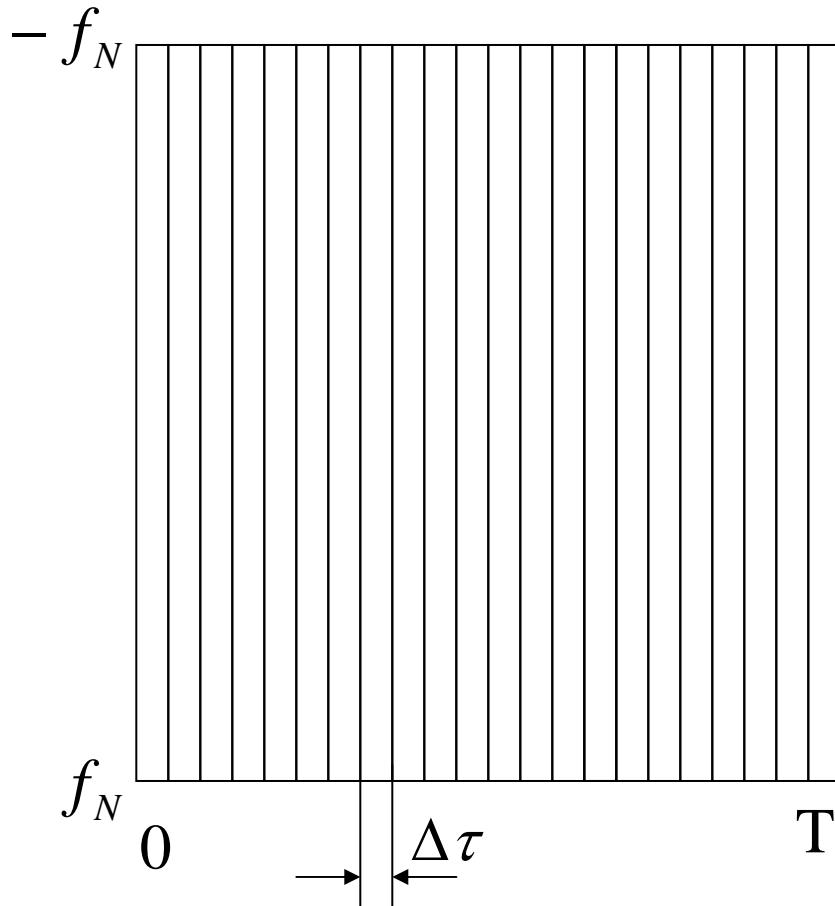
Shannon' sampling theorem:

In the case of $f_s = 1/T_s$, the maximum frequency that can be analysis becomes $f_N = f_s / 2 = 1/2T_s$, we call this frequency Nyquist frequency

Example: $f = 5\text{Hz} > f_N = f_s / 2 = 2\text{Hz}$

It is impossible for catching information of $f=5\text{Hz}$, so the 5Hz was mistaken to the 1Hz.

3) Sampling Seen From Information



Indefinite principle :

The amount of information is regularity and is not changed by the analysis technique.

$$\Delta\tau 2f_N = \text{Const}$$

A rectangular area expresses the amount of information

7.2 Filter Seen from Information

7.2.1 Digital Filter Type

The frequency properties $H(\omega)$ of the digital filter expresses polar coordinates

$$H(\omega) = A(\omega)e^{i\theta(\omega)}$$

where amplitude is $A(\omega)$ and phase is $\theta(\omega)$.

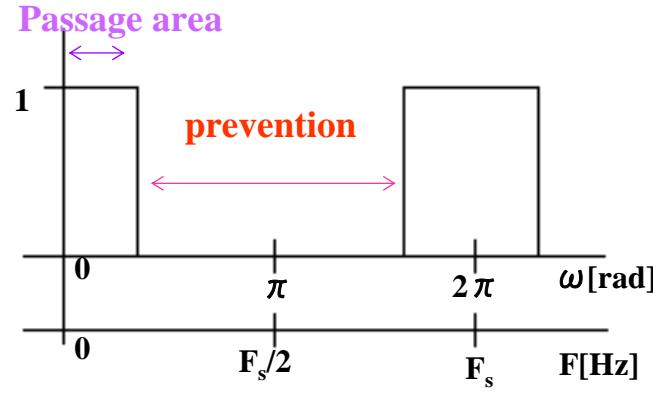
1) Classification by amplitude properties

The filters are classified as follows by the difference in amplitude properties

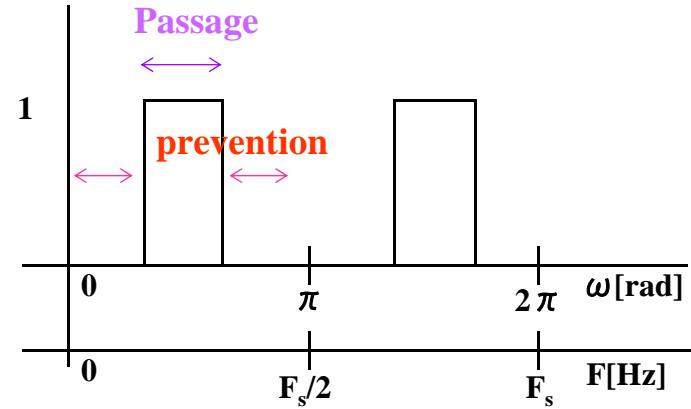
- Low pass filter
- High pass filter
- Band pass filter
- band reject filter

Filter processing (ideal)

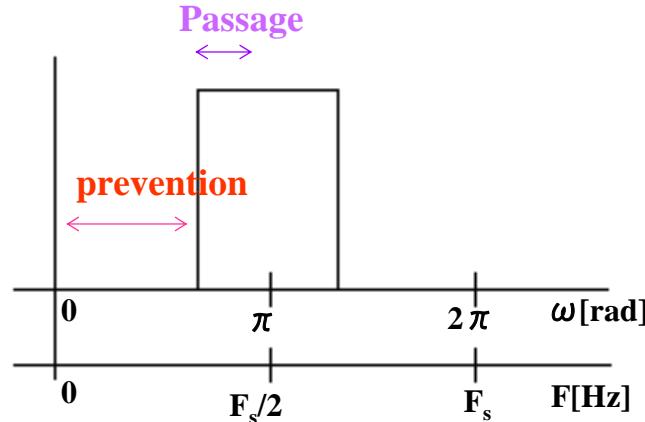
Low pass filter,LPF



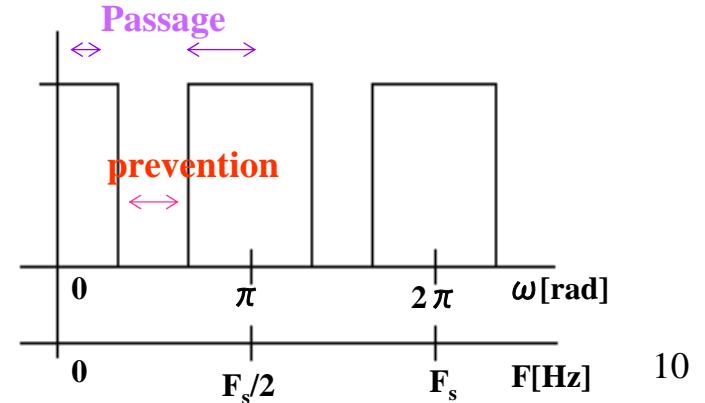
Band pass filter,BPF



High pass filter,HPF



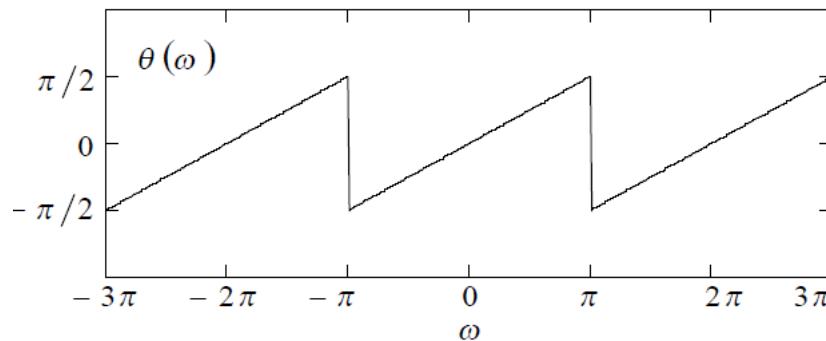
Band reject filter,BRF



2) Classification by phase properties

linear phase is that a phase characteristic changes in proportion to frequency linearly in passage area.

For the linear phase filter, the signals in the pass band have same quantity of group delay, and is not distortion after passed filter.



