

5. Image processing by Wavelet Transform

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4. Discrete Wavelet Transform

4.1 Discrete Wavelet Transform (DWT)

4.2 Features of the DWT

Severe shift dependence.

Poor directional selectivity

4.3 New Design Method for CDWT

Make Real and Imaginary components of the complex

Mother Wavelet must be a Hilbert pair

4.4 New Calculation methods for CDWT

For scaling functions, which shapes of the real and imaginary components are completely same and positions are different from 1/2 sample

4.5 Example of De-noising

Model signals, EEG, Music

Important point of the 1D-DWT

1) Basis and fast algorithm of the DWT

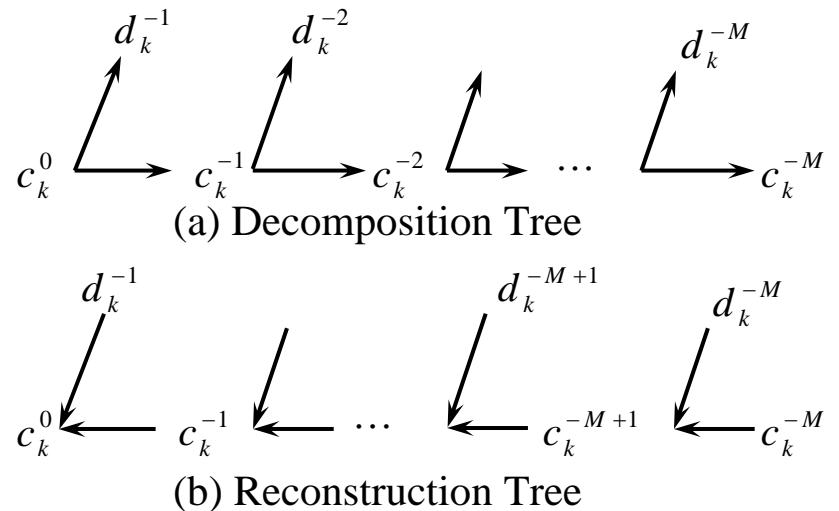
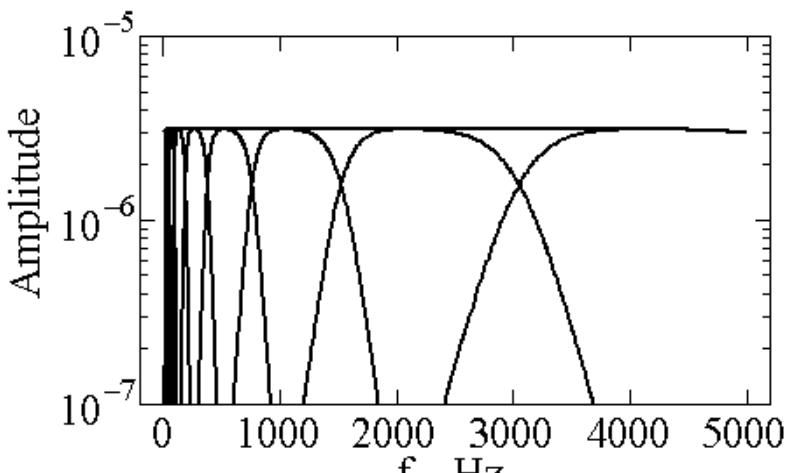


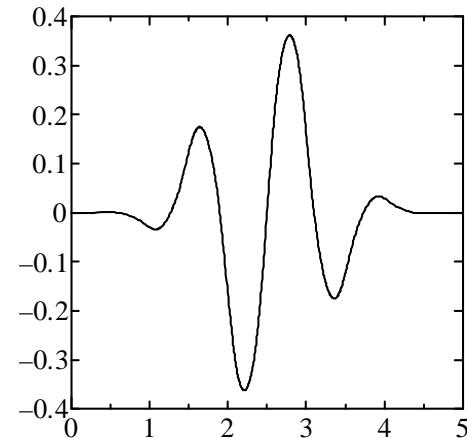
Fig.2 Tree algorithm of MRA

$$d_k^{j-1} = \sum a_{l-2k} c_k^j$$

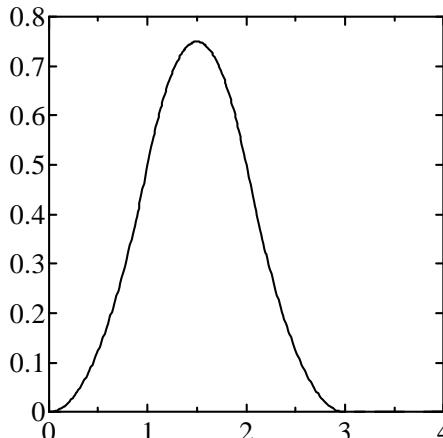
$$c_k^{j-1} = \sum_k b_{l-2k} c_k^j$$

$$c_k^0 = \int_{-\infty}^{\infty} f(t) \phi(t - k) dt$$

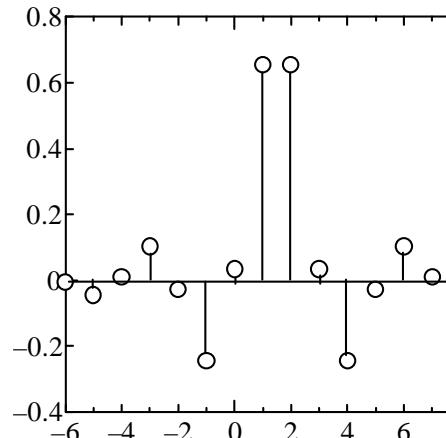
2) Low-pass and High-pass filter and Wavelet, scaling functions



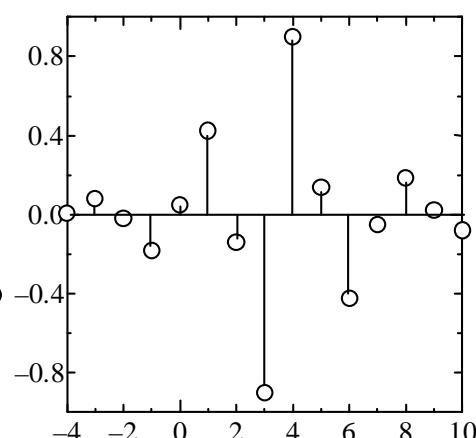
(a)Wavelet



(b)Scaling function

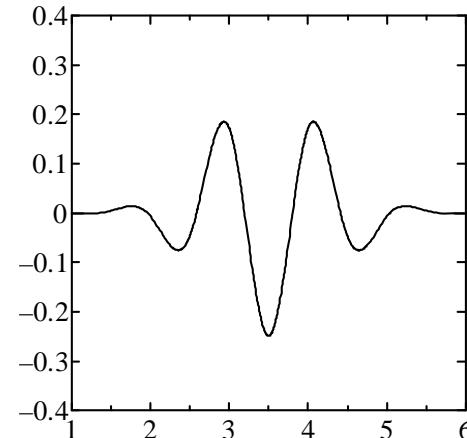


(c) a_k

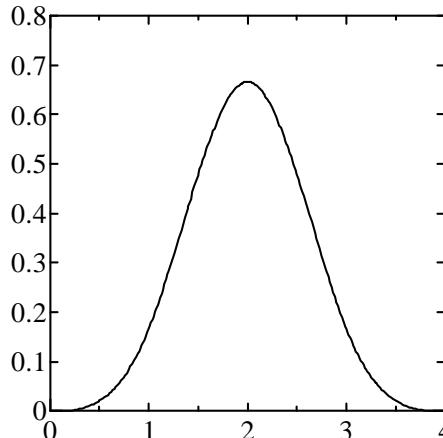


(d) b_k

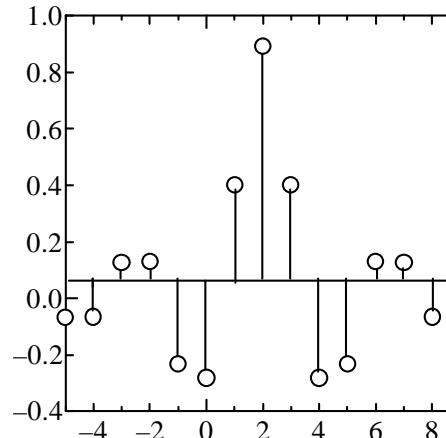
(a) $m=3$ Spline Wavelet



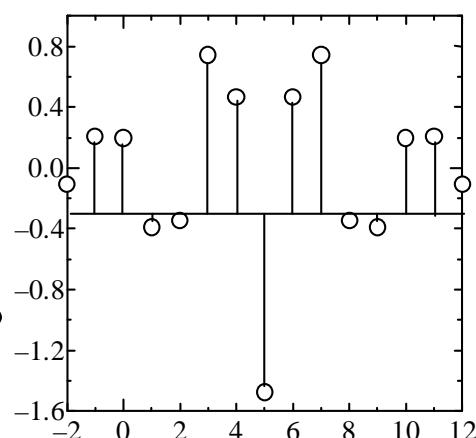
(a)Wavelet



(b)Scaling function



a_k



(d) b_k

(b) $m=4$ Spline waveley

There is a half sample delay problem between two wavelet

3) Severe shift dependence problem

(MW is Daubichies 8)

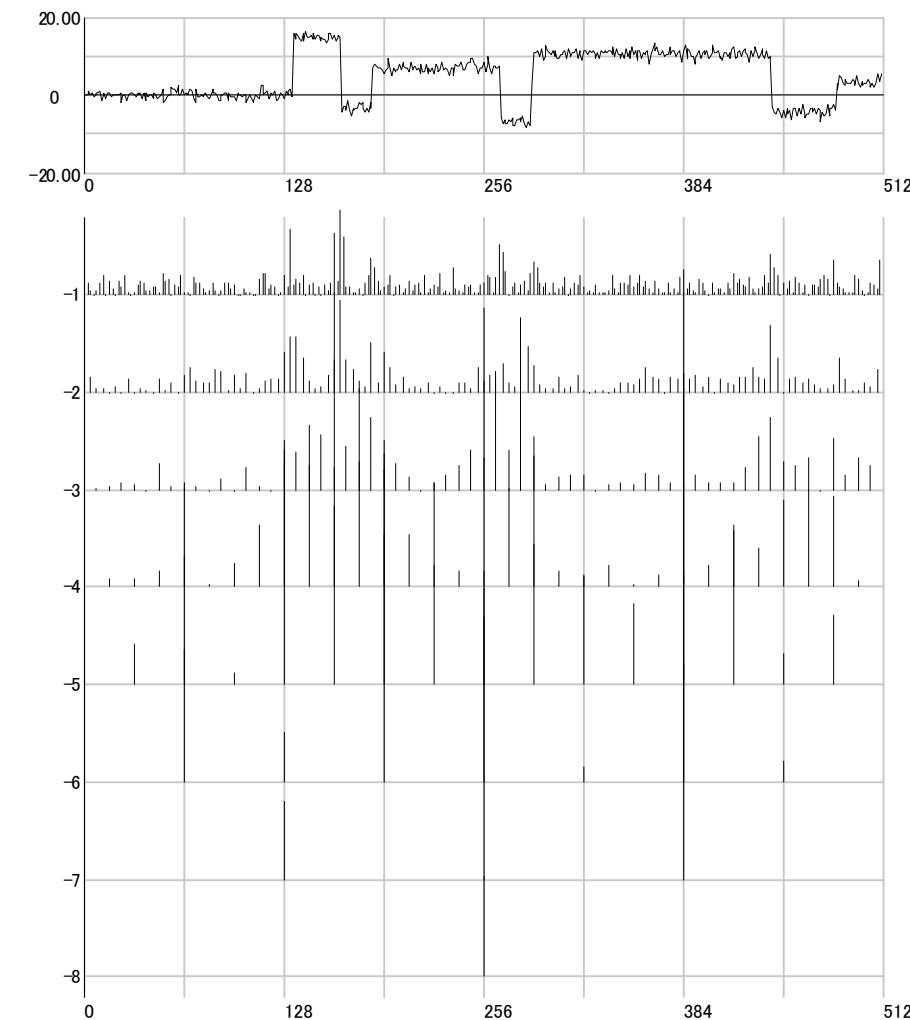


Fig.5 Original signal

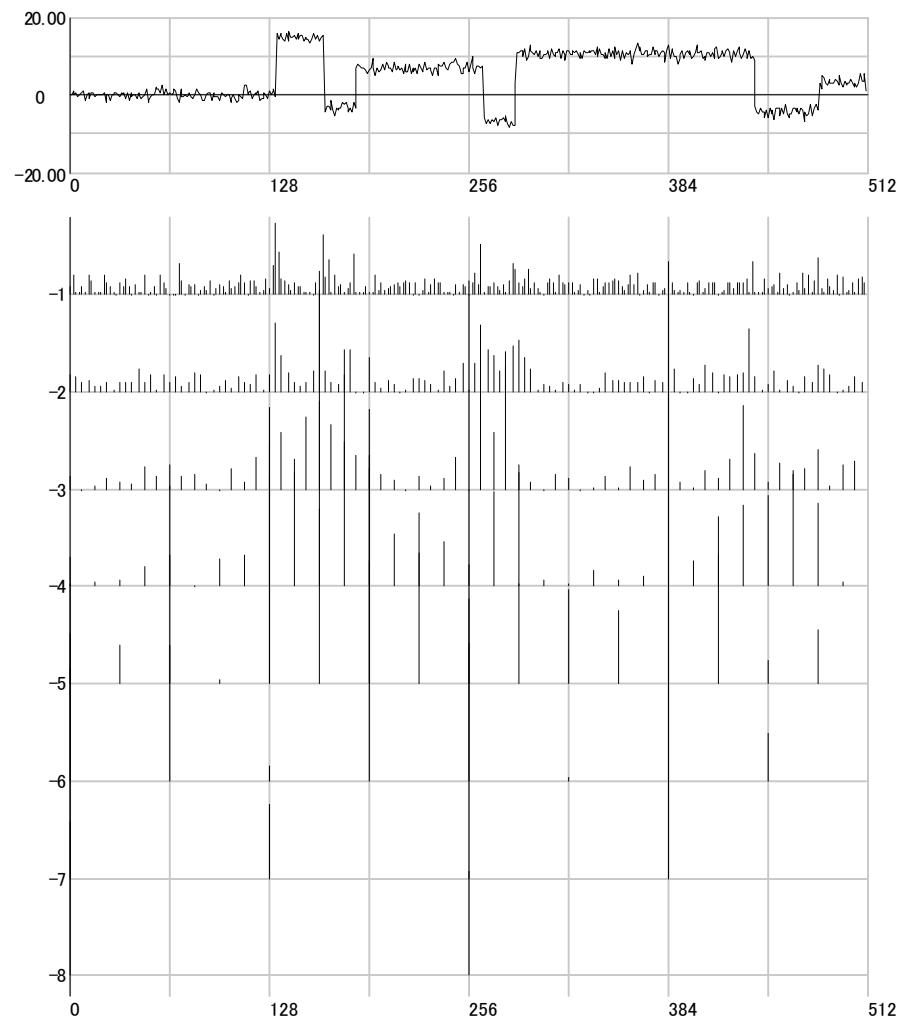


Fig.6 Signal with shift of one sample

4) PTI CDWTの設計

The condition of Perfect Translation Invariance (PTI) :

$\phi^R(t)$ and $\phi^I(t)$ have the same shapes with the 1/2-sample distance.

$$\phi^I(t) = \phi^R(t - 1/2)$$

This PTI condition is achieved by the characteristics of Meyer wavelet

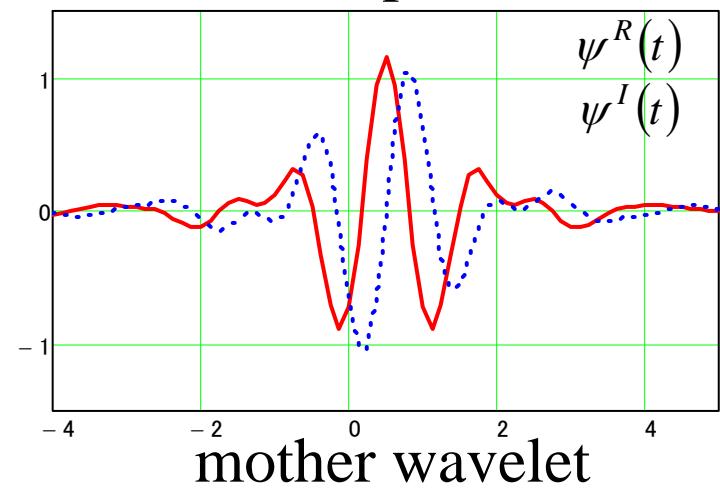
$$\phi^R(t) = \phi^b(t) = \phi^M(t - b)$$

$$\phi^I(t) = \phi^{b+1/2}(t) = \phi^M(t - (b + 1/2))$$

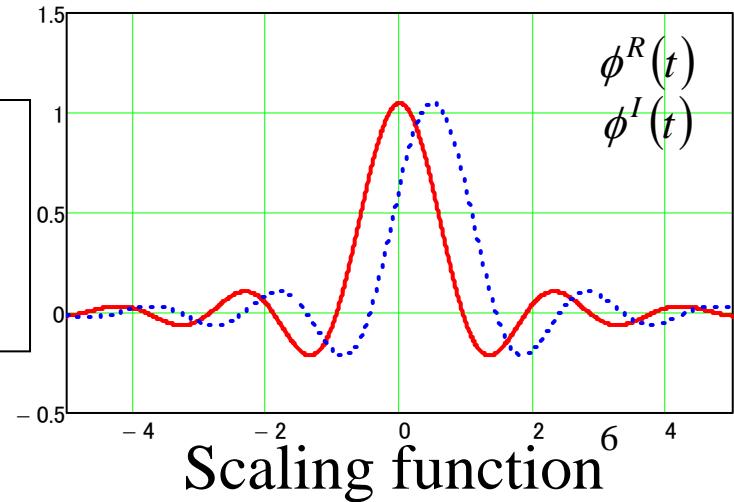
The translated scaling functions of the Meyer wavelet

