3.Contiunance Wavelet Transform

Instrumentation Systems Lab. Prof. Zhang Zhong

2. Signal Type and Analysis Method

2.1 Stationery Signal analysis Methods

2.1.1 Statistics Method

(1) Gaussian Distribution and parameters

(2) Evaluation of different from Gaussian distribution

2.1.2 Statistic Model

2.1.3.Spectrum Analysis

(1)Fourier Transform

(2) Maximum Entropy Method (MEM)

2.2 Non-stationary Signal Analysis Method

2.2.1 Short Time Fourier transform(STFT)

2.2.2 Continuance Wavelet Transform (CWT)

2.3 Example: sound source separation

Sound source separated By ICA and CDWT

Important point of last lesson

1) Signal Type and Analysis Method

Signal Type and Comparing Analysis Methods

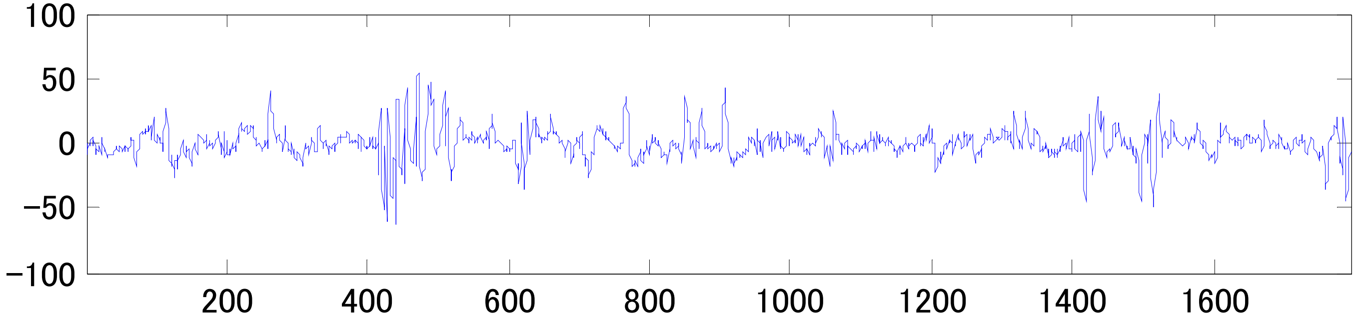
Method / Wave	Statistics Method	Statistics Model	Spectrum Analysis (FFT, MEM)	STFT, WT
Sine wave	○	○	◎	○
Compound cycle	○	○	◎	○
Steady（Ergodic ）	○	◎	○	○
non-Ergodicity	×	×	×	◎
Unsteady	×	×	×	◎

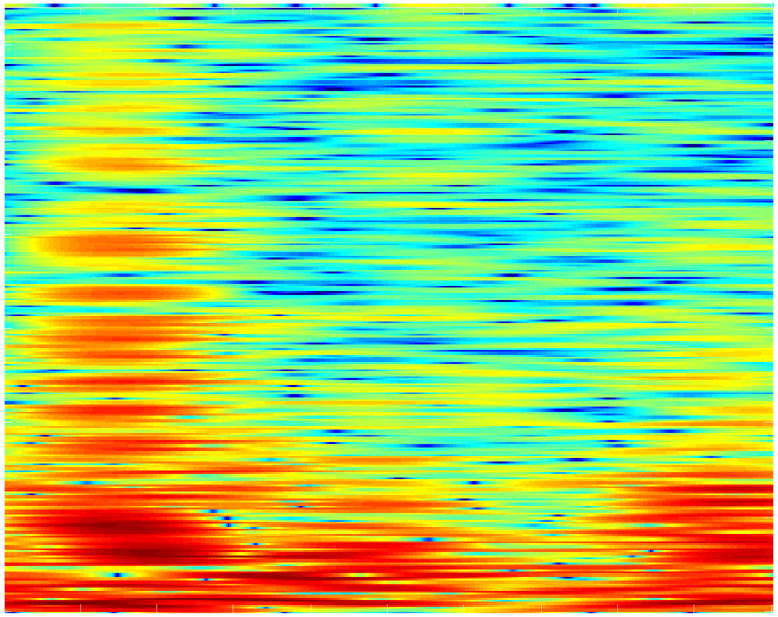
*Method adapted for unsteady signal analysis：