The quantity of group delay is the degree of leaning of the phase characteristic, which is differentiated phase at each frequency and is expressed as follows:

$$n_d = -\frac{d\theta(\omega)}{d\omega}$$

3) IIR and FIR filters

Example of digital filter: $y(n) = \sum_{k=0}^N h(k)x(n-k)$

IIR: Infinite impulse response filter

- Realizing a digital filter as IIR system

FIR: Finite impulse response filter

- Realizing a digital filter as IIR system

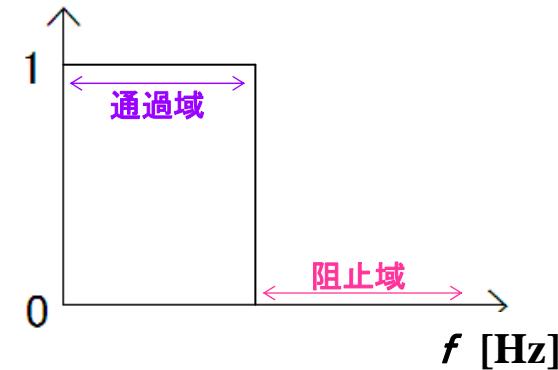
	IIR filter	FIR filter
Stability	Attention is necessary	<u>Always stable</u>
Linear phase	Realization is difficult	<u>Completely feasible</u>
Degree of the transmission function	<u>Low</u>	High

There are both good points and bad points in each filter, and it is necessary to use it properly as needed

4) Ideal filter and real filter

Ideal filter

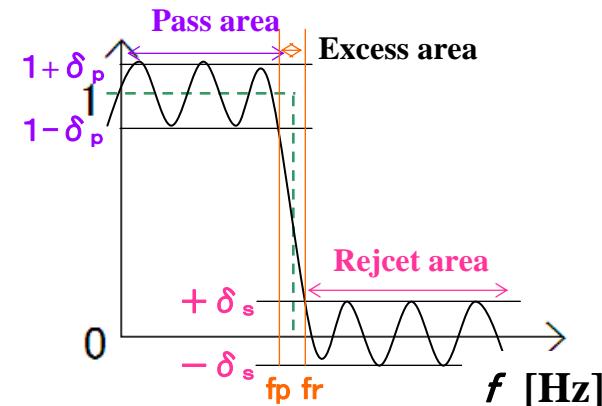
- Amplitude of passage area is constant
- Amplitude of reject area is zero
- Discontinuous from passage to reject area
- linear phase properties



Real filter

- Amplitude of the passage area fluctuates,
Passage area error : δ_p
- Amplitude of the reject area fluctuates ,
Reject area error : δ_s
- There is a excess area from passage area
to reject area

(f_p : Passage area edge frequency
 f_r : Reject area edge frequency



7.2.2 Filter Design

1) Step of filter design

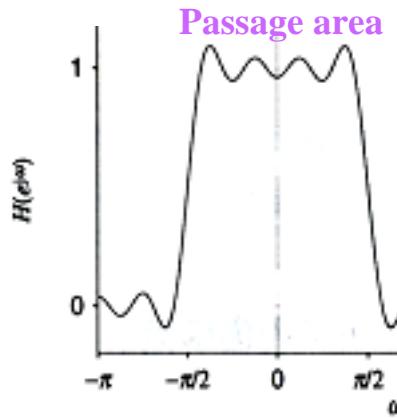
- (1) Approximation of the filter : designing the transmission function with a desired characteristic
- (2) Circuit implementation : constructing a circuit by a condenser, a coil, op-amp and so on

For the filter design, LPF becomes basic and other filters can be calculated from LPF.

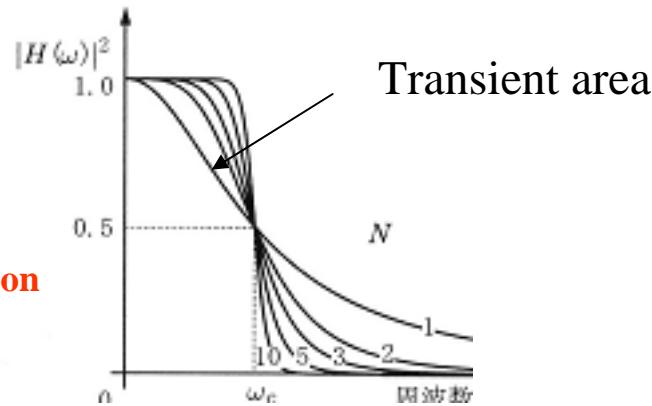
$$\text{LPF: } y(n) = \sum_k h(k)x(n-k)$$

$$\text{HPF: } f(n) = x(n) - \sum_k h(k)x(n-k)$$

2) Typical FIR Filter



(a) Moving average filter



(b) Butterworth filter

(a) Non-regression filter (moving average filter)

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$$

(b) Regression filter (Butterworth filter)

$$y(n) = \sum_{k=0}^N h(k)x(n-k) + \sum_{k=1}^N g(k)y(n-k)$$

3) Realization of the linear phase filter

Non-regression filter :

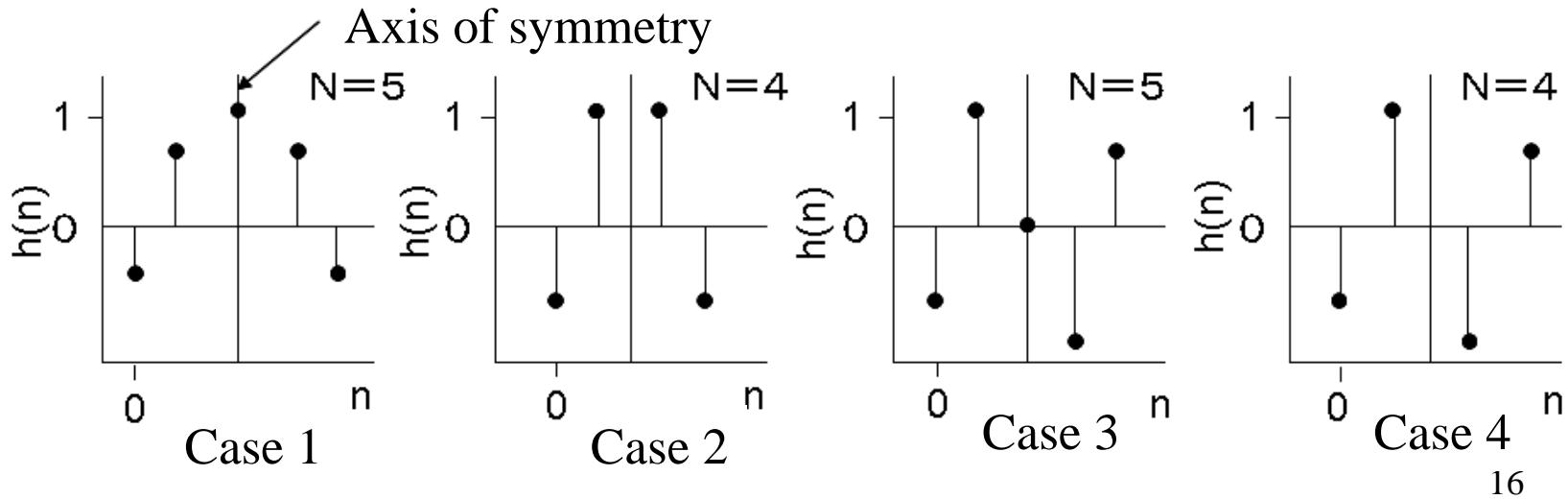
$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$$

$h(n)$: impulse response

N : filter's coefficients number

h(n) : Symmetricalness

- Necessary and sufficient condition for an FIR to have linear phase





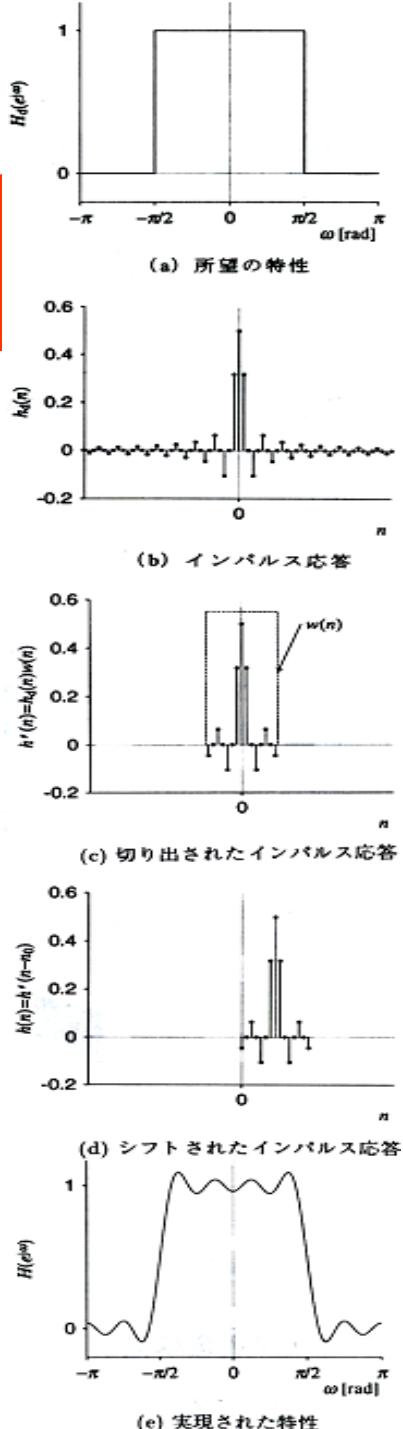
Design method in the frequency domain:

Purpose: designing the FIR filter with linear phase

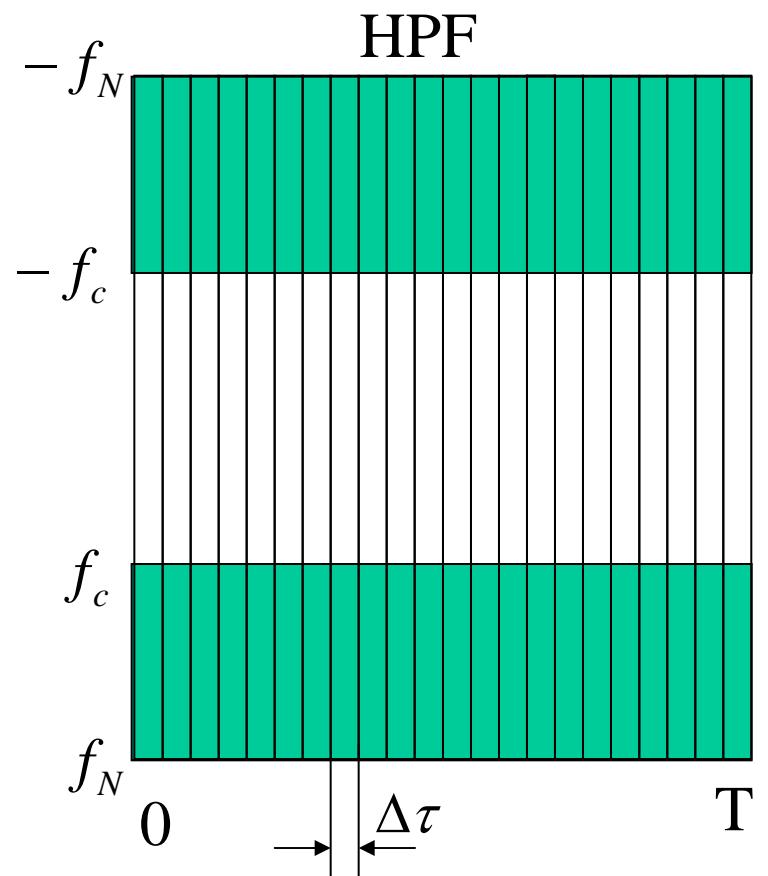
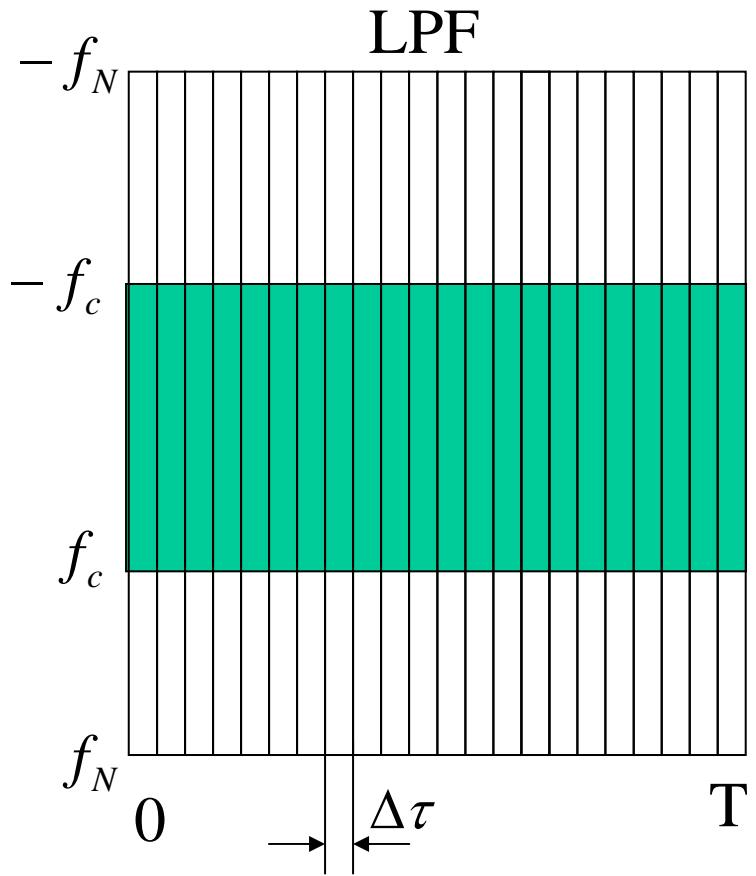
$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$$

Procedure:

1. Deciding desired amplitude properties.
2. Calculating $h(n)$ by using IFFT.
3. Taking window function $w(n)$ of the N point and cutoff limited area.
4. Shifting $h(n)$ to satisfy causal relation.
5. Confirming amplitude properties $h(n)$ by FFT



7.2.3 Filter Seen from Information



$$\Delta\tau 2f_N = \text{Const}$$

7.3 Fourier Transform Seen from Information

1) Fourier Transform

$$C(n) = \int_{-L}^L x(t) e^{-i2\pi \frac{n}{2L} t} dt$$

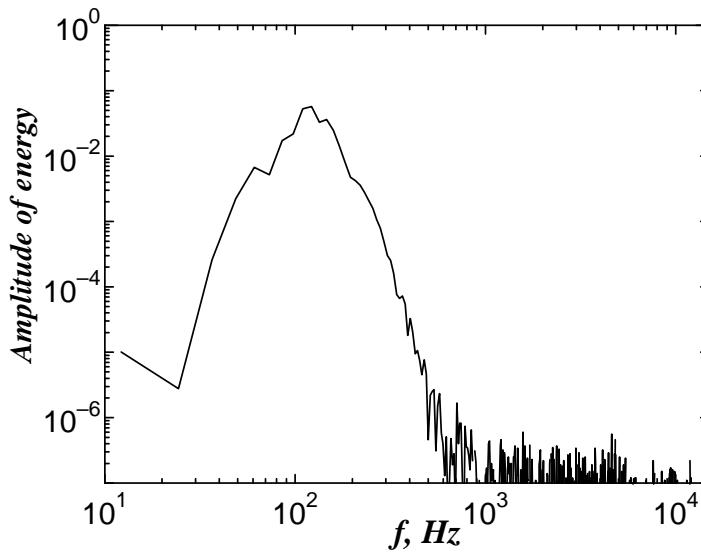
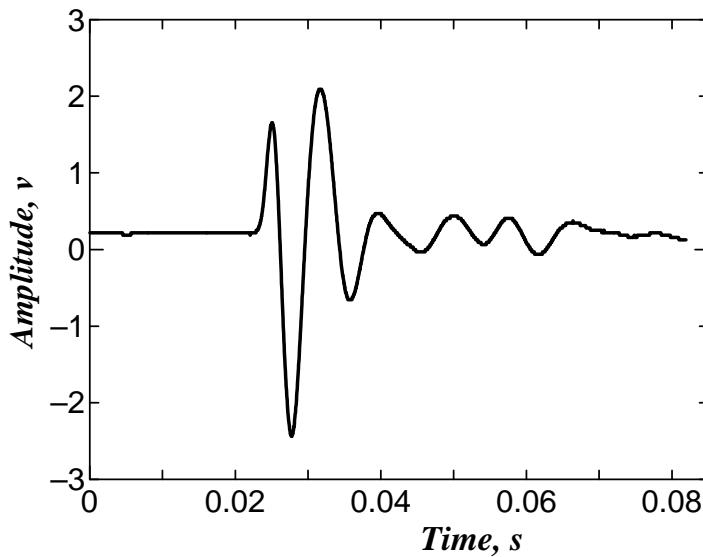
$$x(t) = \frac{1}{2L} \sum_{n=-N}^N C(n) e^{i2\pi \frac{n}{2L} t}$$