$$p_n^b = \phi^M\left(\frac{n-b}{2}\right)$$

The example of $b = 0$



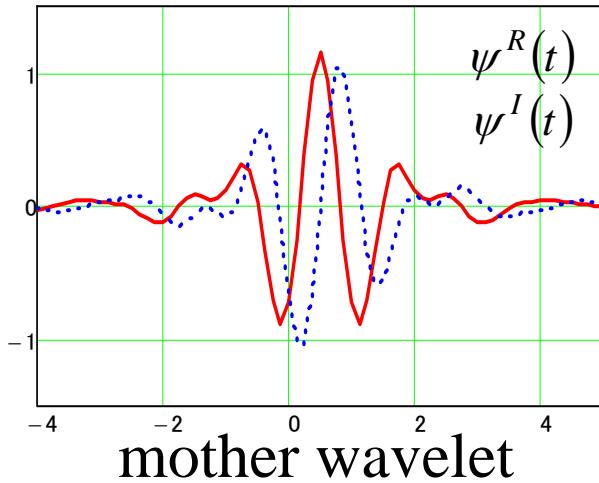
mother wavelet



Scaling function

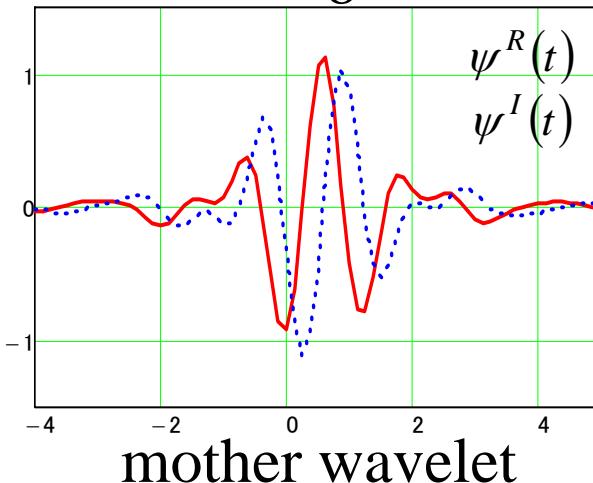
Example of the PTI complex wavelets

$$b = 0$$



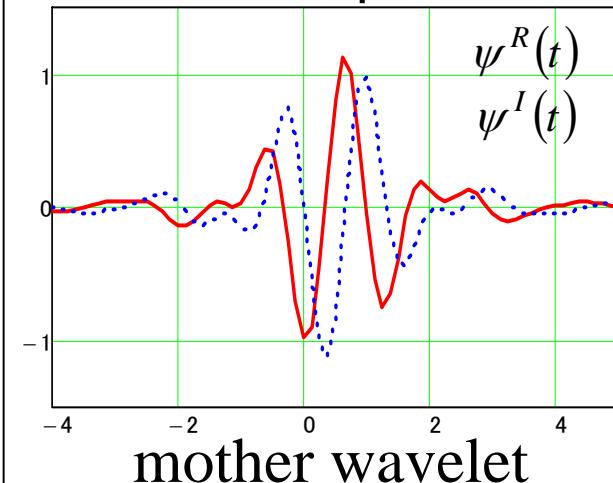
mother wavelet

$$b = \frac{1}{8}$$



mother wavelet

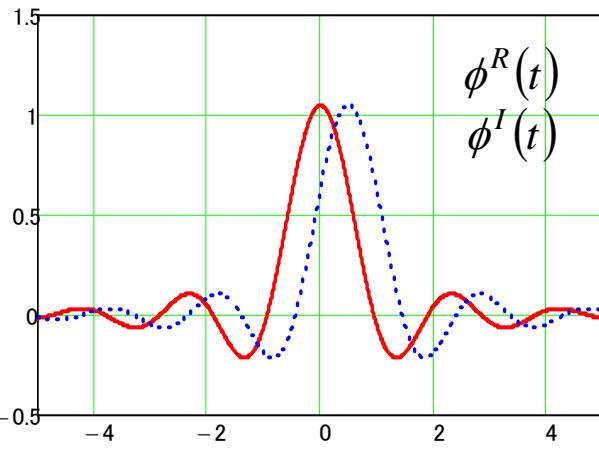
$$b = \frac{1}{4}$$



mother wavelet

$$\phi^R(t)$$

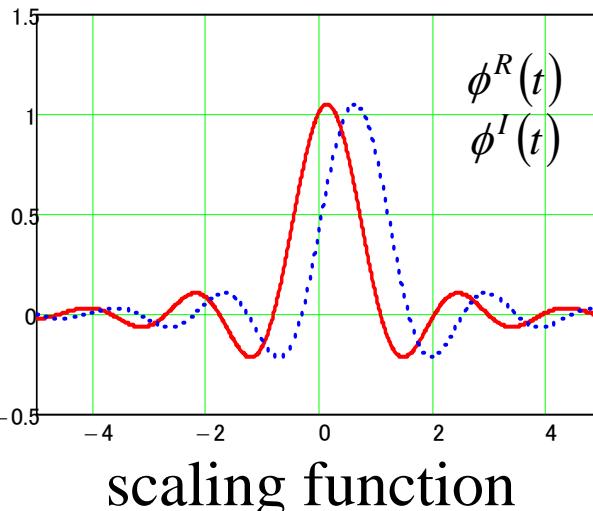
$$\phi^I(t)$$



scaling function

$$\phi^R(t)$$

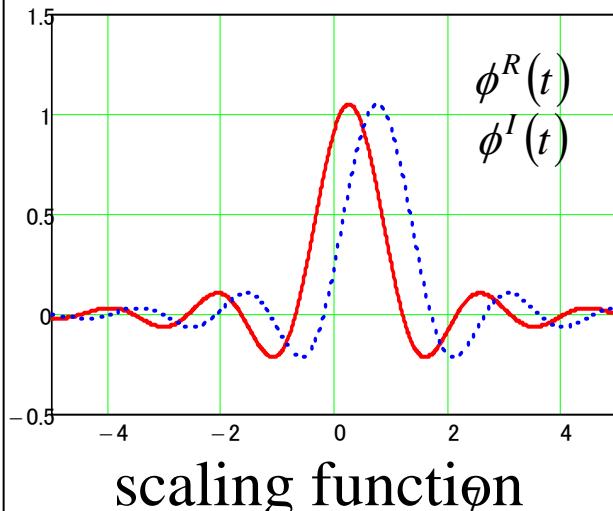
$$\phi^I(t)$$



scaling function

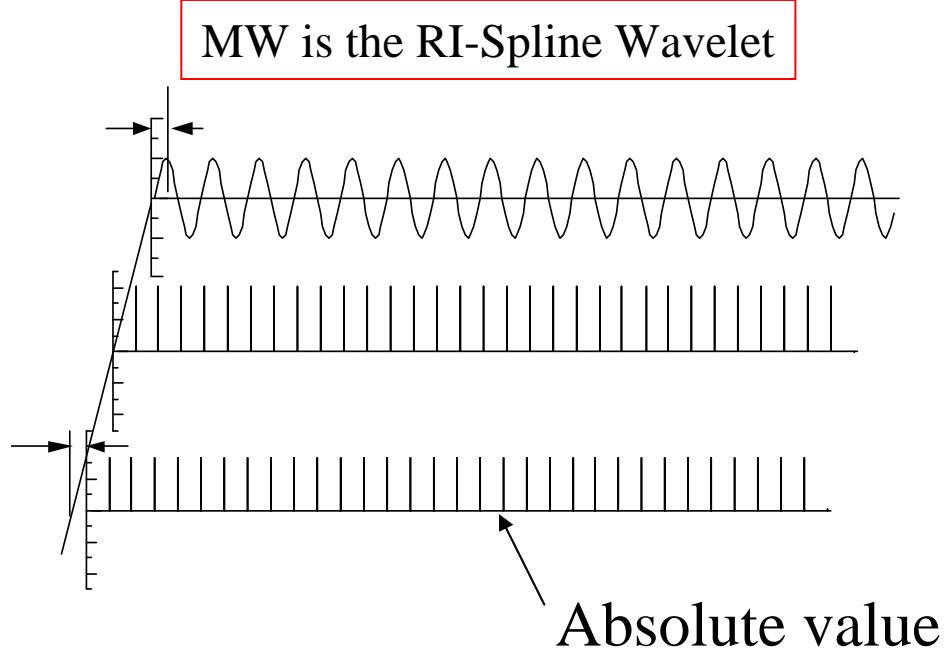
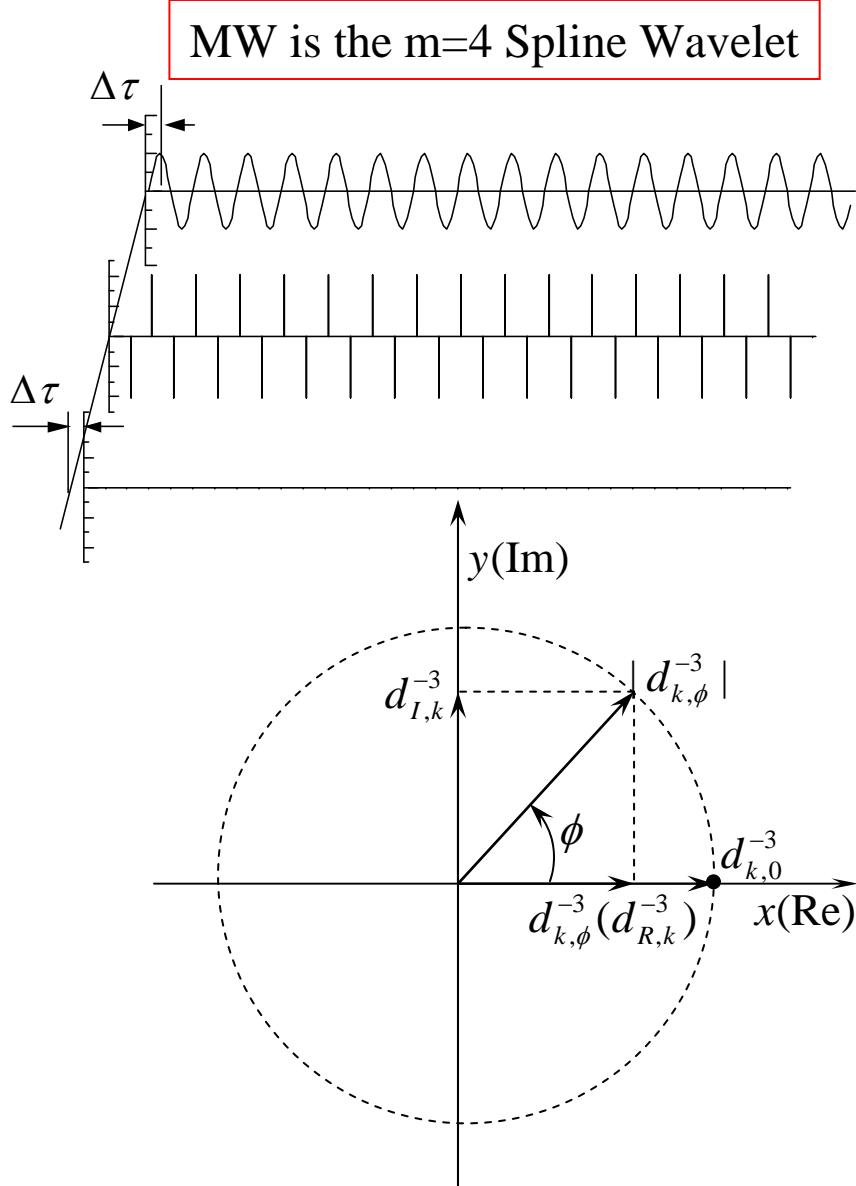
$$\phi^R(t)$$

$$\phi^I(t)$$



scaling function

Comparing DWT and CDWT

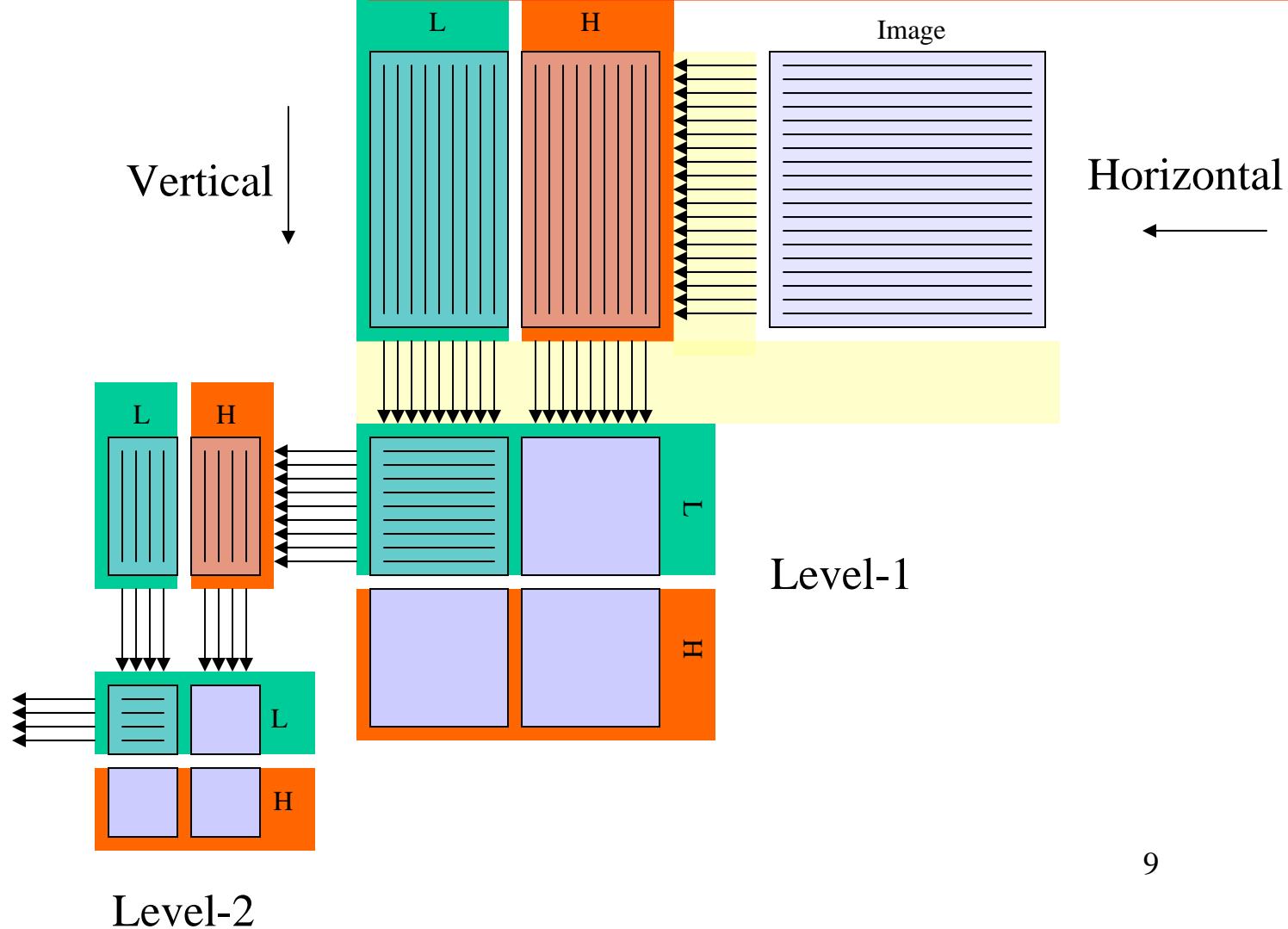


$$|d_{k,\phi}^{-3}| = \sqrt{(d_{I,k}^{-3})^2 + (d_{R,k}^{-3})^2}$$

5.1 2D-Discrete Wavelet Transform

1) Calculation method

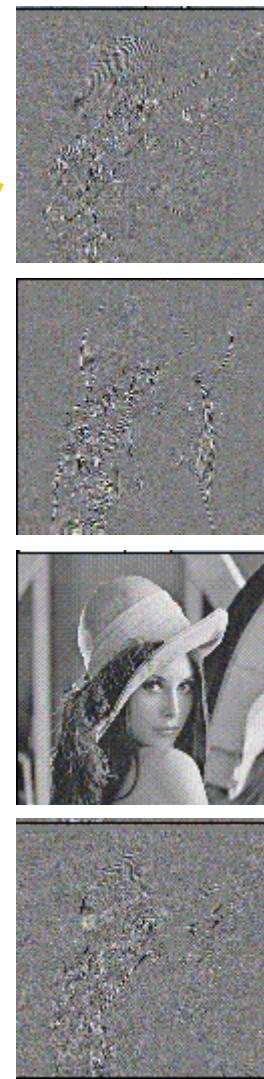
Orthogonal wavelet: $\psi(x, y) = \psi(x)\psi(y)$



Example of a Model Image

→ Decomposition

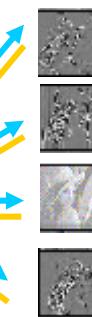
← Reconstruction



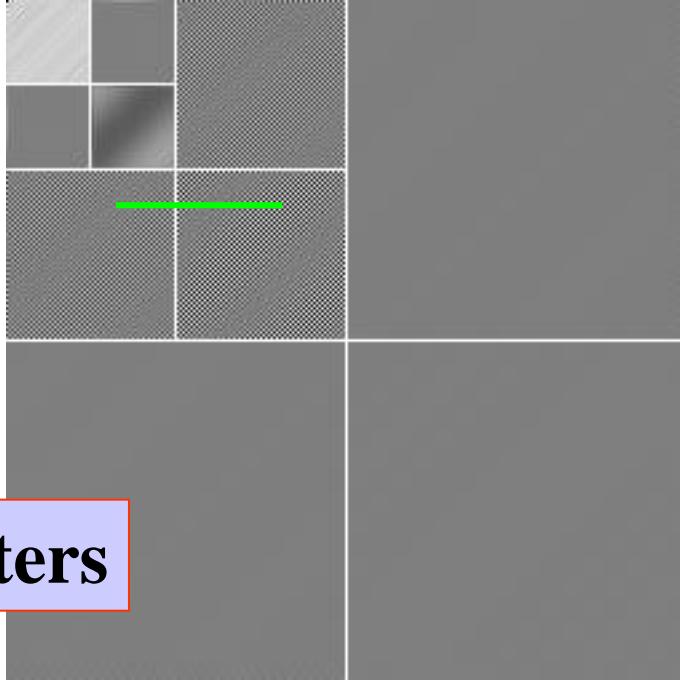
LH

HL

HH



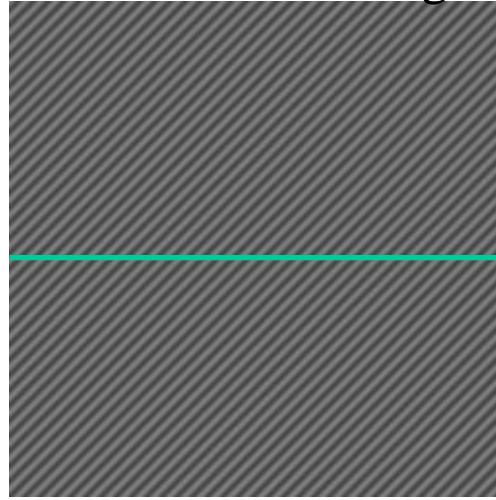
2) Characters



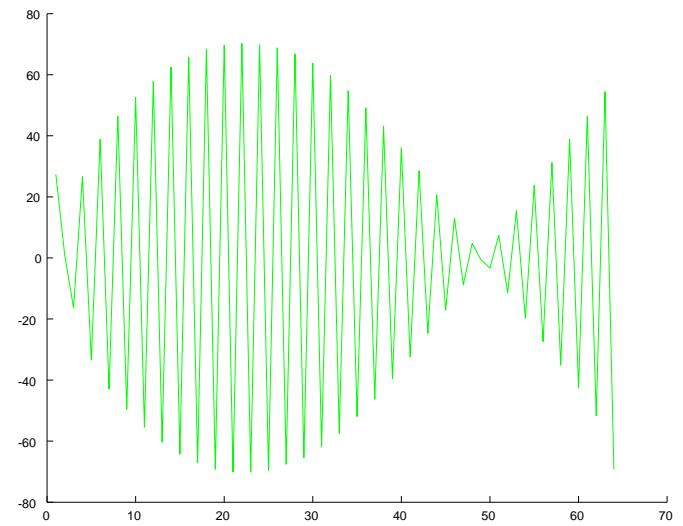
	3-HL		
3-LH	3-HH	2-HL	1-HL
		2-LH	2-HH
		1-LH	1-HH

Cloth Surface Image

Determination component

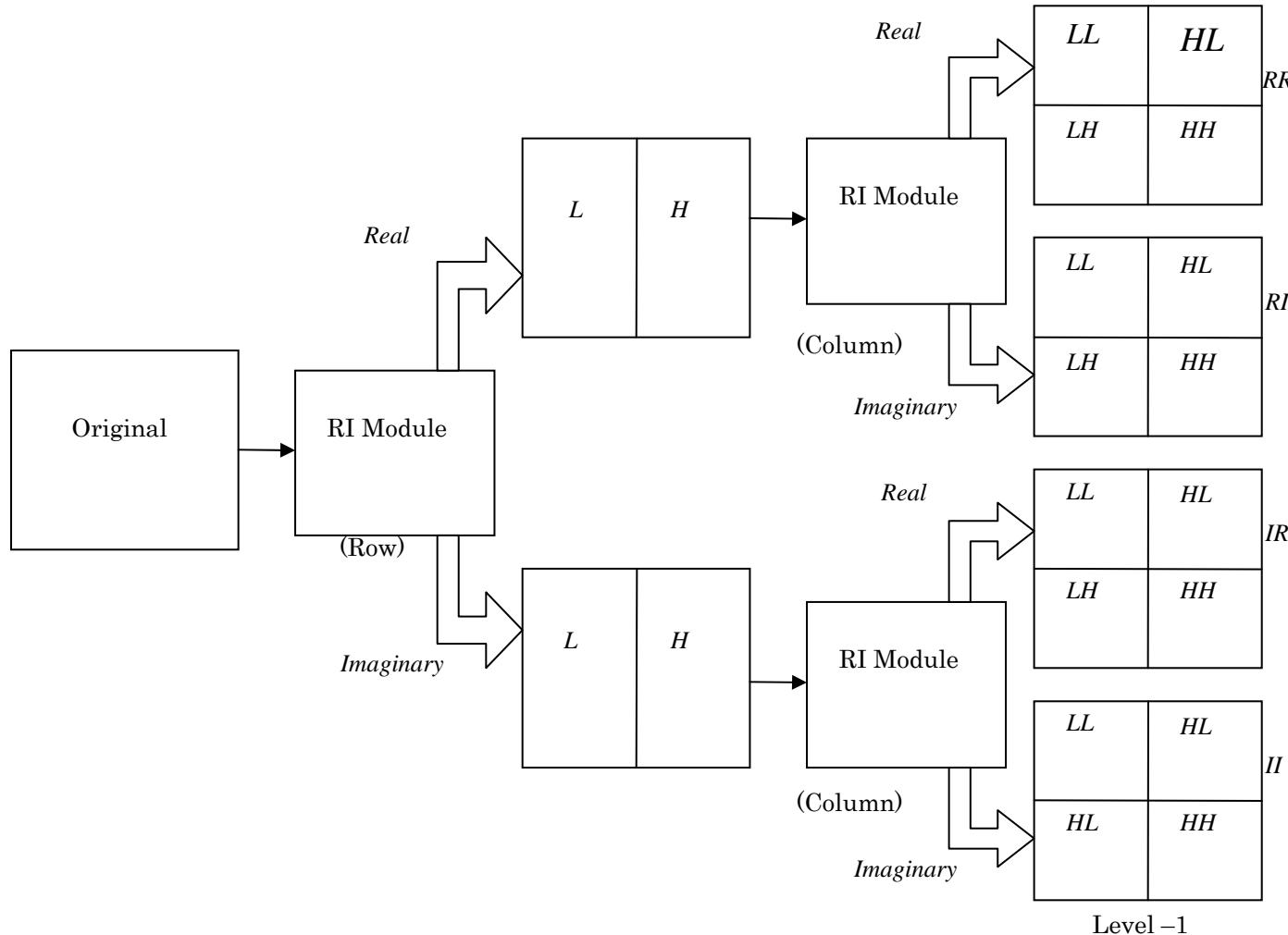


Result obtained in level-3



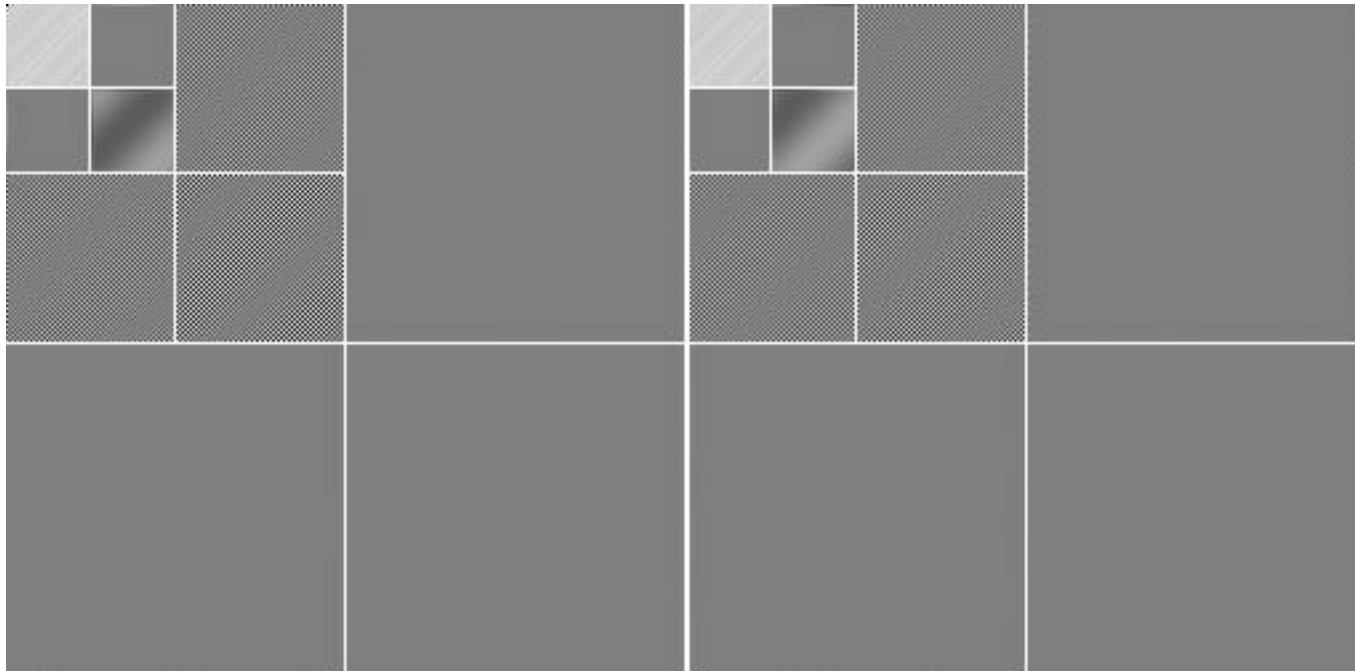
5.2 2D-Complex Wavelet Transform

1) Calculation method



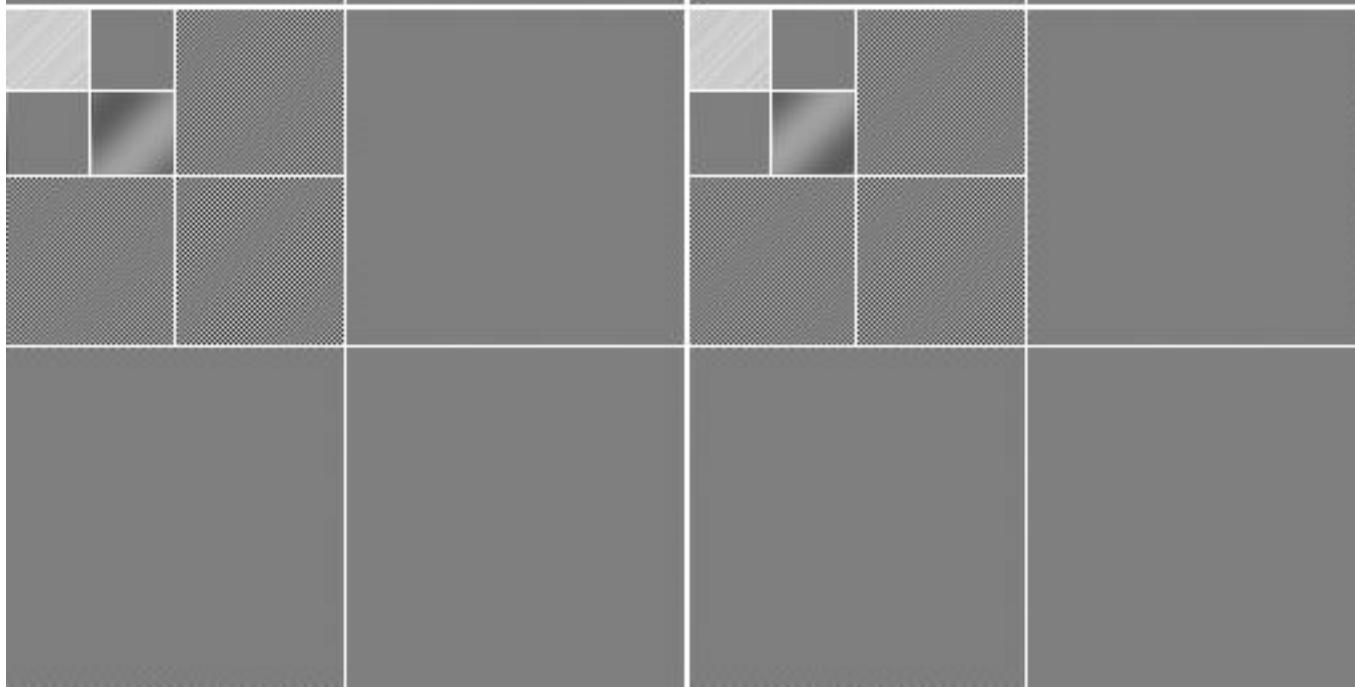
(a) Block diagram of level -1

RR

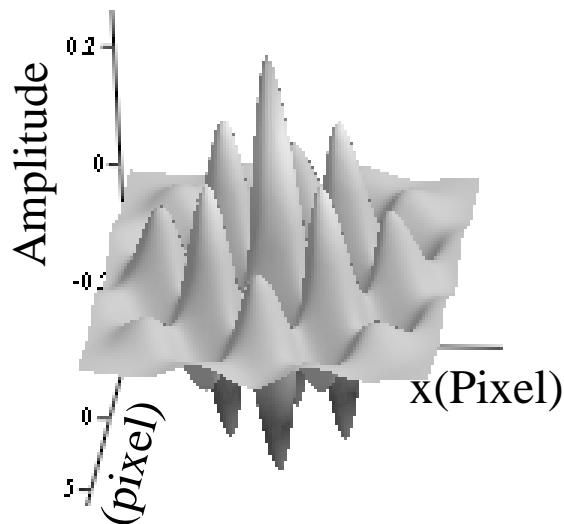


IR

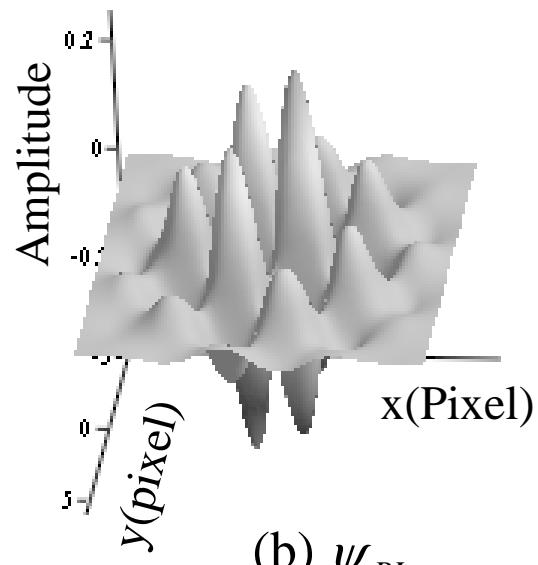
RI



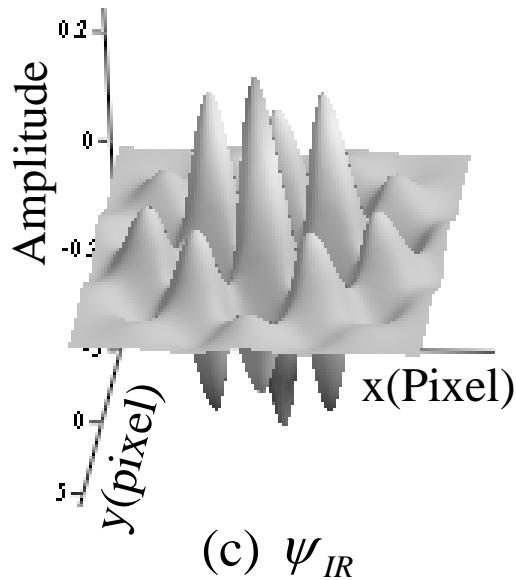
2) 2D-Mother Wavelets



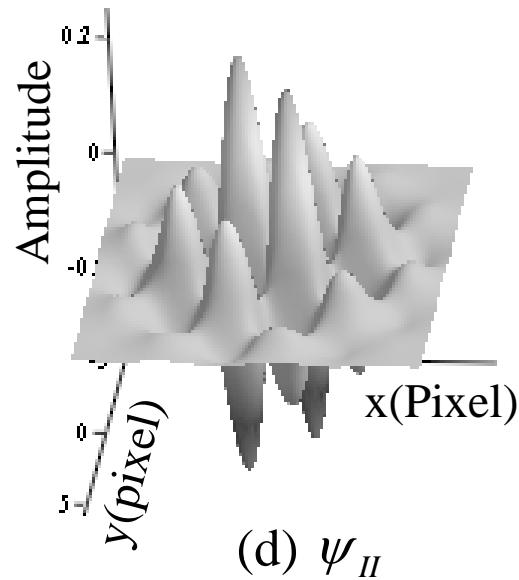
(a) ψ_{RR}



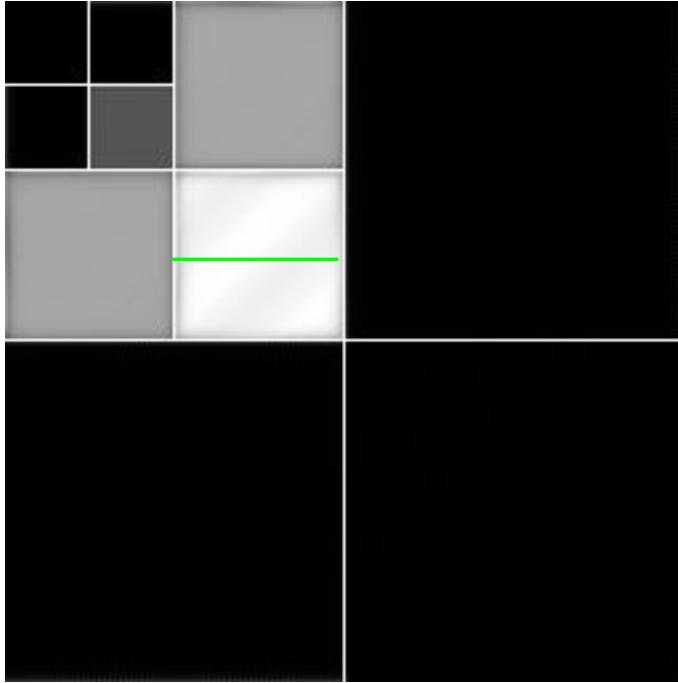
(b) ψ_{RI}



(c) ψ_{IR}



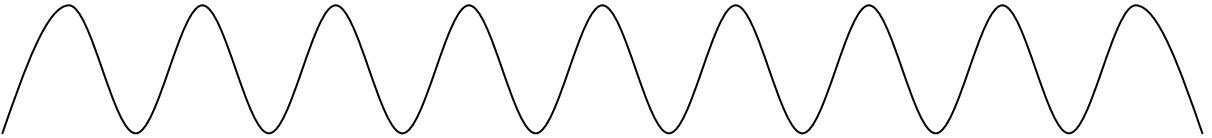
(d) ψ_{II}



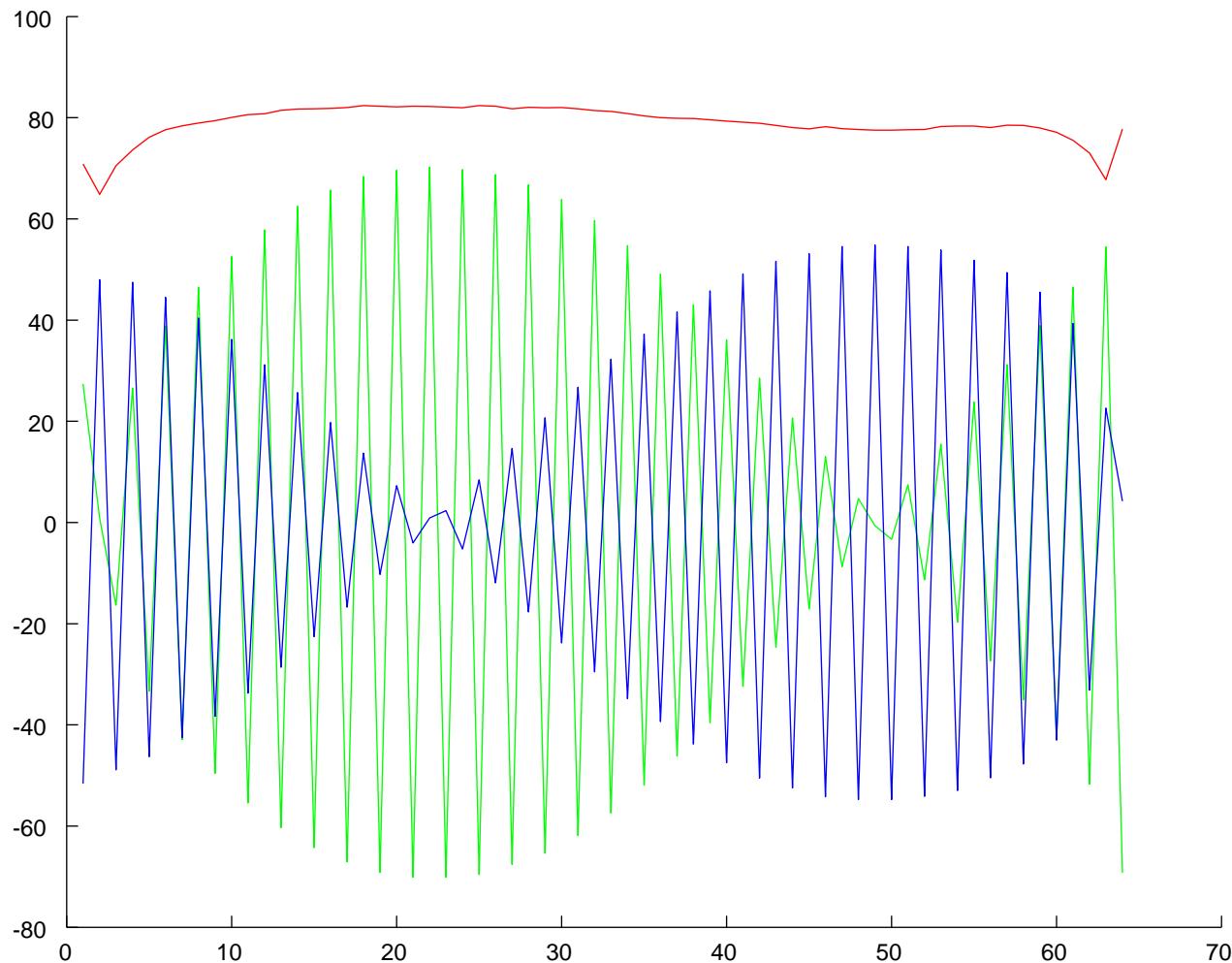
	3- HL	2-HL	
3- LH	3- HH		1-HL
2-LH	2-HH		
		1-LH	1-HH

Carrying out root sum square of coefficients of RR, IR, RI and obtaining Translation invariant coefficient (TI) $d_{kx,ky}^j$

$$d_{kx,ky}^j = \sqrt{(d_{RR}^j)^2 + (d_{RI}^j)^2 + (d_{IR}^j)^2 + (d_{II}^j)^2}$$



Determination component



Results obtained by RR

Result obtained by II

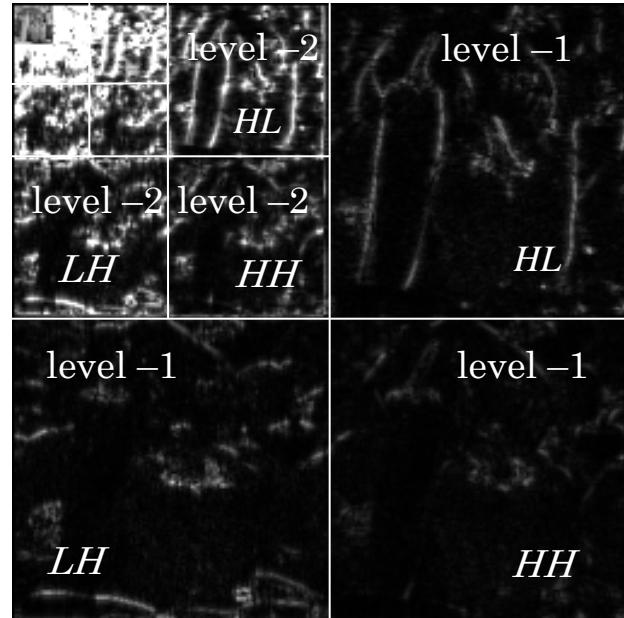
Result obtained by
TI $d_{kx,ky}^j$

Result of Level -2, HH, Horizontal direction

2) Translation invariance property of 2D-CDWT



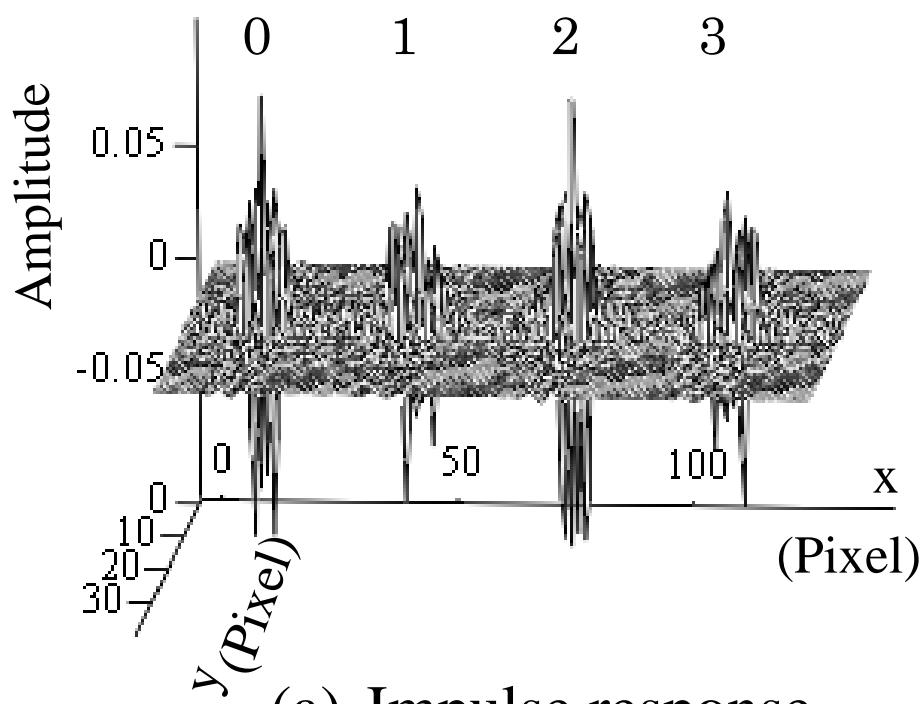