$$\Delta f = \frac{1}{2L}, \quad f_N = N \Delta f$$

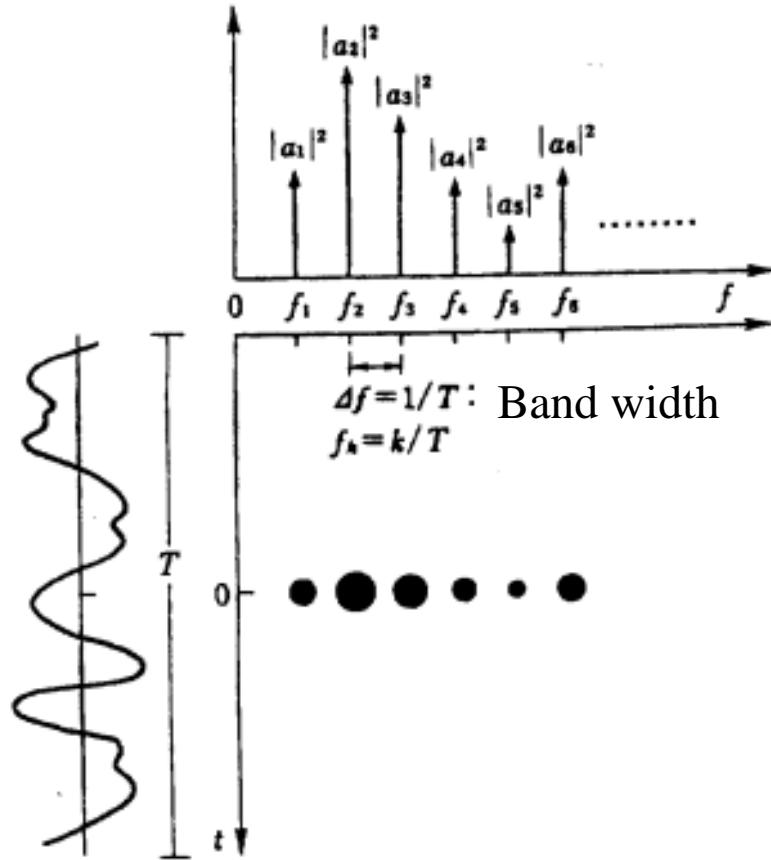
Power

$$E(f) = \frac{1}{2L} |C_n|^2$$

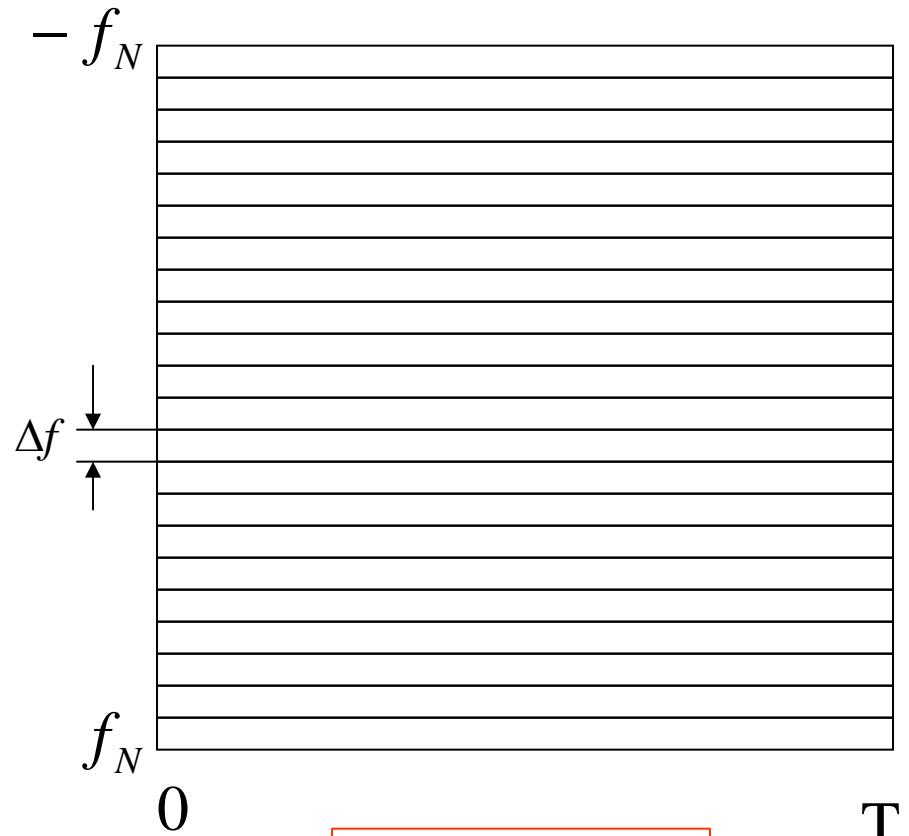
$$f = n \Delta f$$



2) Fourier Transform Seen from Information



Signal analysis by Fourier transform



$$\Delta f T = \text{const}$$

Information is regularity₀

7.4 STFT Seen from Information

1) Sort Time Fourier Transform (STFT)

Transform:

$$G_w(b, f) = \int_{-\infty}^{\infty} x(t) \overline{w(b-t)} e^{-i2\pi ft} dt$$

Window function: $w(t)$

Inverse Transform:

$$x(t) = \frac{1}{\|w\|} \int_{-\infty}^{\infty} G_w(b, f) w(b-t) e^{i2\pi fb} db df$$

$$\|w\| = \left[\int_{-\infty}^{\infty} w(t) \overline{w(t)} dt \right]^{1/2}$$

$\overline{w(t)}$: Complex conjugate of $w(t)$

$$w(t)(\cos(2\pi ft) + i \sin(2\pi ft))$$

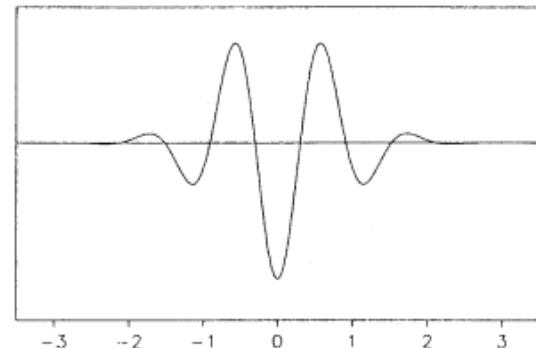
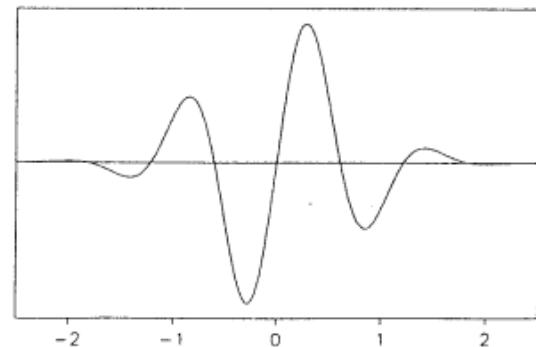
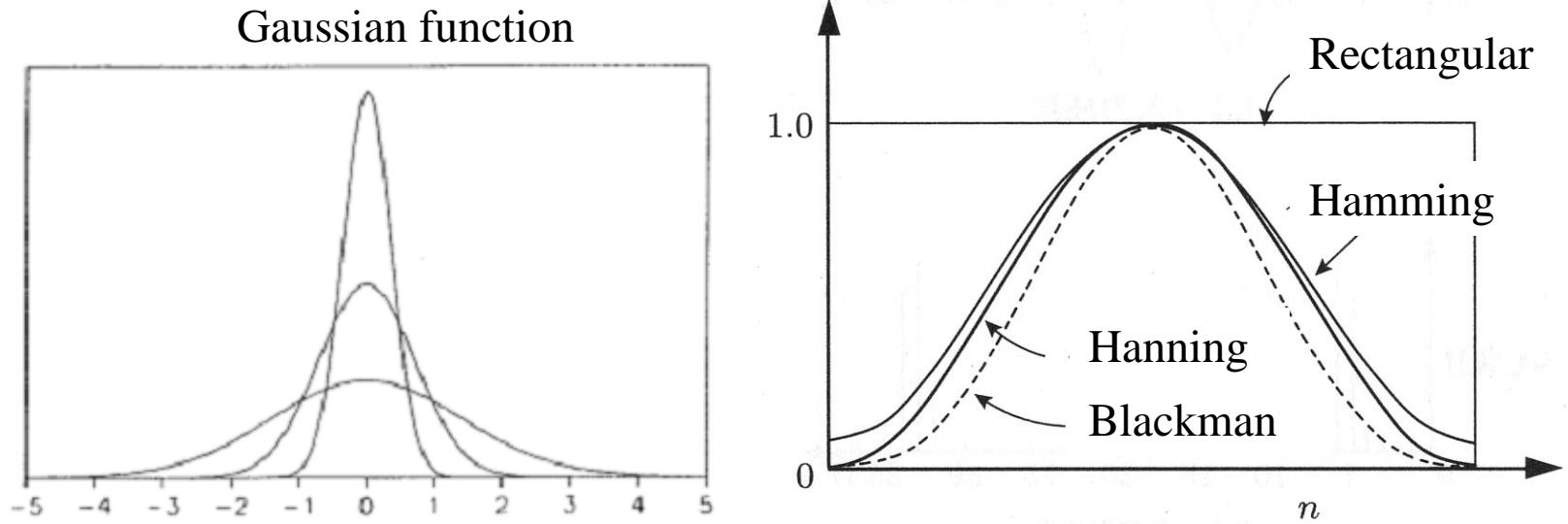