(a) 256X256 Peppers Image



(b) Characteristic Image by using
RI-Spline wavelet's 2-D CDWT

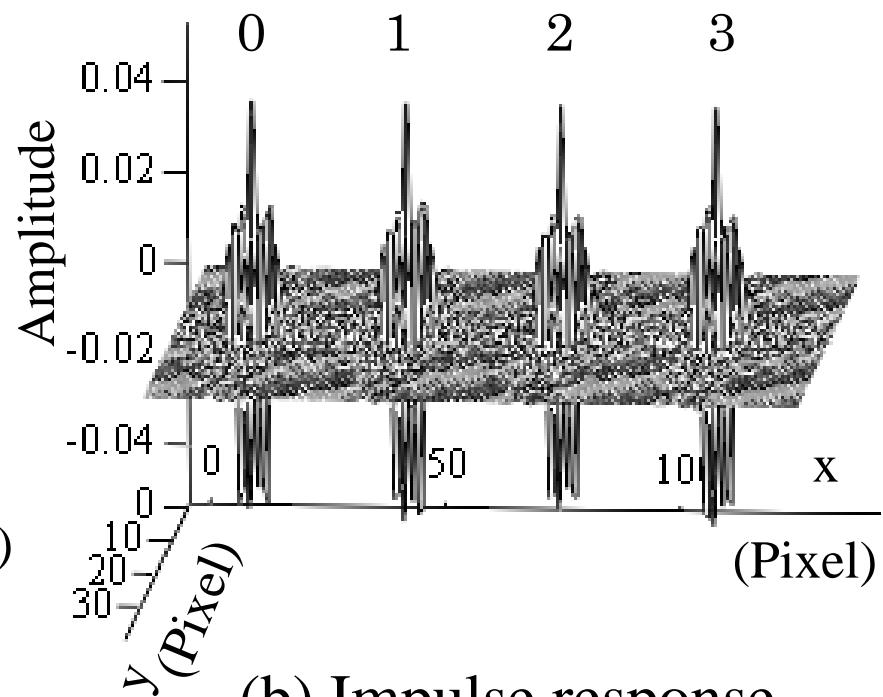
$$d_{kx,ky}^j = \sqrt{(d_{RR}^j)^2 + (d_{RI}^j)^2 + (d_{IR}^j)^2 + (d_{II}^j)^2}$$

Impulse shift



(a) Impulse response
of the DWT

Impulse shift



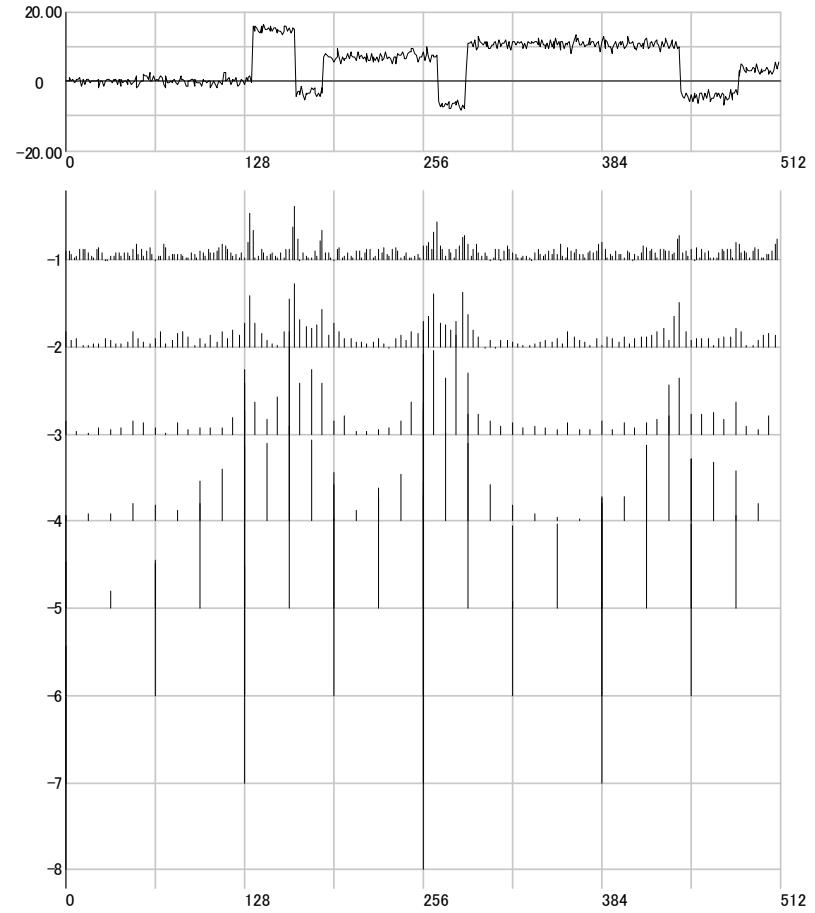
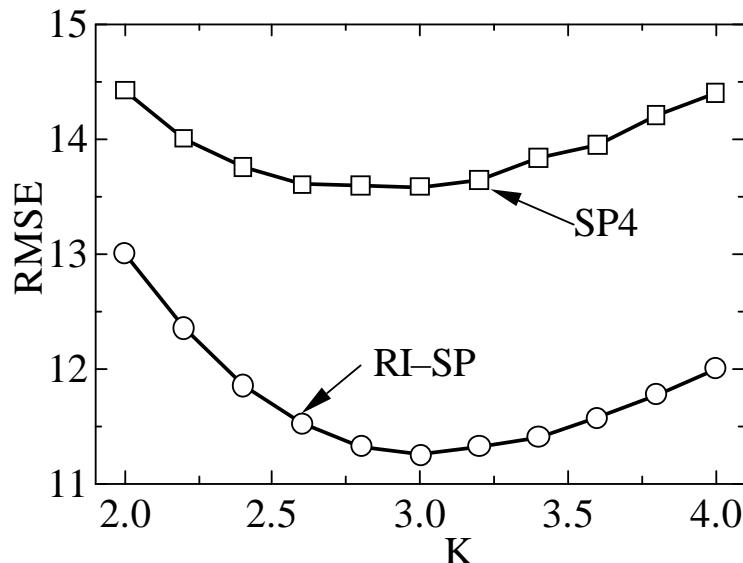
(b) Impulse response
of the CDWT

5.3 Application on de-noising

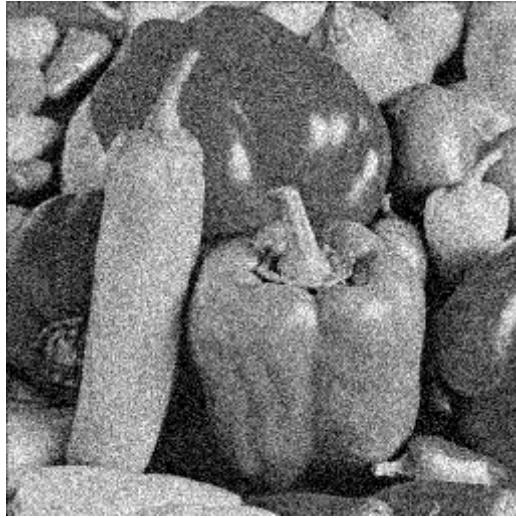
$$d_{kx,ky}^j = \sqrt{(d_{RR}^j)^2 + (d_{RI}^j)^2 + (d_{IR}^j)^2 + (d_{II}^j)^2}$$

$$\hat{d}_{kx,ky}^j = \begin{cases} d_{kx,ky}^j - \lambda, & d_{kx,ky}^j > \lambda \\ 0 & , d_{kx,ky}^j \leq \lambda \end{cases}$$

$$\lambda = K\sigma$$



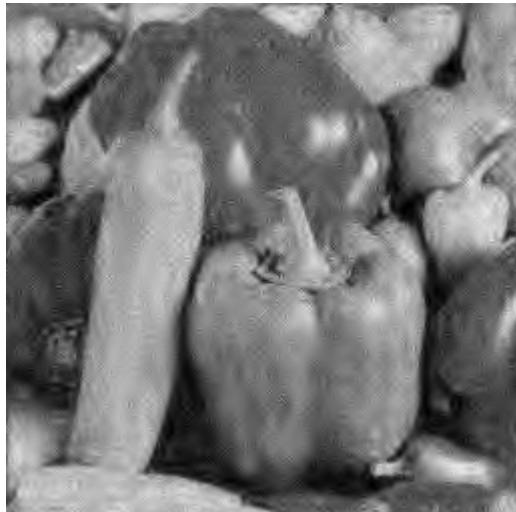
1) Example of de-noising by model image



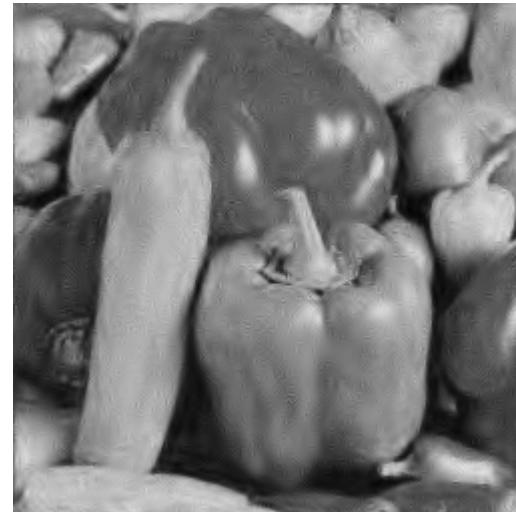
(a) Image with Gaussian noise
SNR=6.0dB



(b) Smoothing filter (5X5 pixels),
SNR=13.06dB

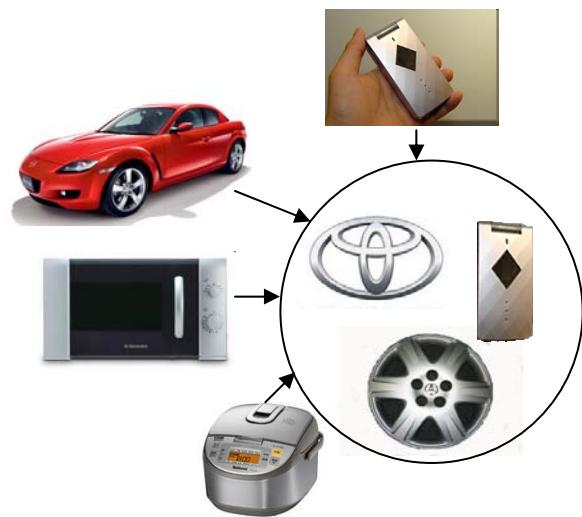


(c) The $m=4$ Spline wavelet,
SNR=11.86dB



(d) The $m=4,3$ RI-Spline wavelet,
SNR=13.49dB

2) Application in Inspection of metallic surface



A lot of metal plating parts are used for the car and the house electric appliance, etc.



These parts are manufactured from the automated production line

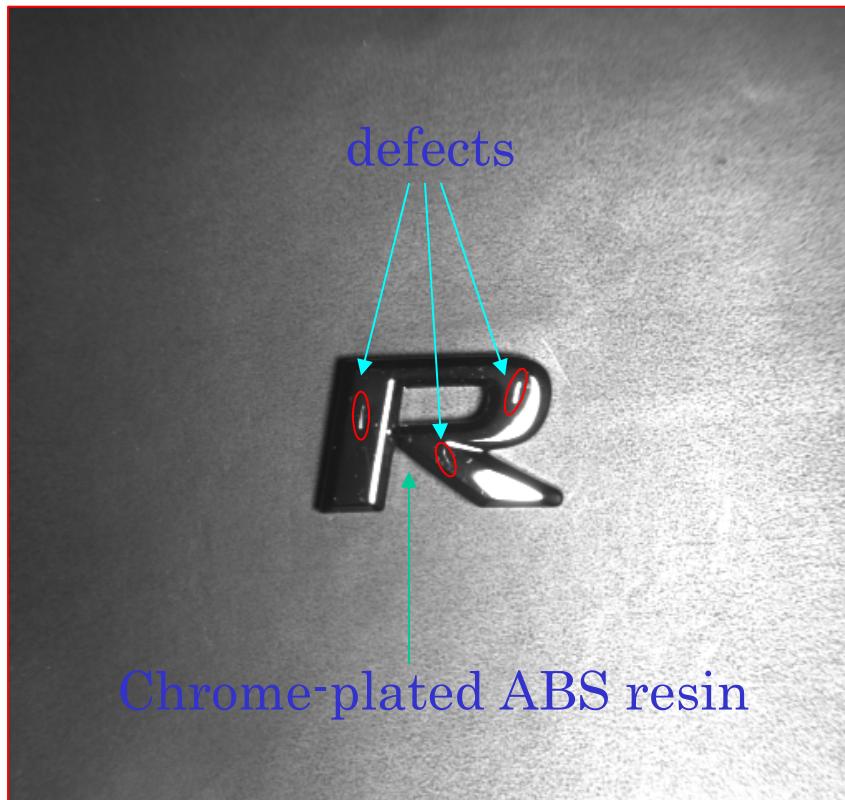
Automotive inspection is very difficult



The surface inspection of the product is almost done by human inspectors

Objective

The overall objective is to design a new automated inspection method of metallic surface.



Original image

Proposed Method

Images measuring



Reflection processing



Image filtering



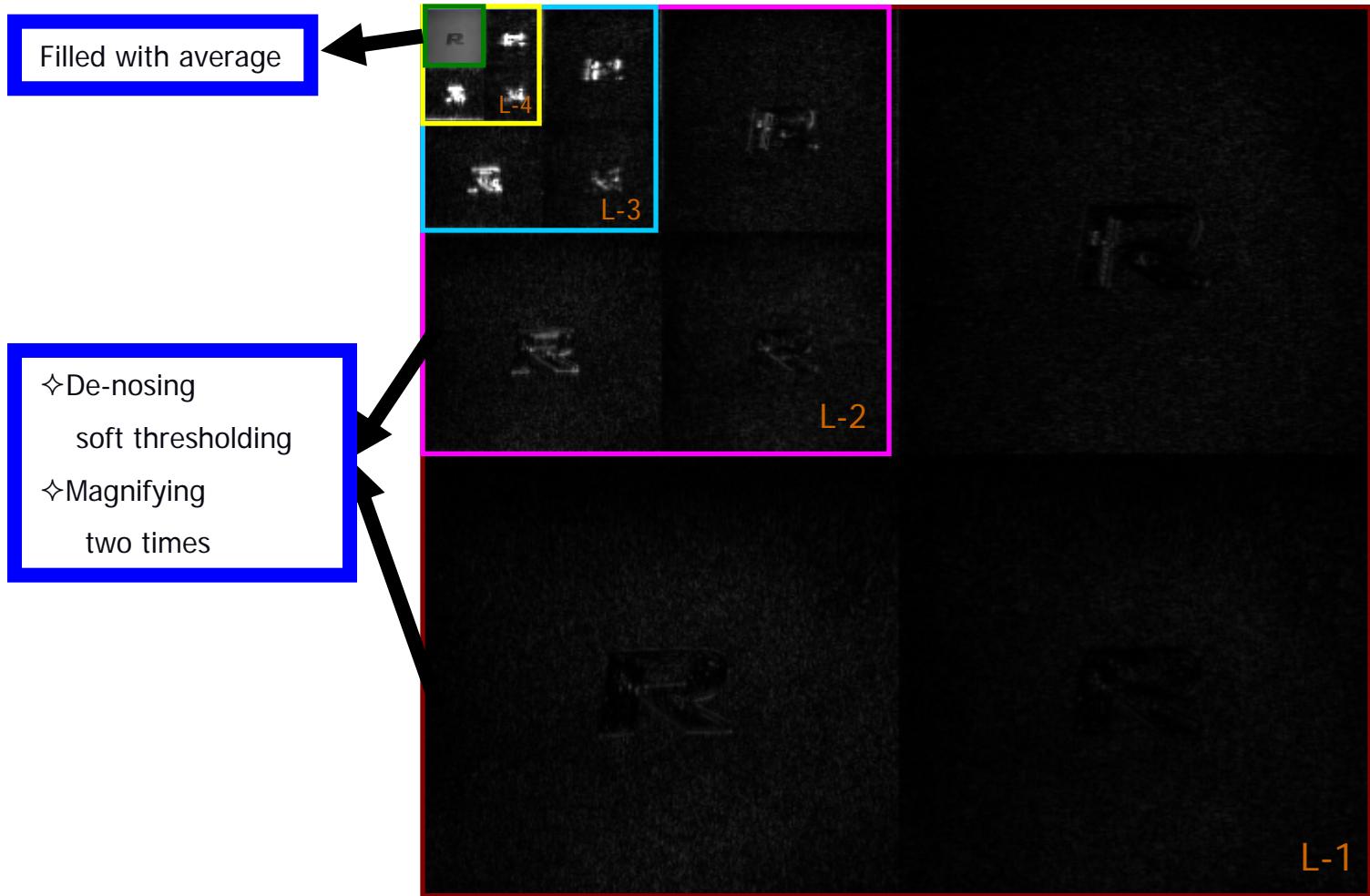
Defects detection



Note: using template matching method.

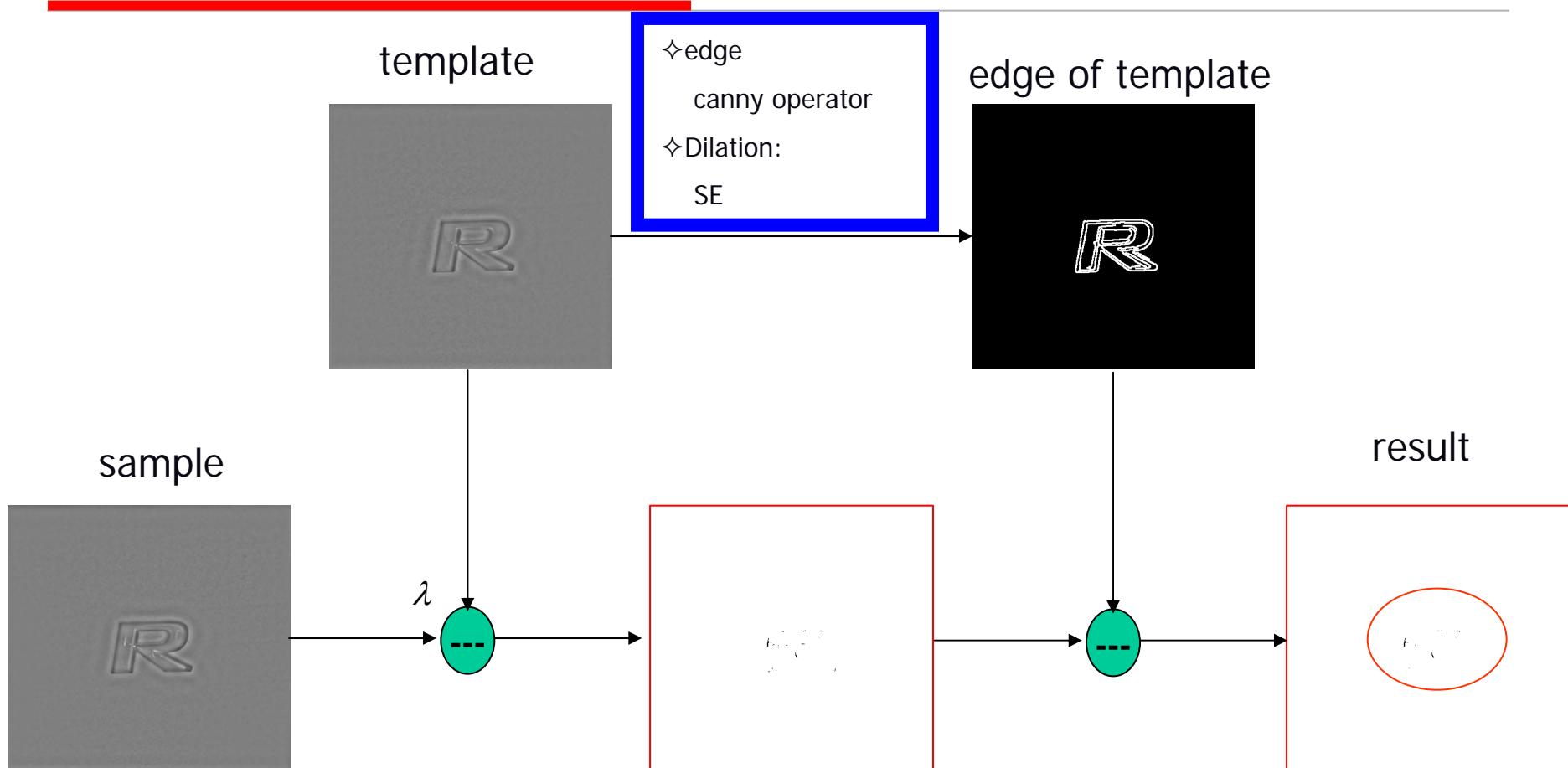
Image Filtering

processing by $m=4,3$ RI-Spline WT



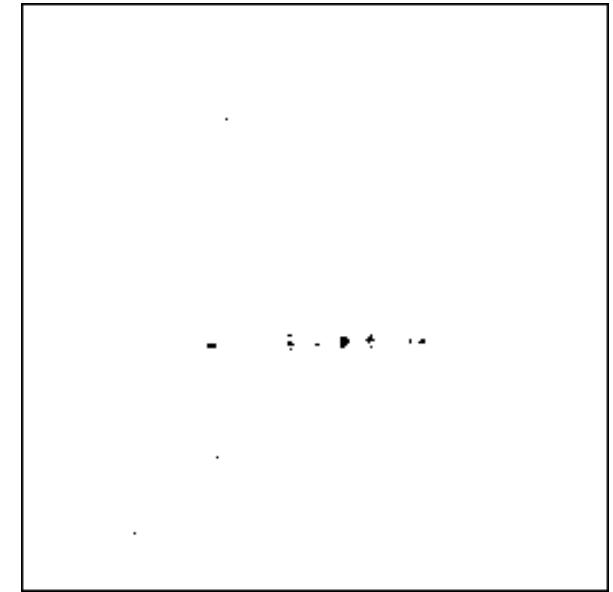
L-1

Defects Detection



Extraction of a cloth surface crack

Extracted surface crack



Cutting texture

5.4 Direction detection property of 2D-CDWT

1) Calculation method

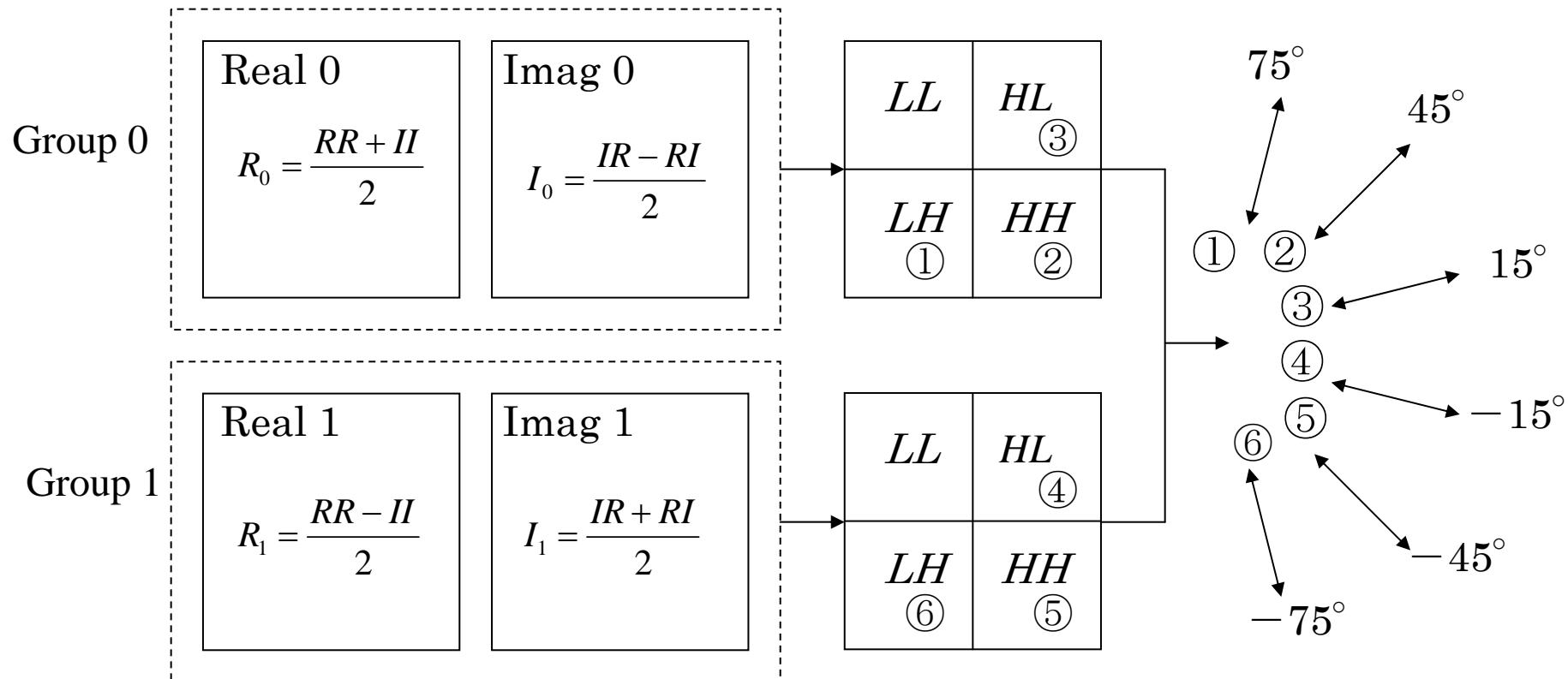
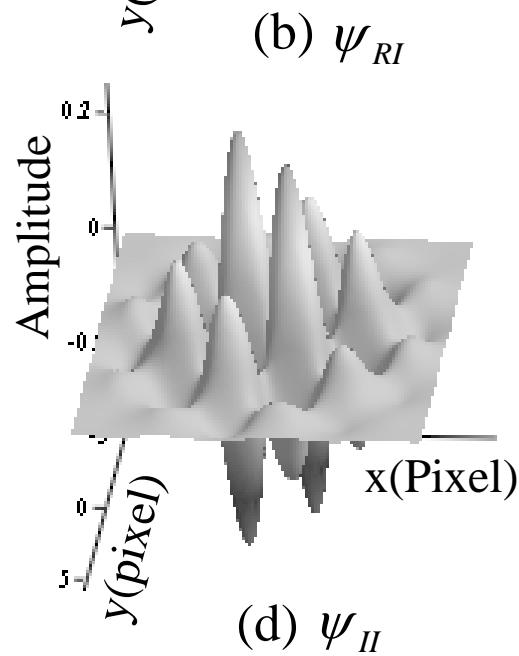
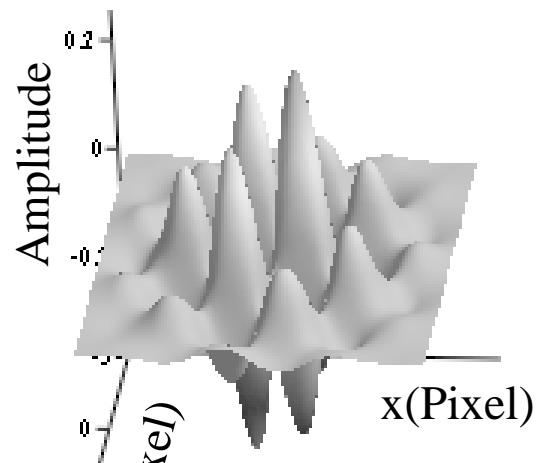
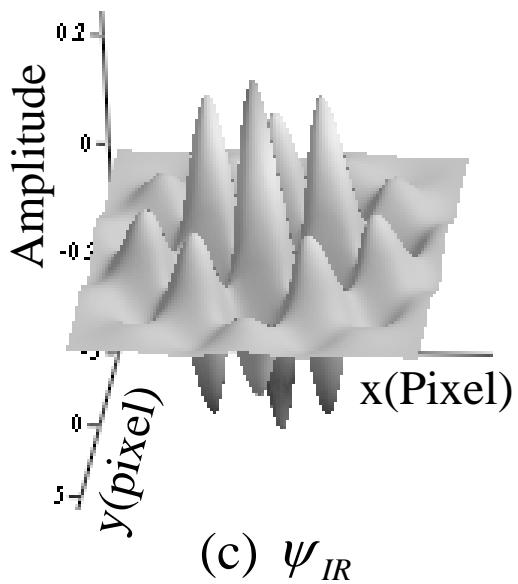
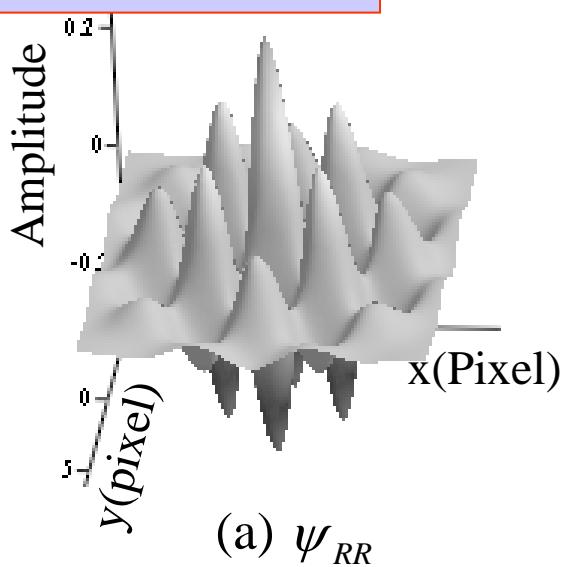