图 3.1.1 $\text{Re } G_{0,2\pi}^a$, $a=0.2925$

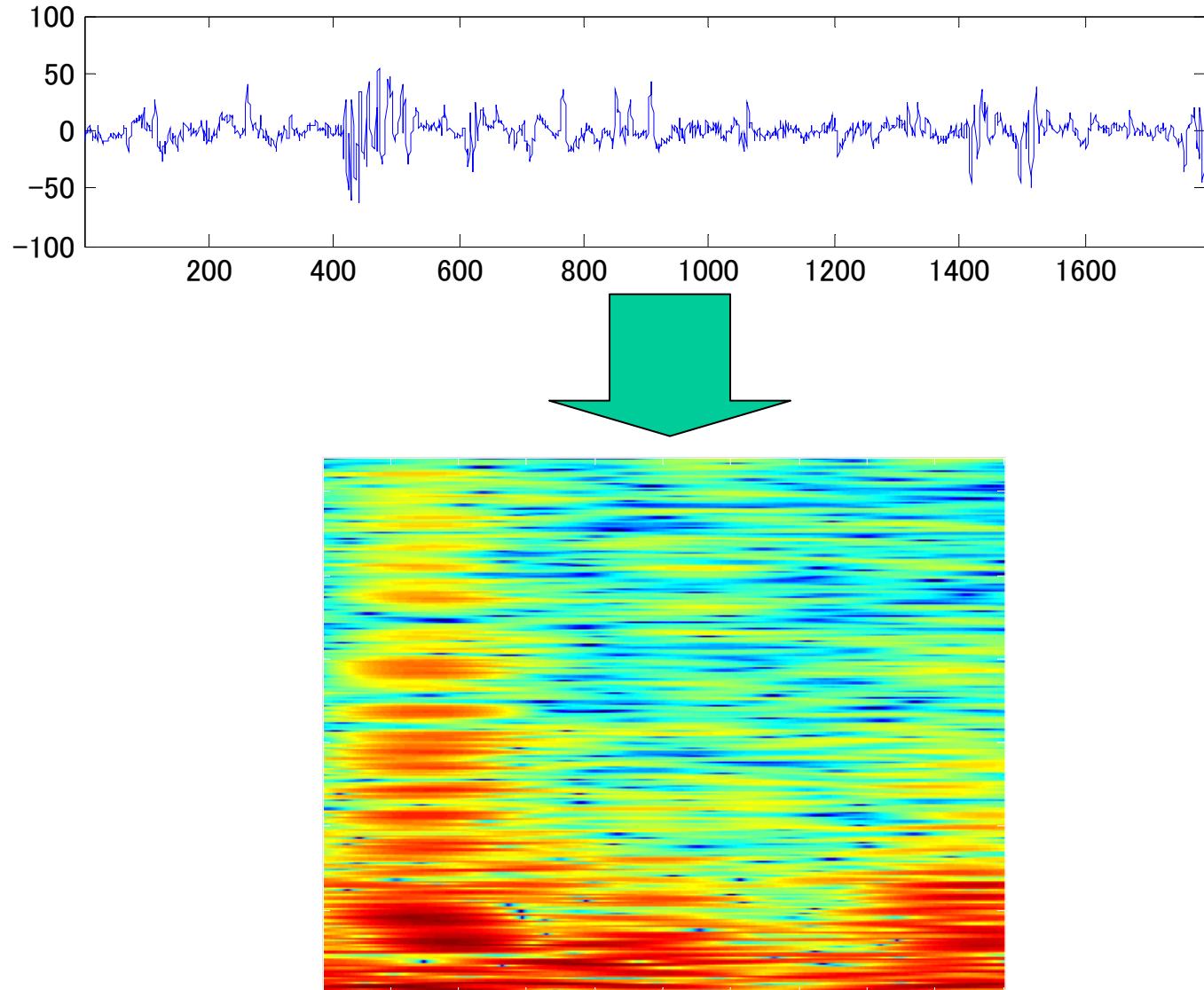


2) Example of window function



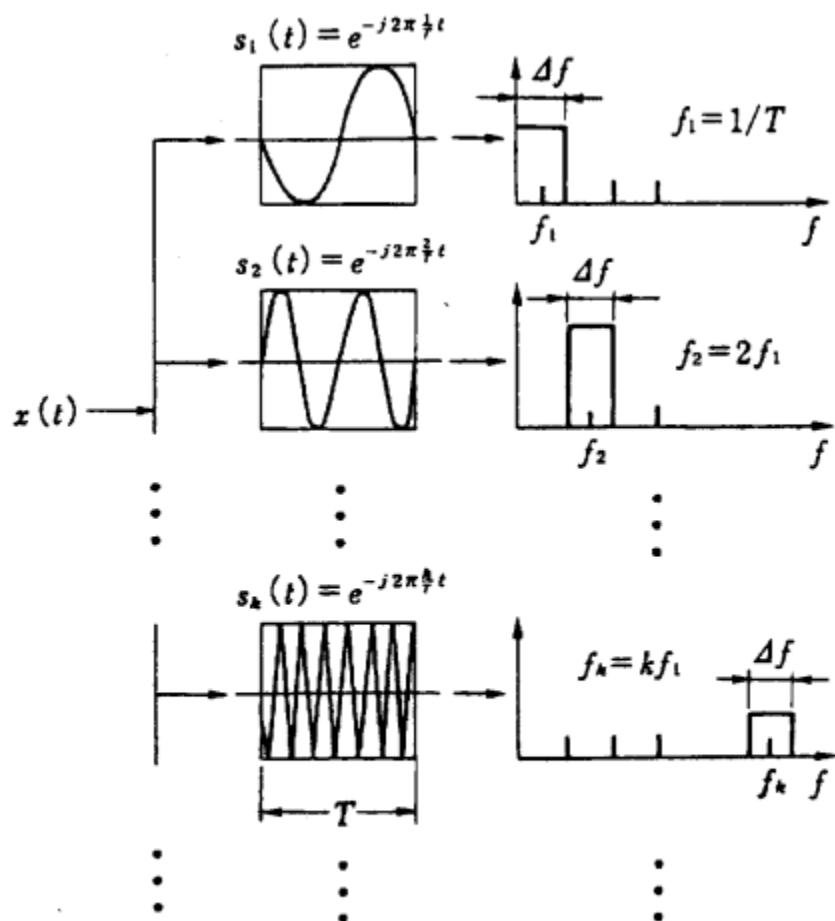
Name	Window function	Feature
Hamming window	$\omega(n) = \begin{cases} \alpha - (1-\alpha) \cos \frac{2\pi n}{N-1} & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$ <p>$0 \leq \alpha \leq 1$ とくに $\alpha = 0.50$ Hanning $\alpha = 0.45$ Hamming</p>	<p>Steep in transient area</p> <p>Attenuation is not big in Prevention area</p>
Blackman window	$\omega(n) = \begin{cases} 0.42 - 0.50 \cos \frac{2\pi n}{N-1} + 0.80 \cos \frac{4\pi n}{N-1} & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$	<p>Attenuation is big in Prevention area</p> <p>It is not steep in transient area</p>

3)Feature of STFT :

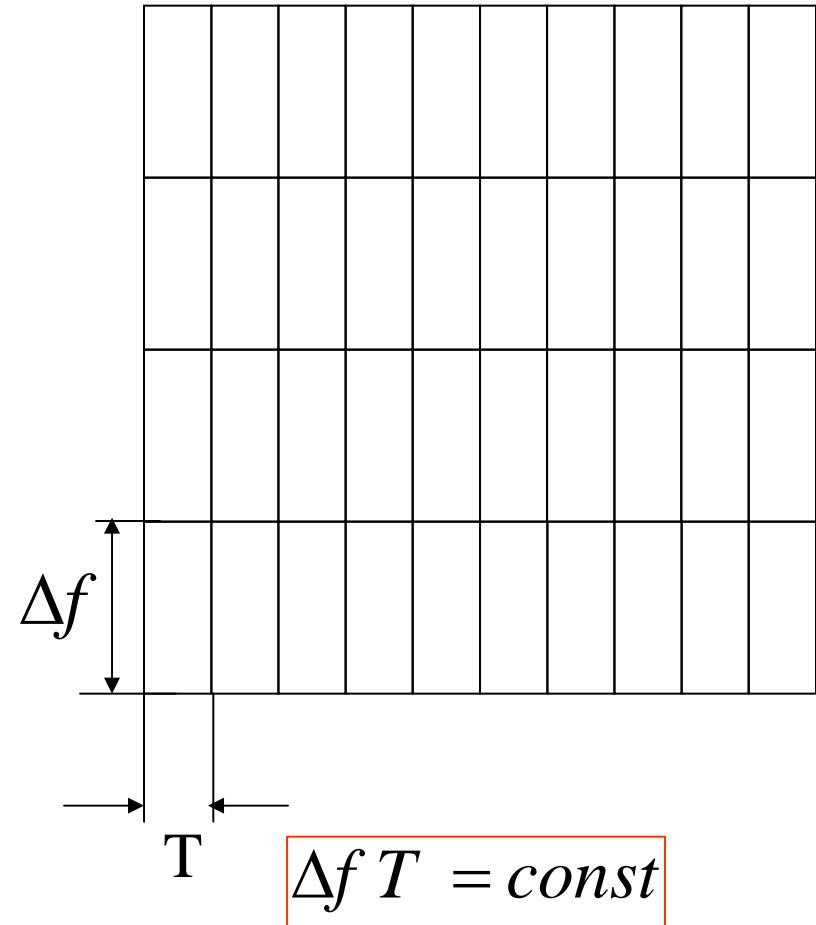


STFT N=512

3) STFT Seen From Information



Frequency resolution is fixed
STFT: $\Delta f = 1/T(\text{Const})$



Information is regularity

7.5 WT Seen From Information

1) Wavelet Transform (WT)

$$w(b, a) = \int_{-\infty}^{\infty} x(t) \overline{\psi_{a,b}(t)} dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