Fig. 3 Calculations of the Directional Selection

2D-Mother Wavelet



2D-Mother Wavelet with Direction selection

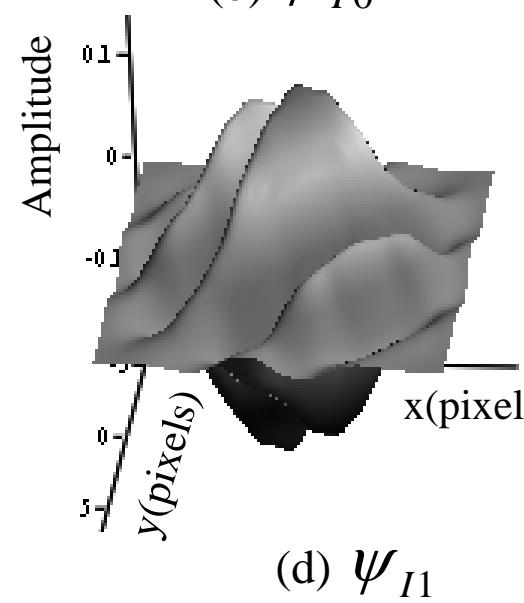
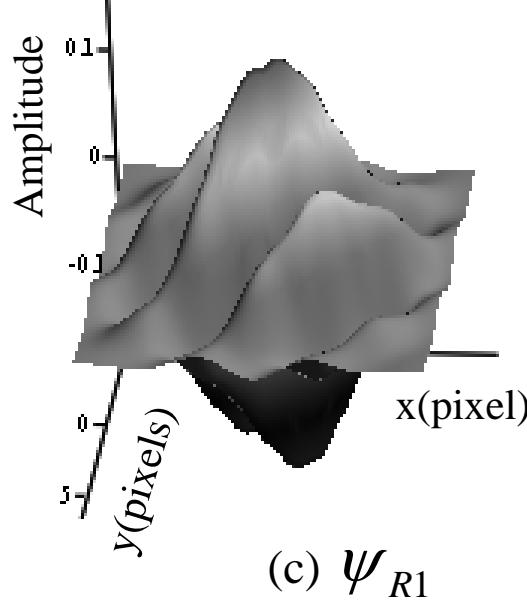
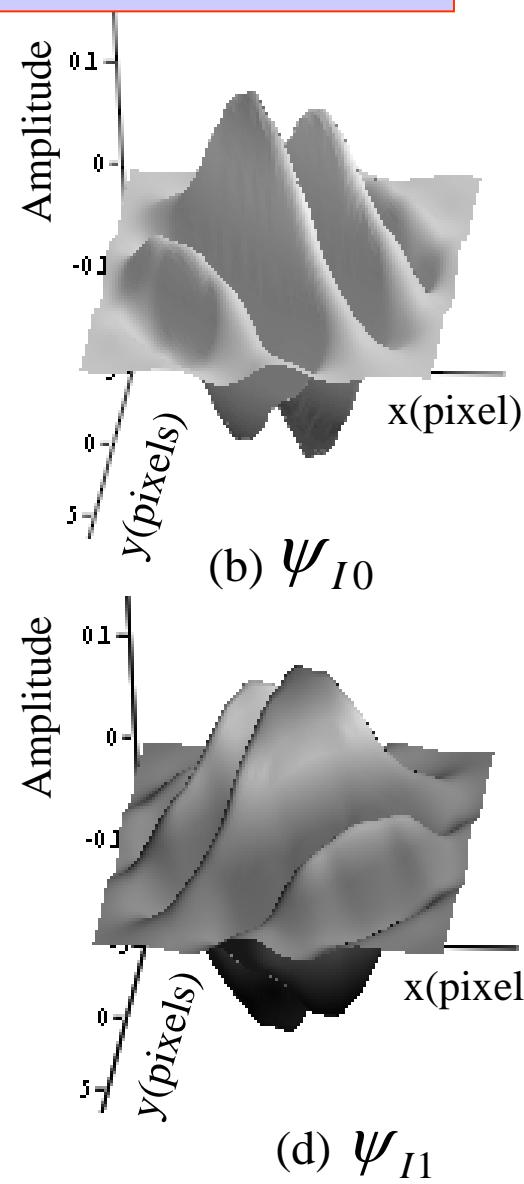
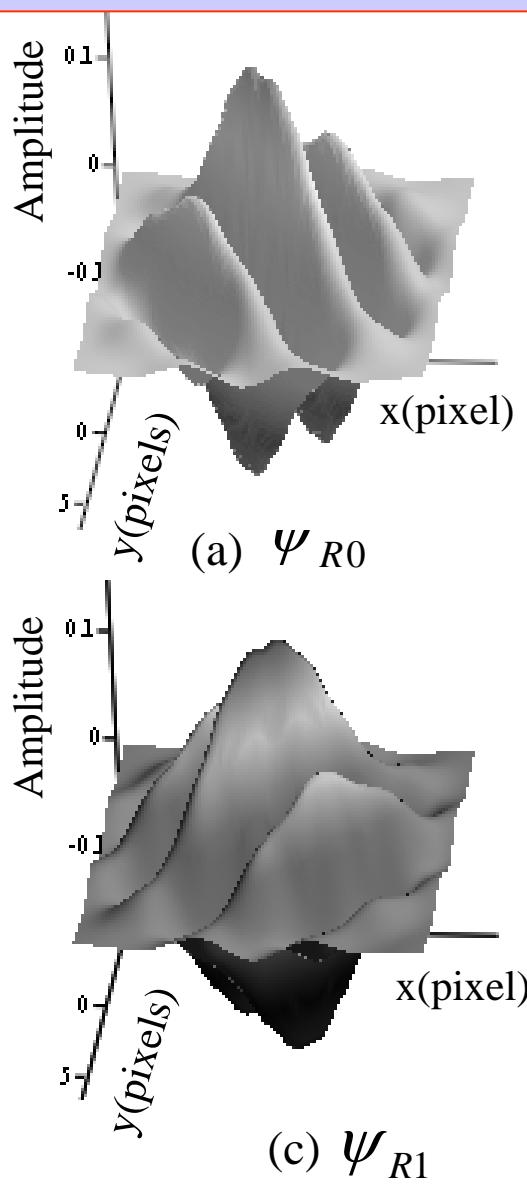


Fig. 4 The 2-dimentional wavelets on level -2 (45° , -45°)

Impulse responses

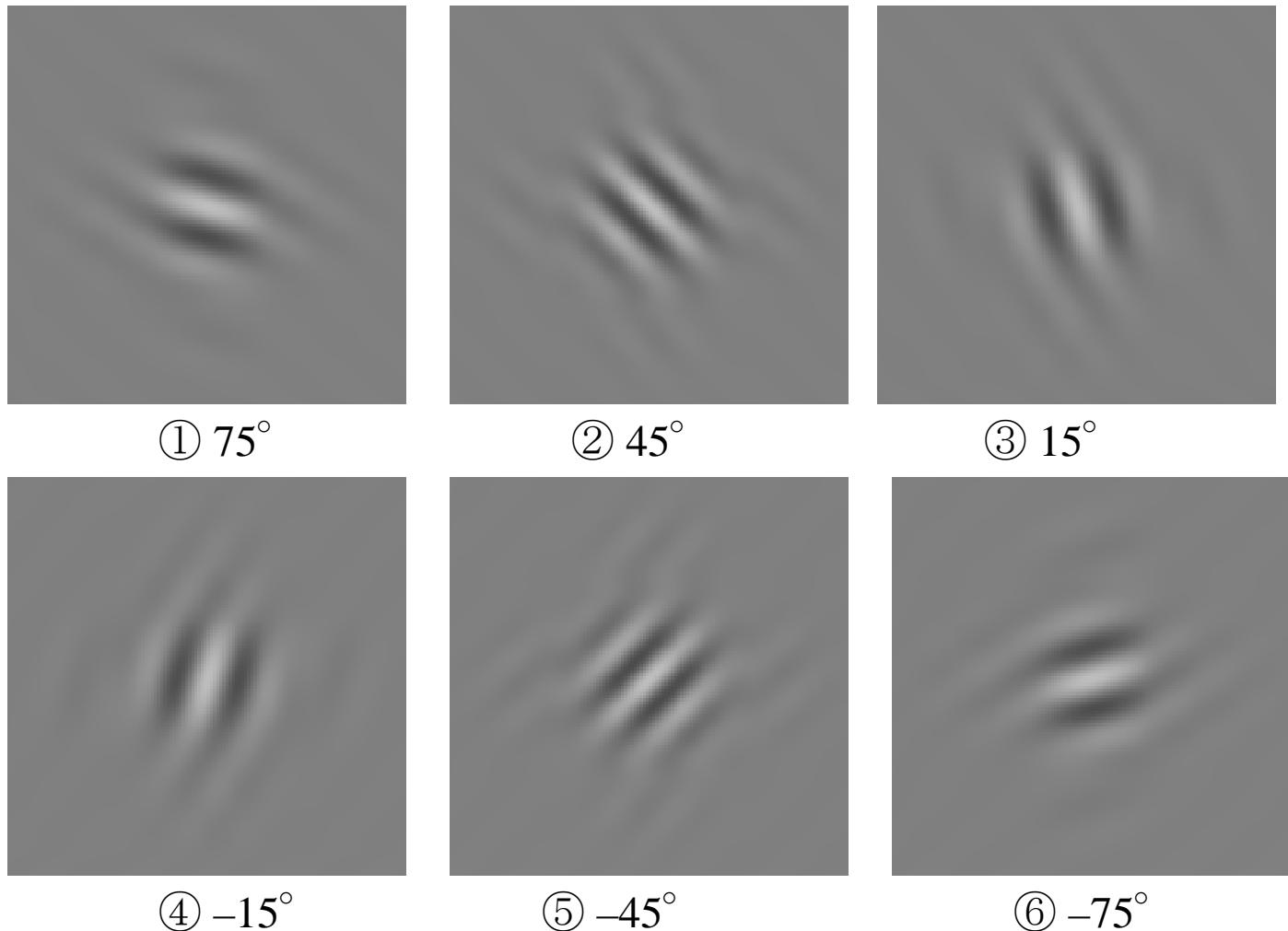


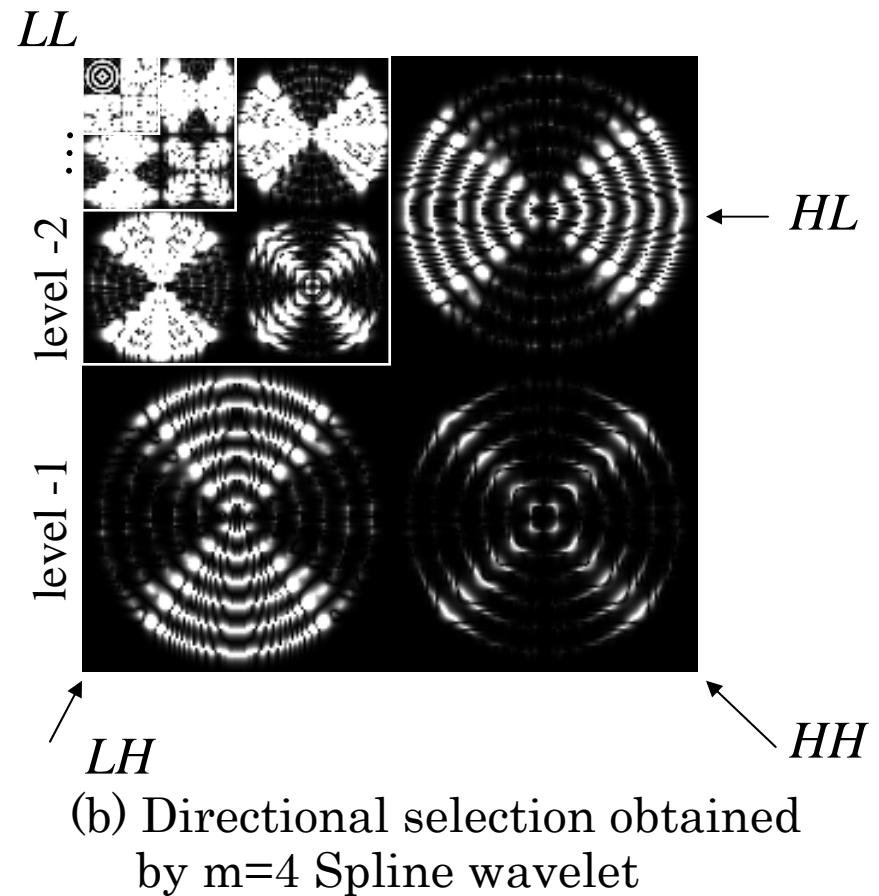
Fig.5 Impulse responses on level -4

2) Direction Selection Results

(1) Example of DWT



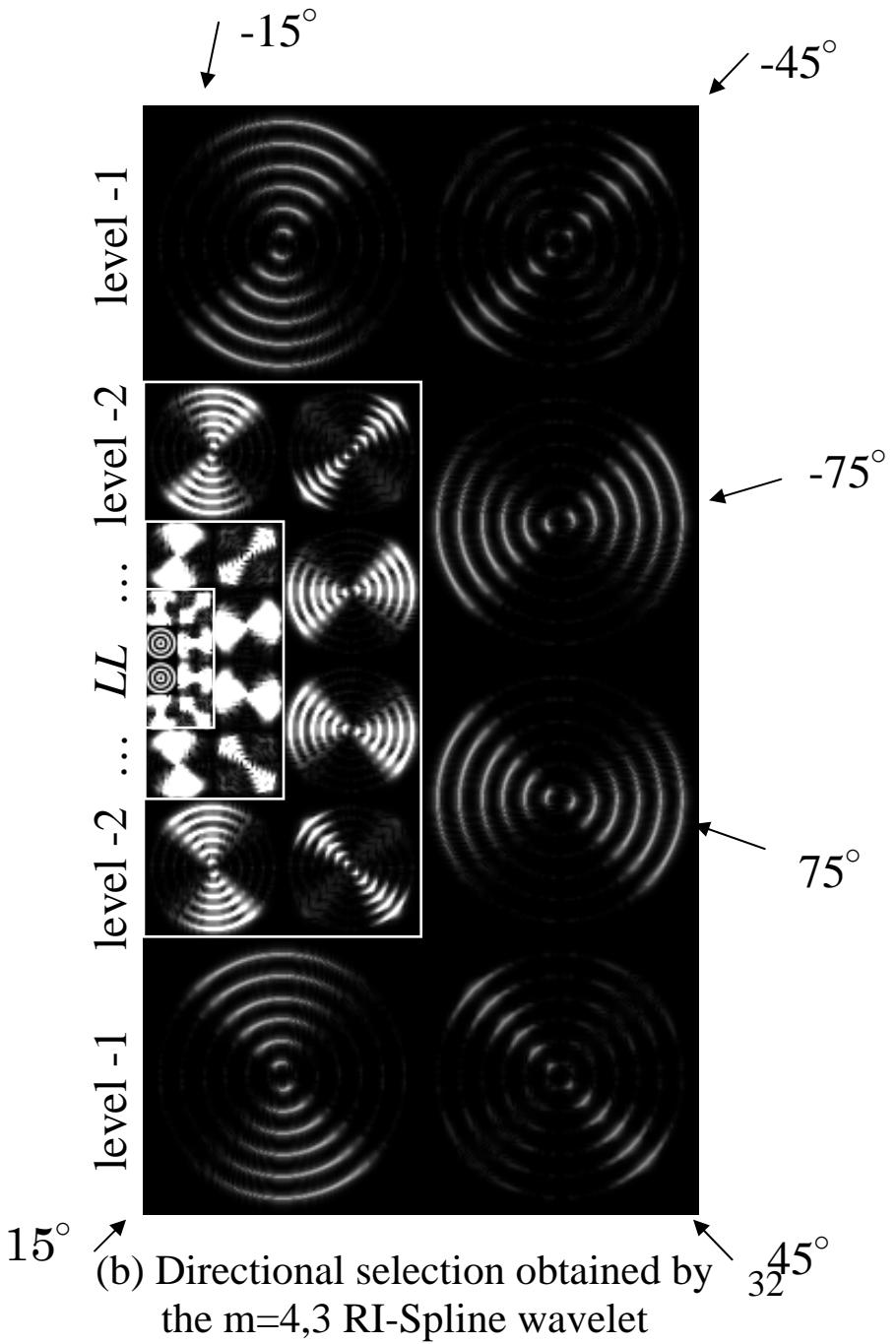
(a) Circle Image (256×256)