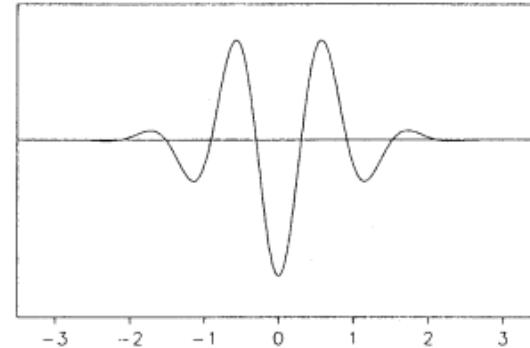
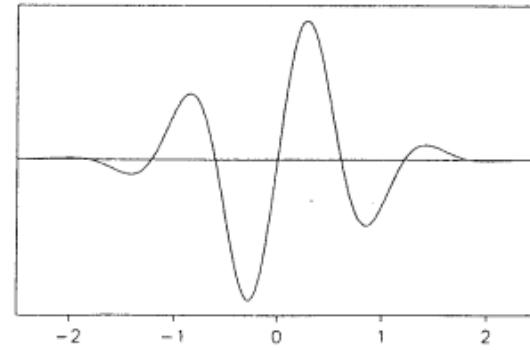


図 3.1.1 $Re G_{0,2\pi}^a$, $\alpha = 0.2925$

Mother Wavelet: $\psi(t)$

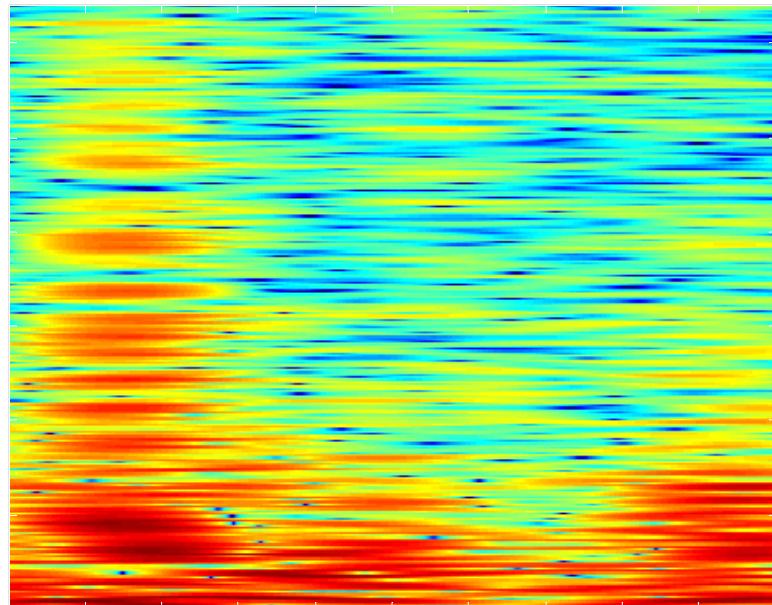
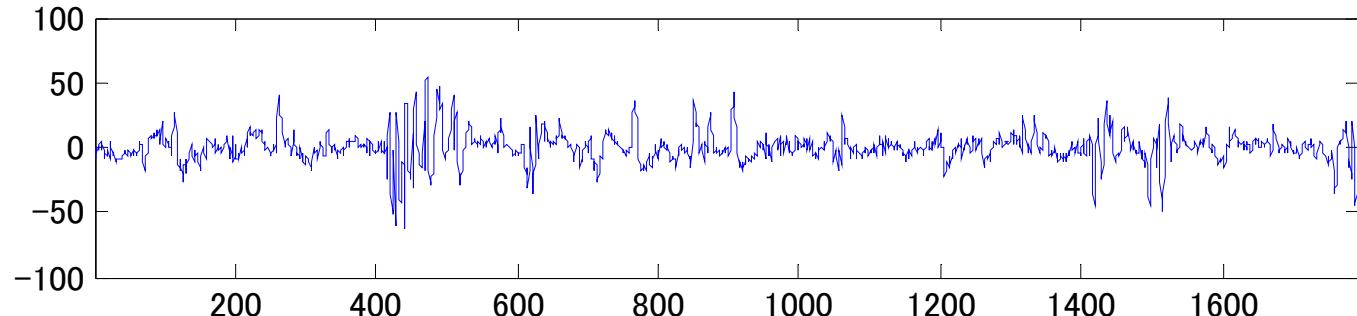
$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} w(a, b) \psi_{a,b}(t) \frac{da db}{a^2}$$

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega$$

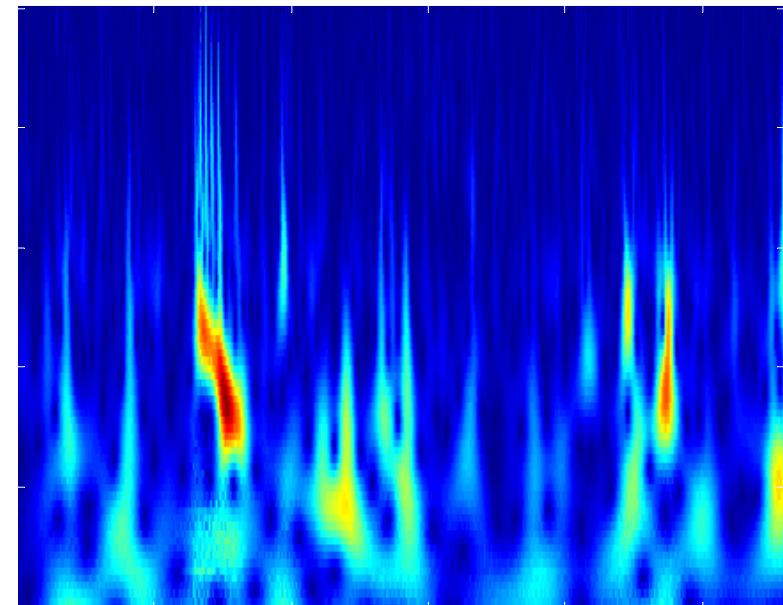


Shape of the Mother wavelet
is same as the core of STFT

2)Feature of the WT:

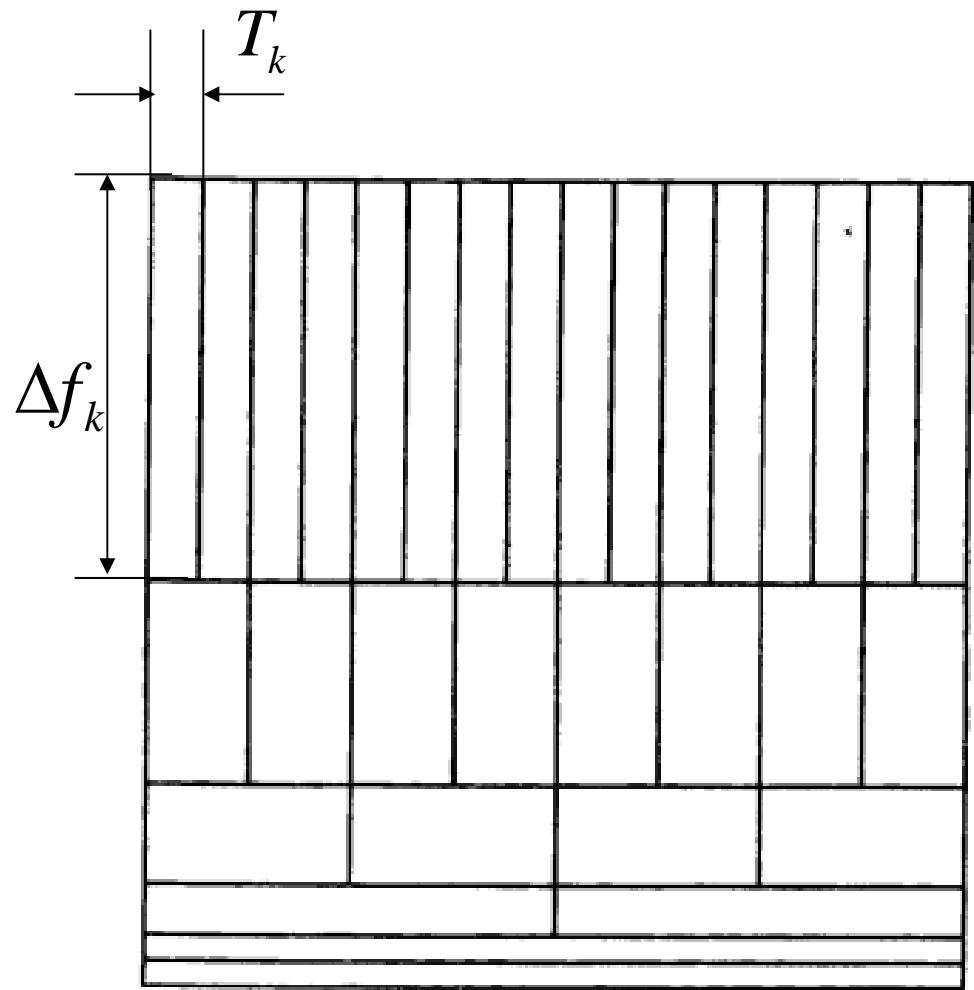
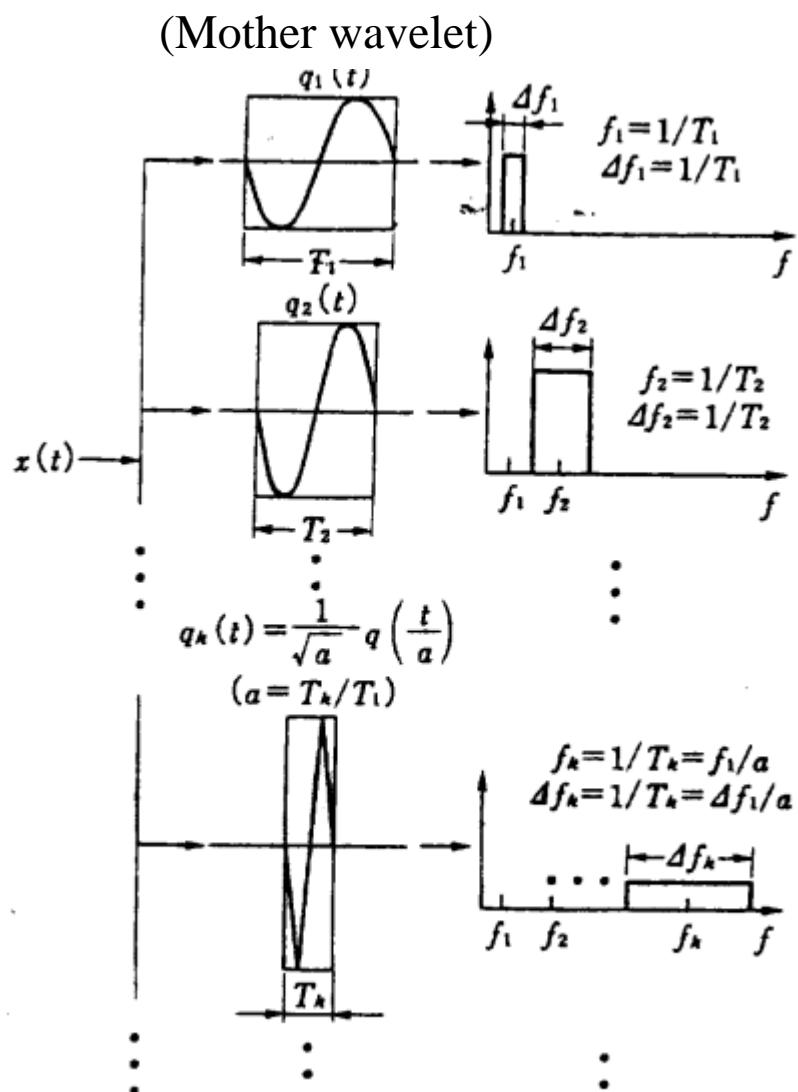


STFT N=512



CWT(Continus Wavelet Transform)²⁶

3) WT Seen Form Information



$$\Delta f_k T_k = \text{const}$$

Milt-frequency resolution
 WT: $\Delta f = \Delta f_1 / a$

Information is regularity ₂₇

Why ?

Short time Fourier Transform:

$$G_w(b, f) = \int_{-\infty}^{\infty} x(t) \overline{w(b-t)} e^{-i2\pi ft} dt$$

Window function: $w(t)$

Wavelet Transform:

$$w(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \overline{\psi}(\frac{t-b}{a}) dt$$

MW: $\psi(t)$

$$w(b-t) e^{-i2\pi ft} \approx \psi(t-b) \neq \psi(\frac{t-b}{a})$$

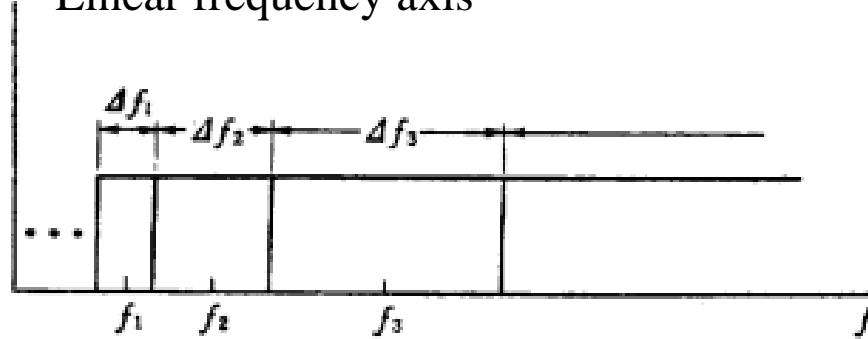
4) Regularity Q value frequency analysis

1) Regularity Q value frequency analysis

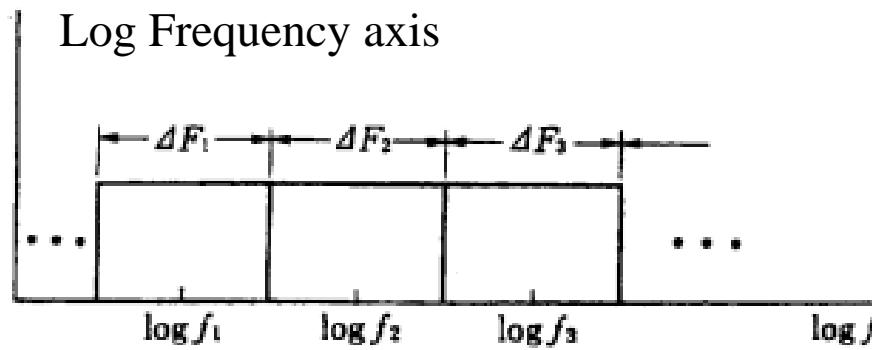
$$Q = \frac{f_1}{\Delta f_1} = \frac{f_2}{\Delta f_2} = \dots = \text{const}$$

$$\Downarrow$$
$$\frac{f_2}{f_1} = \frac{f_3}{f_2} = \dots = \gamma = \text{const}$$

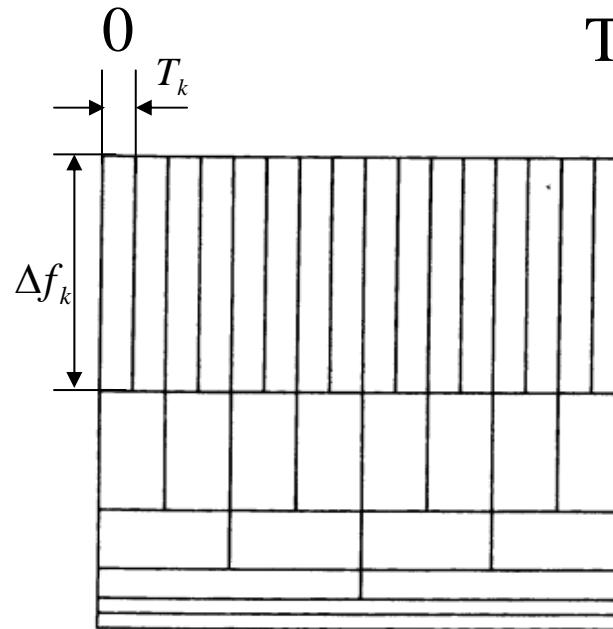
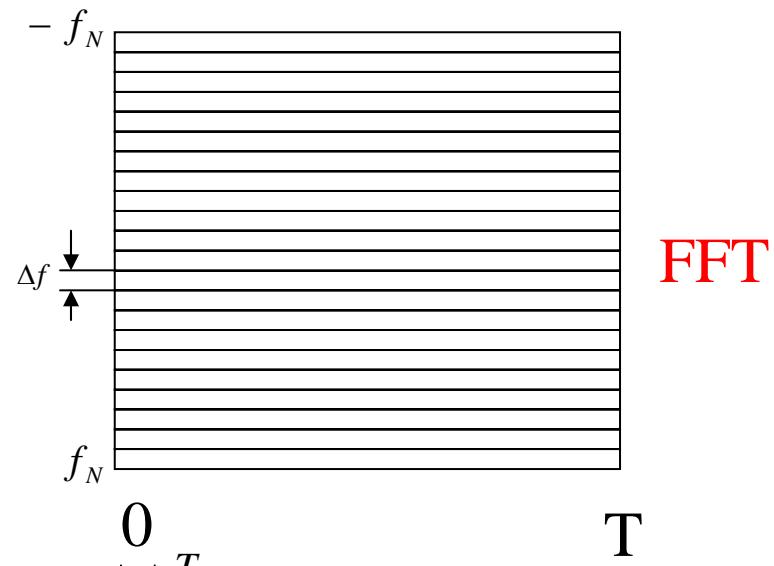
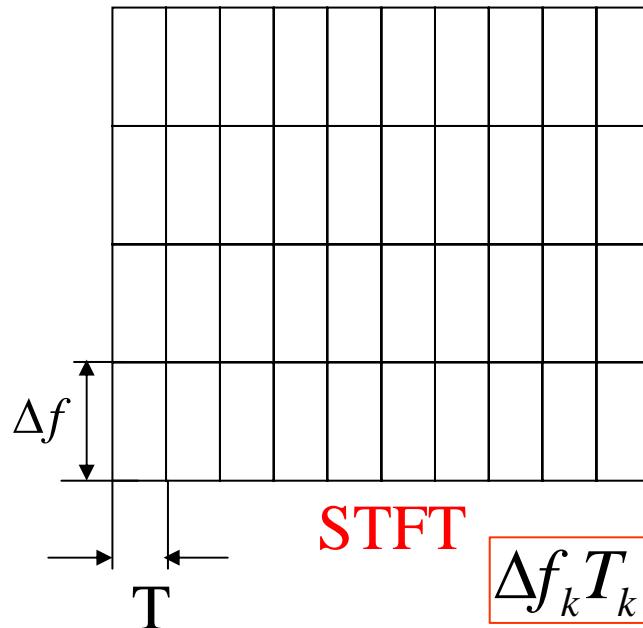
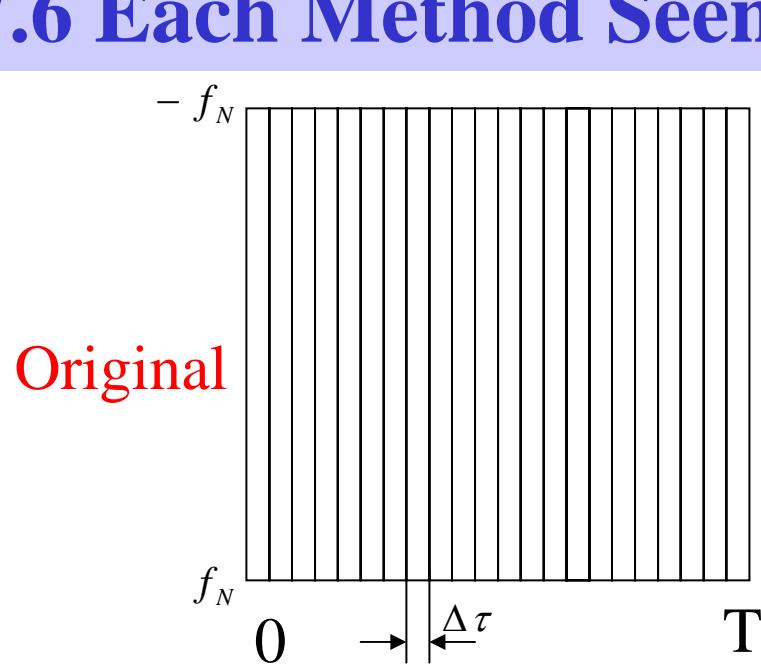
Linear frequency axis



Log Frequency axis



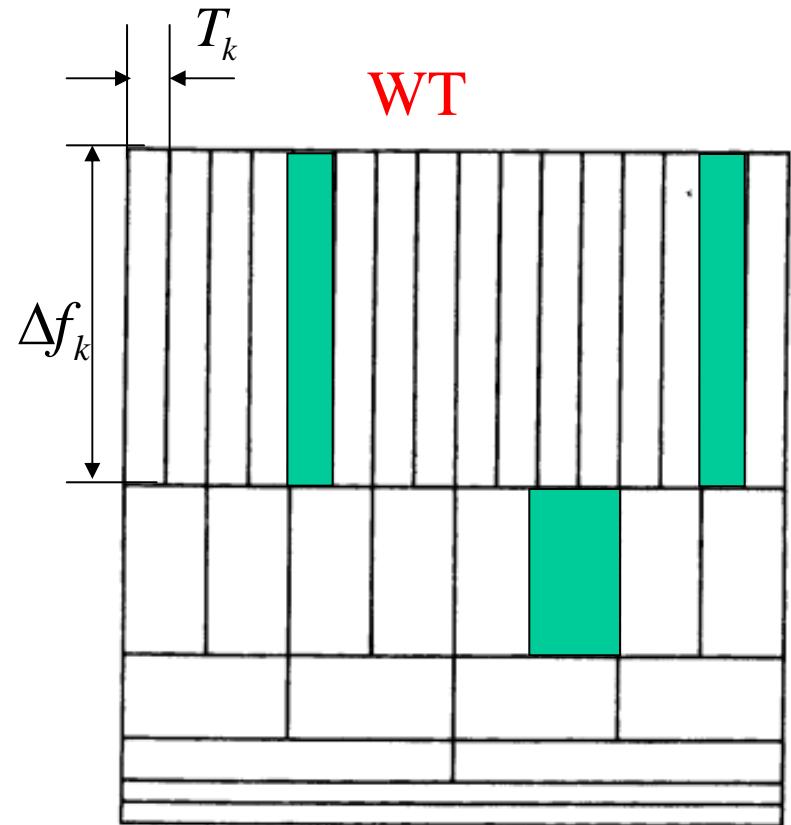
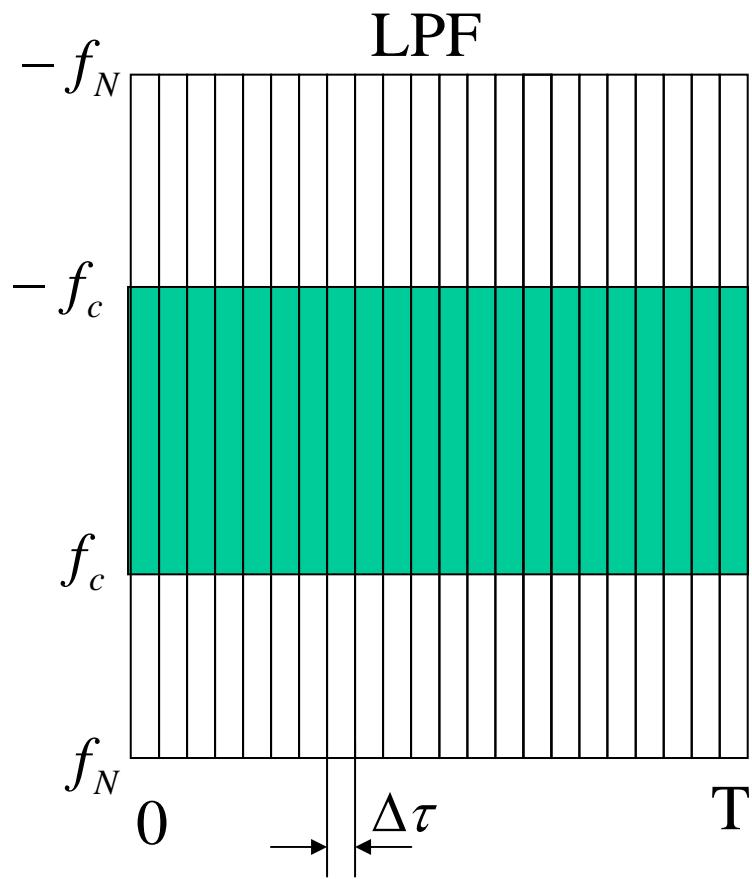
7.6 Each Method Seen From Information



$$\Delta f_k T_k = \text{const}$$

WT
Information is regularity³⁰

7.7 Different between LPF and WT in the case of de-noising



Summary :

- 1) WT's mother wavelet width changes with frequency. Comparing to it, STFT' window width is fixed, and it is optimum for only one frequency.
- 2) STFT has same frequency resolution for every frequency. But WT has high frequency resolution for low frequency and high time resolution for high frequency.
- 3) WT is a regularity Q value frequency analysis.
(it suit to the feature of man's ear)
- 4) Signal analysis can not occur any new information. The difference of the analysis methods is that the viewpoint is different.