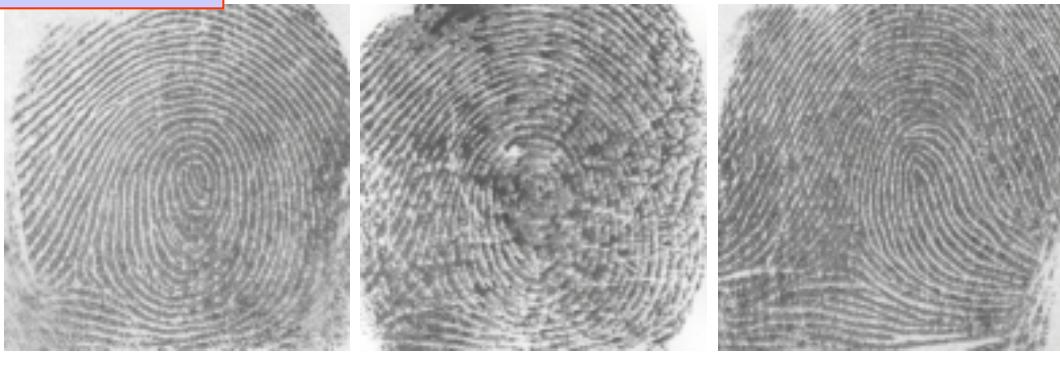
(2) Example of CDWT



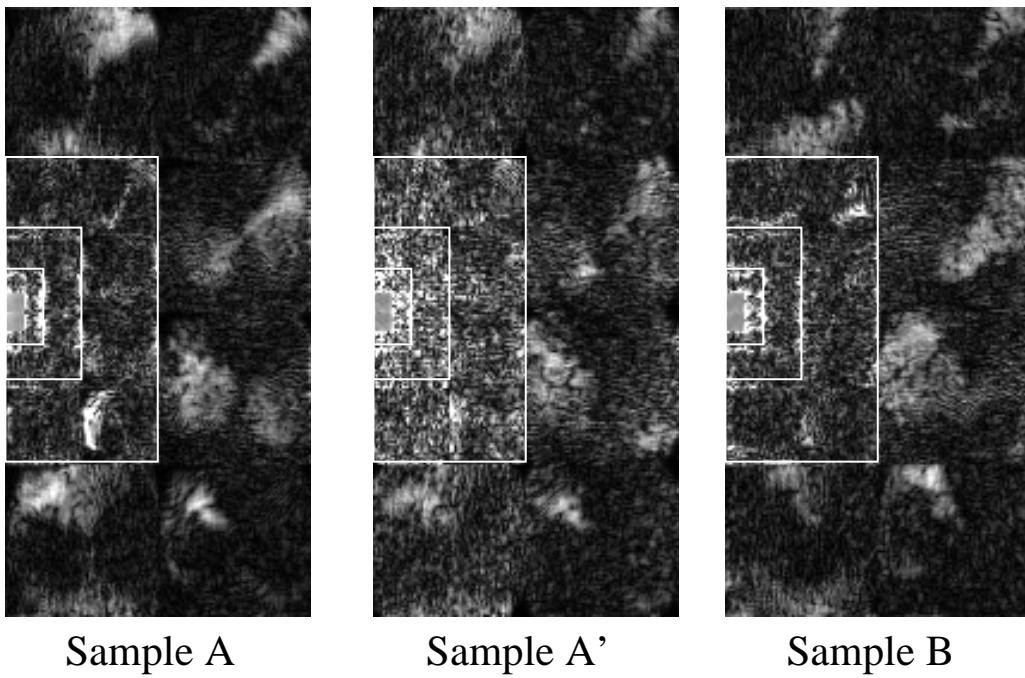
(a) Circle Image (256×256)



(3)Fingerprint extracting

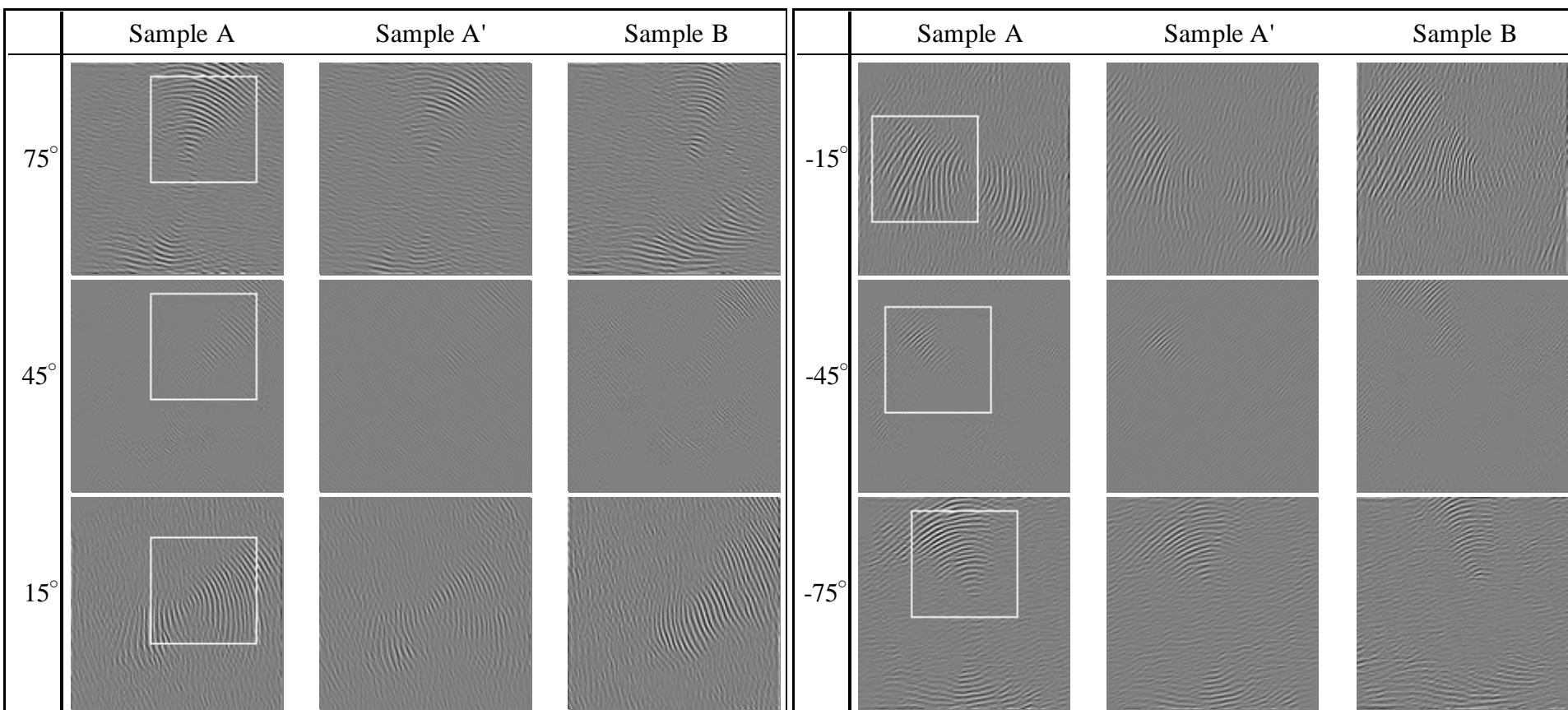


(a) Samples of fingerprints (128×128)



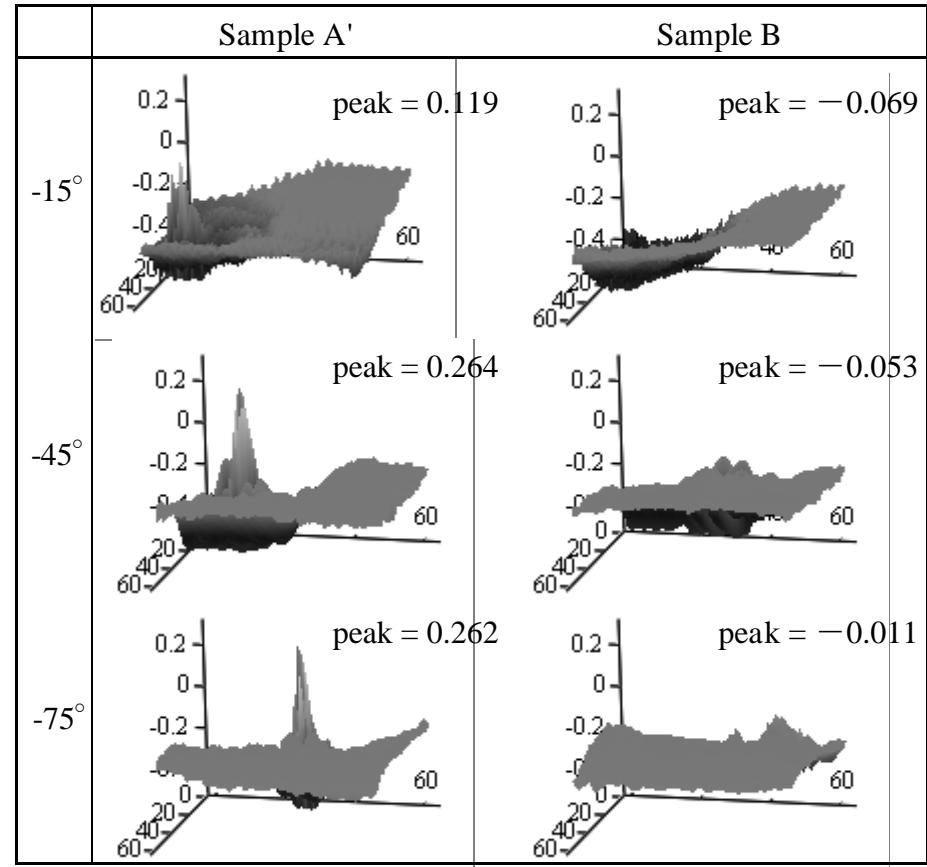
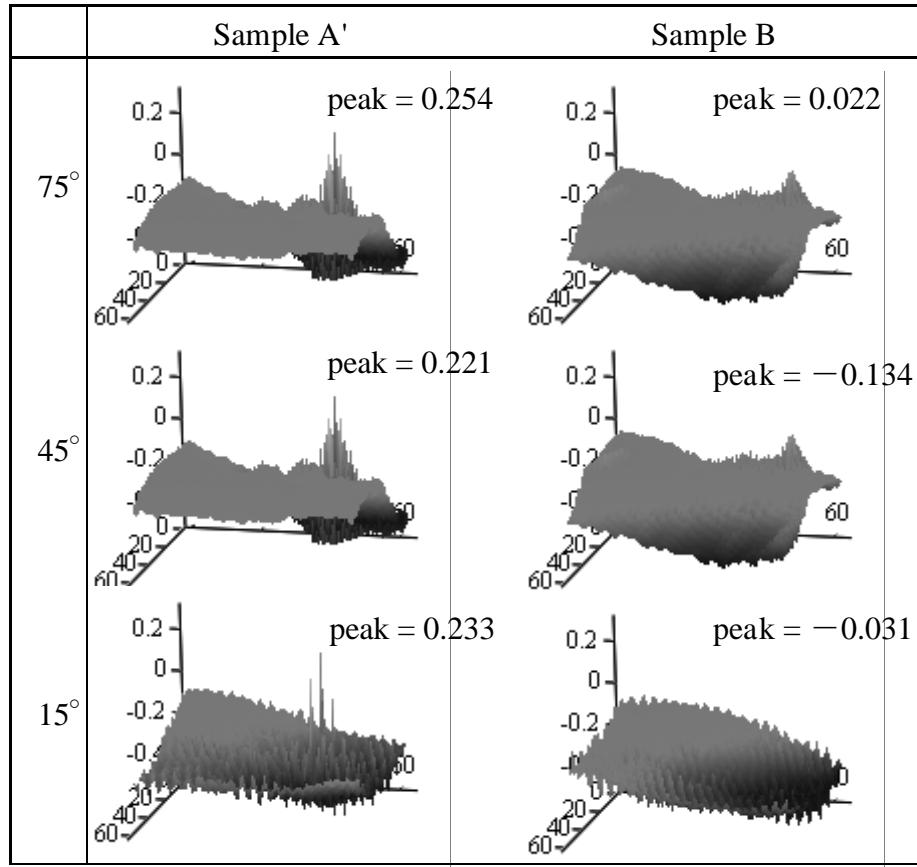
(b) CDWT on a scale of $1/2$
Fig. Analyzing results of fingerprints

Fingerprint analysis results by RI-Spline



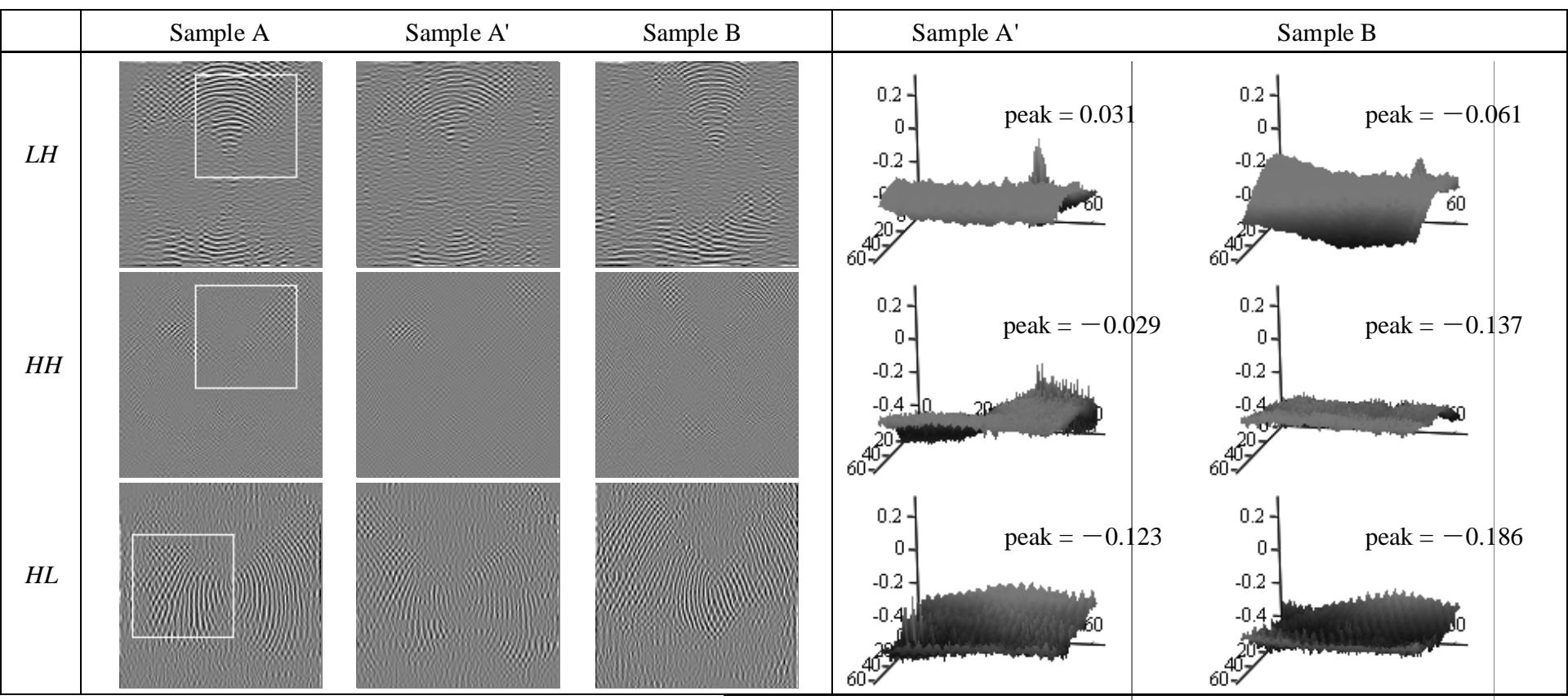
Results by RI-O-Spline wavelet

Detection results using RI-Spline



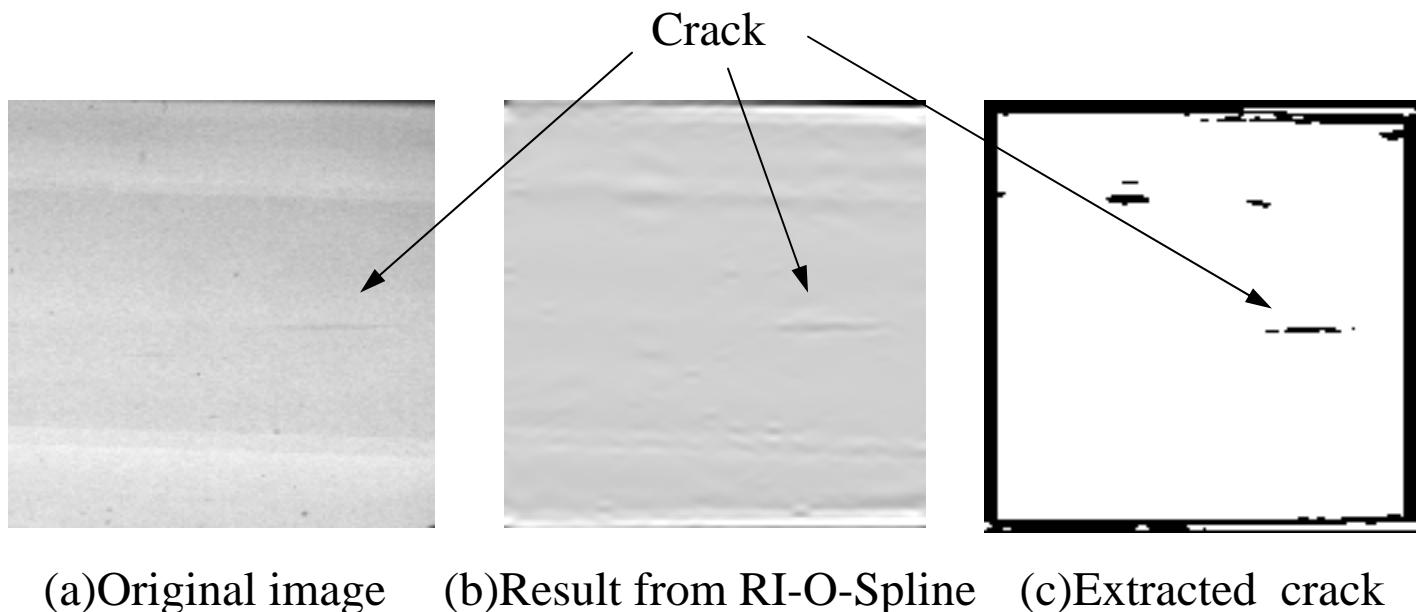
Results by using RI-O-Spline wavelet

Fingerprint analysis results by Spline wavelet



Results by O-Spline 3 wavelet

4) Extracting crack in a grass Braun table

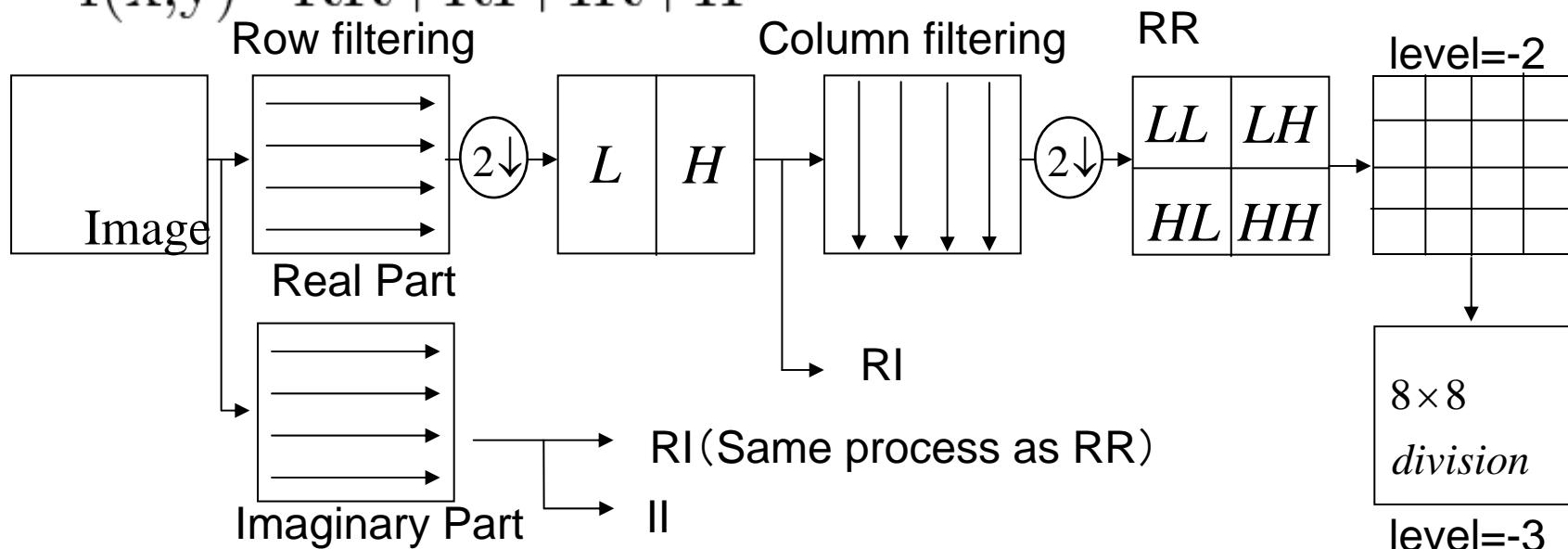


5.5 Direction detection by 2D-Wavelet Packet Transform

5.5 2D-Wavelet Packet Transform

- 2D-CWPT decompose each frequency component using filter

$$f(x,y) = RR + RI + IR + II$$



Direction selection

Each Wavelet
Coefficient
RR,RI,IR,II

LL	LH
HL	HH

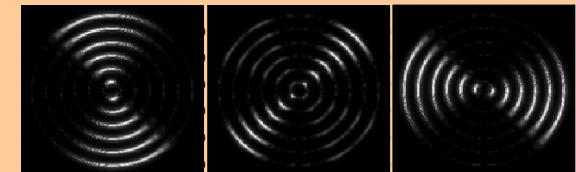
Group0

$$R_0 = \frac{RR + II}{\sqrt{2}}, I_0 = \frac{RI - IR}{\sqrt{2}}$$

Group1

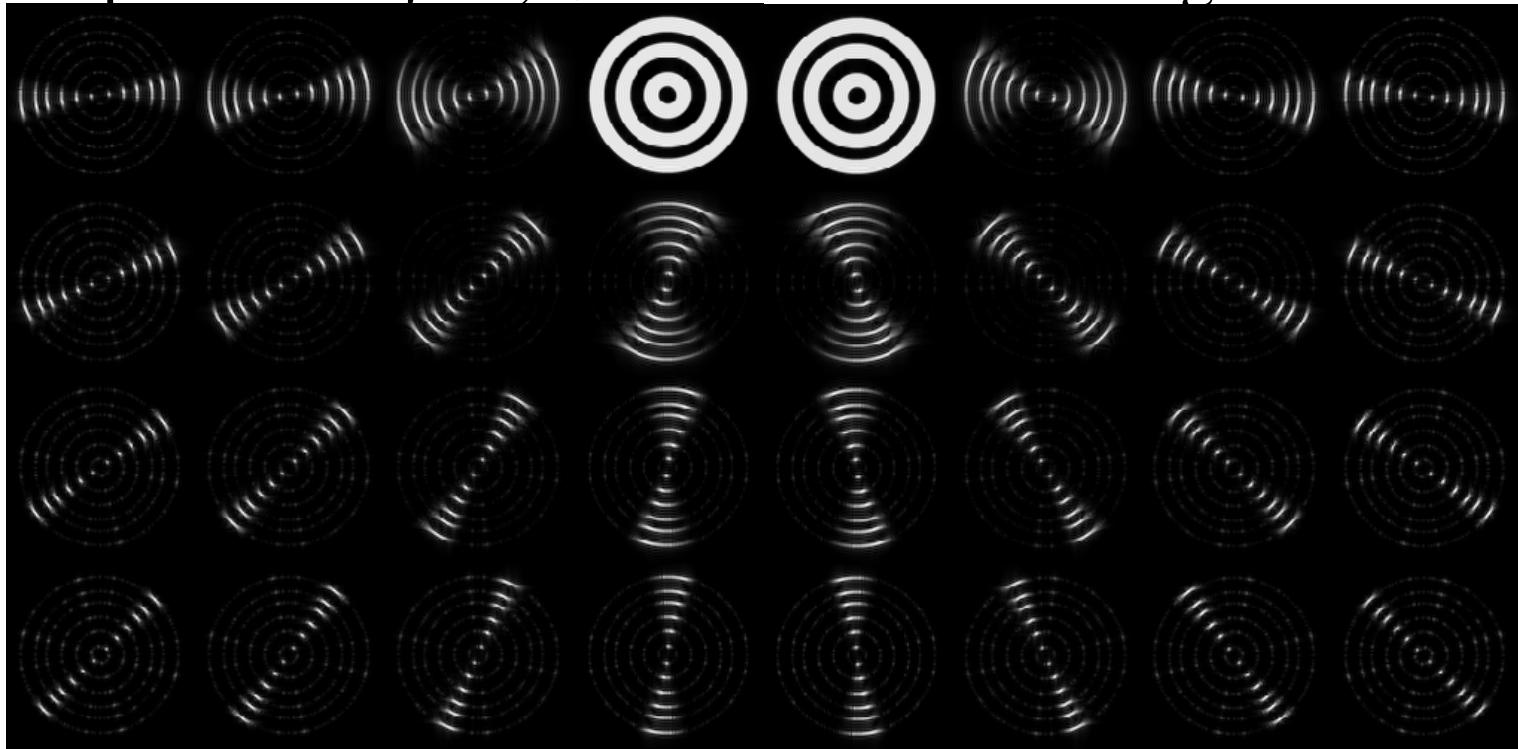
$$R_1 = \frac{RR - II}{\sqrt{2}}, I_1 = \frac{RI + IR}{\sqrt{2}}$$

This calculation can
Get each directional
component



Result of Direction Selection by 2D-CWPT

- 2D-CWPT leads 30 directions of edge
- Decompose level: $j=-3$, (this results shows range of $\omega < \pi/2$)

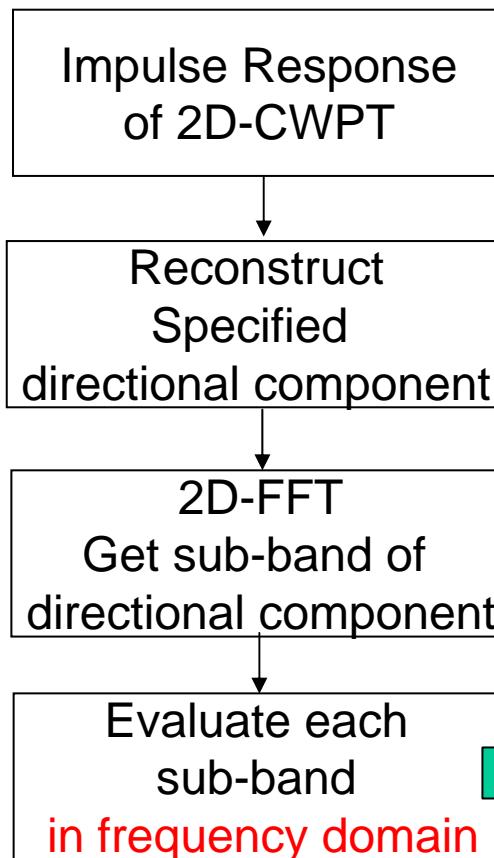


- Next step is that we **need to define the direction and resolution**
 - Because in previous research, definition of directions and resolutions **are NOT clear**

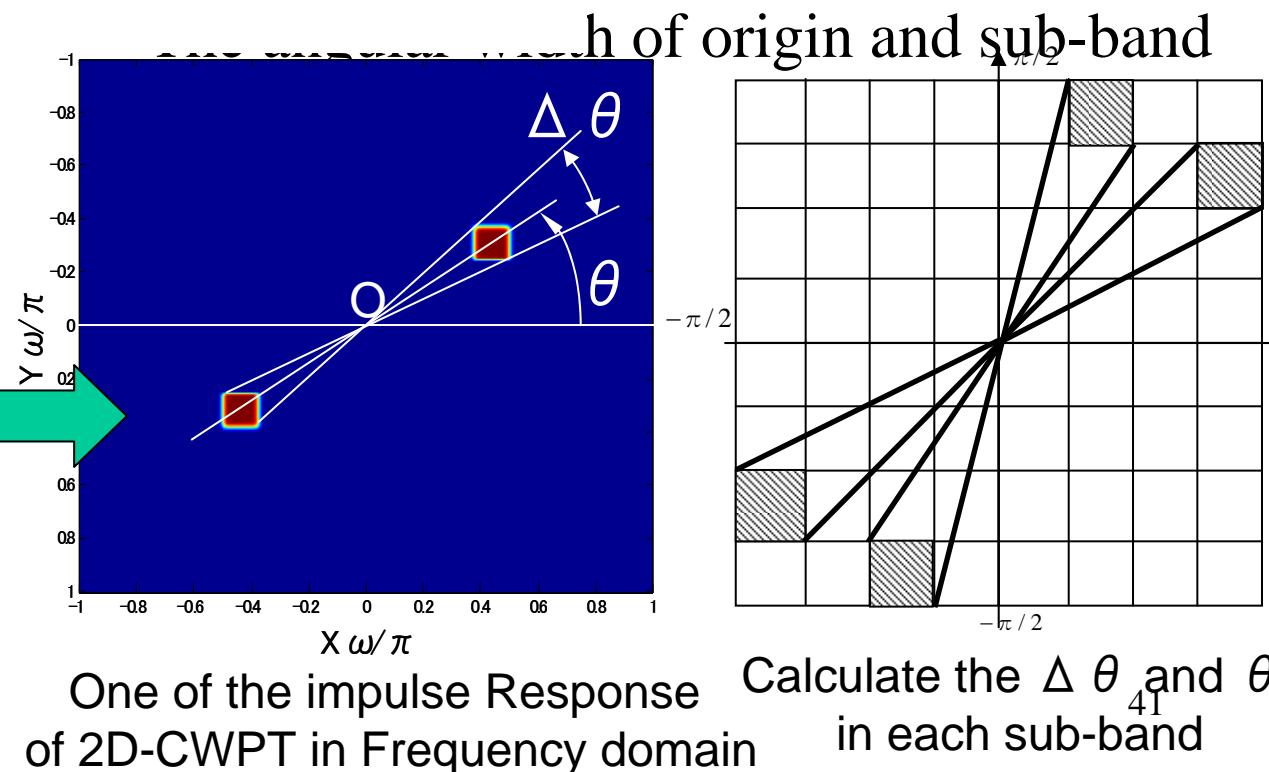


A New Evaluation Method

- So in order to make clear, we proposed new evaluation method.
 - This method uses impulse signal for result is not affected by model signal
 - **Definition of each parameter**

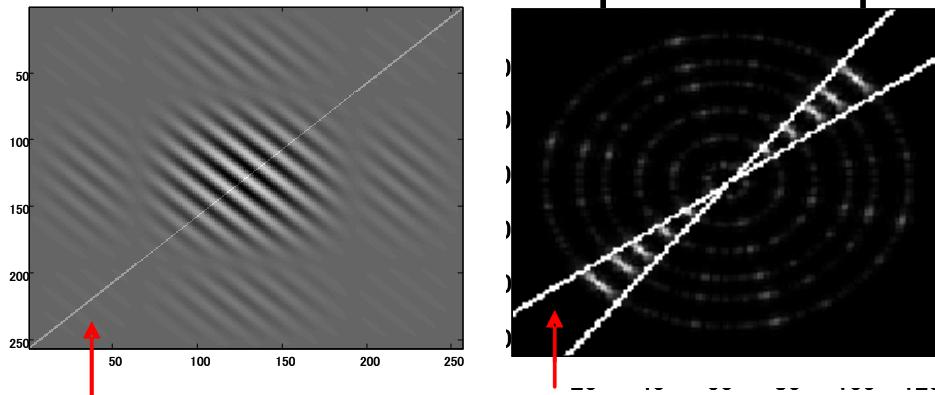


- Direction (θ):
The angle of origin and center of sub-band
- Resolution ($\Delta \theta$):



Result of evaluation

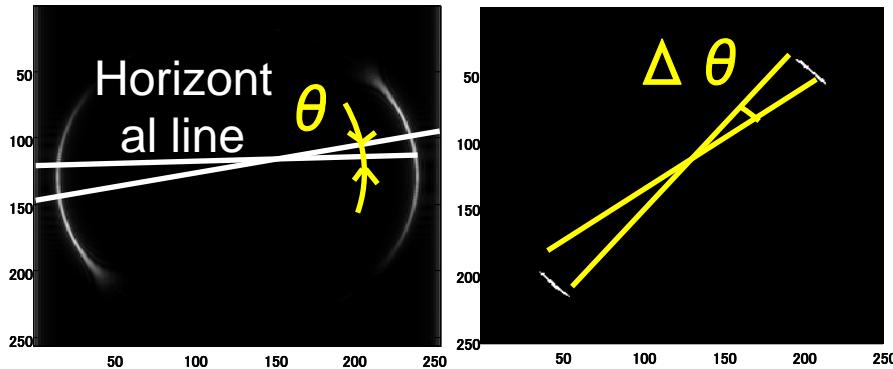
- Defined directions and resolutions are drawn in space domain with Impulse response and model image



Defined θ and $\Delta \theta$ capture the direction and resolution in impulse response and model image

Defined line (θ) Defined resolution ($\Delta \theta$)

- Comparison of factual instrument result

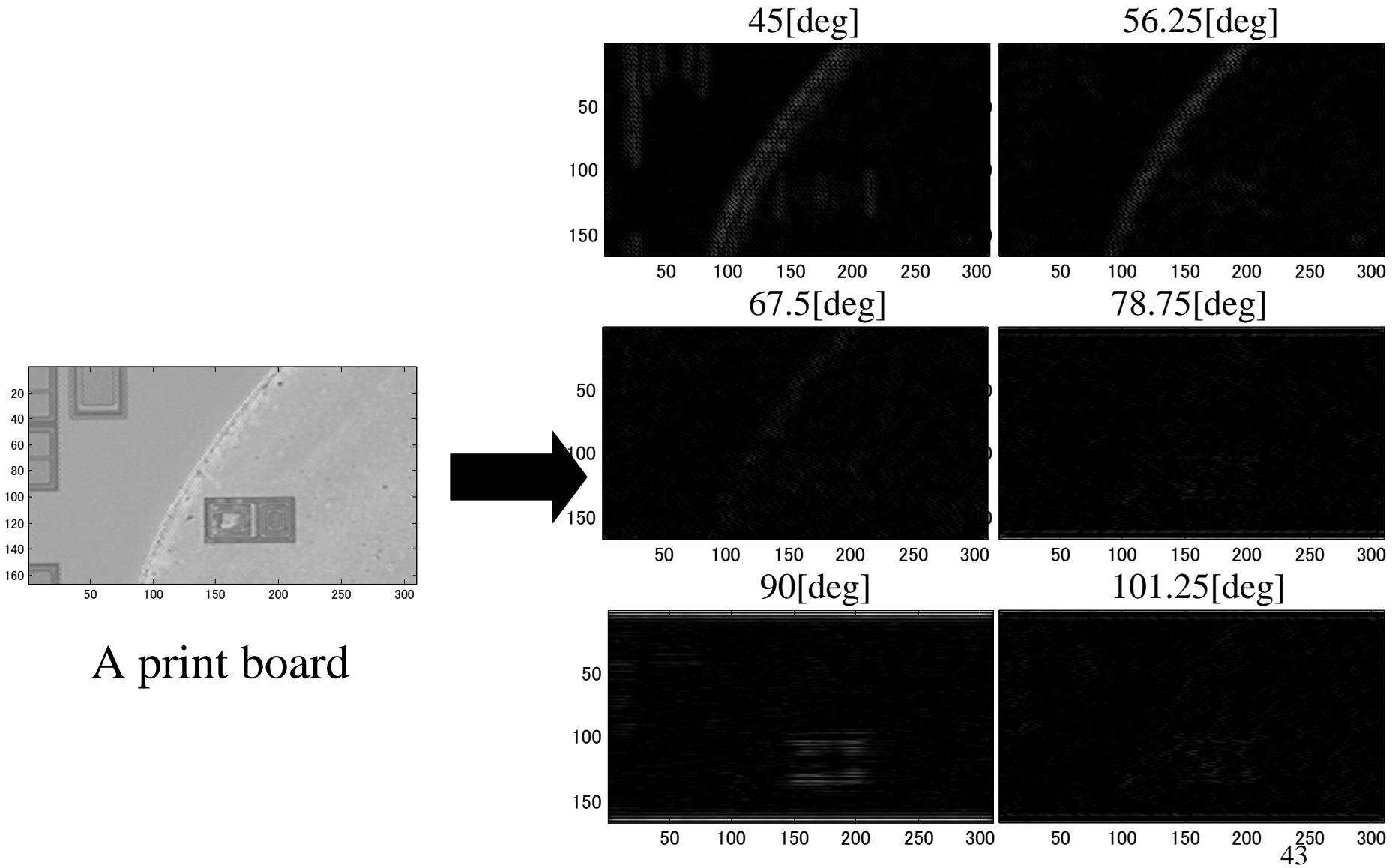


	Error of average
Direction	8.04%
Resolution	6.53[deg]

There is the error, but it also can get same tendency at factual instrument result

We confirm the validation of evaluation method

Example of extracting direction components



5. Image processing by Wavelet Transform

5.1 2D-Discrete Wavelet Transform

1) Calculation method 2) Characters

5.2 2D-Complex Wavelet Transform

**Calculation method, 2D-Mother Wavelets,
Translation invariance property of 2D-CDWT**

5.3 Application on de-noising

Model Image, Inspection of metallic surface

5.4 Direction detection

**Calculation method, Extracting fingerprint and
crack in a grass Braun table**