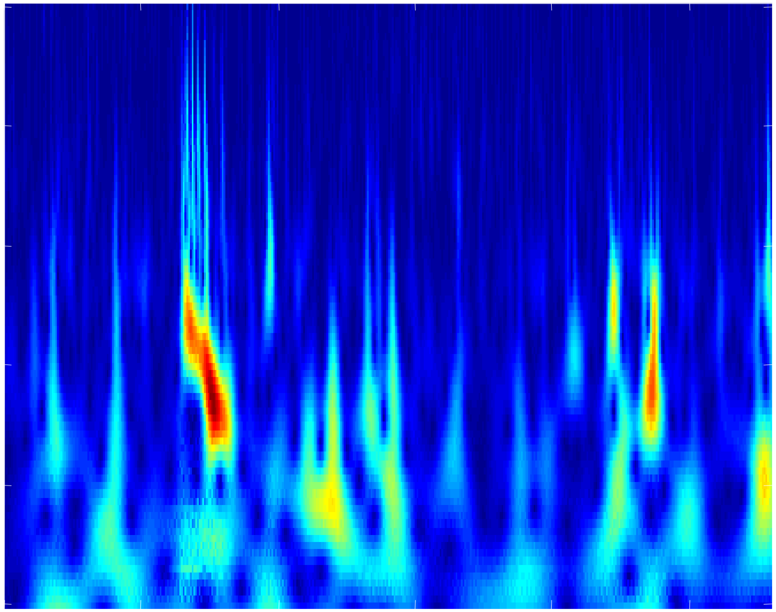
1) Short Time Fourier transform, STFT

2) Wavelet Transform,WT

2)Characteristics of the CWT and STFT







STFT N=512

CWT（Continuum Wavelet Transform）

3.1 Introducing Wavelet Transform

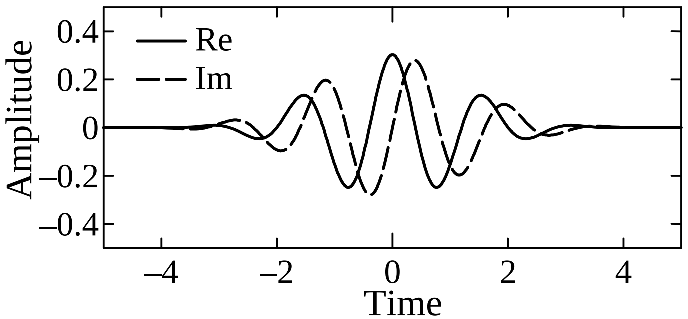
3.1.1 Continuance Wavelet Transform (DWT)

1) Definition of the CWT

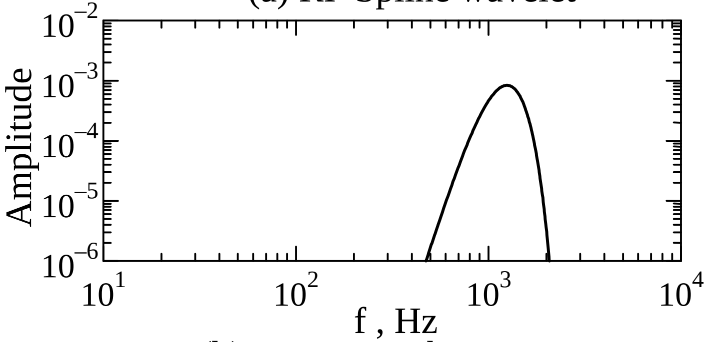
$$w(a,b) = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother Wavelet (MW)
a : Scale ($1/a$ Frequency)
b : Time



(a) RI–Spline wavelet

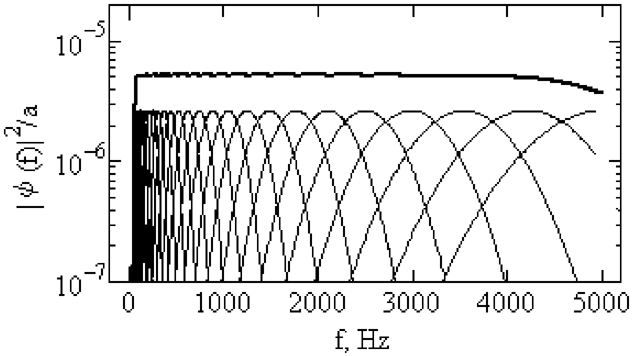


(b)Frequency characteristic

2) Features of the CWT

Advantages:

- Transforming signal to time-frequency plane and making its nature clearly (illustrating)
- Time and frequency resolution can be changed with frequency



(b) Basis of CWT for 6 octave, 4 divided

Problems:

- Redundancy bases
- How do select Mother wavelet?
- There isn't a common fast algorithm for calculation.

3) Example of the CWT

$$w(a,b) = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother Wavelet (MW)
a : Scale (1/a Frequency)
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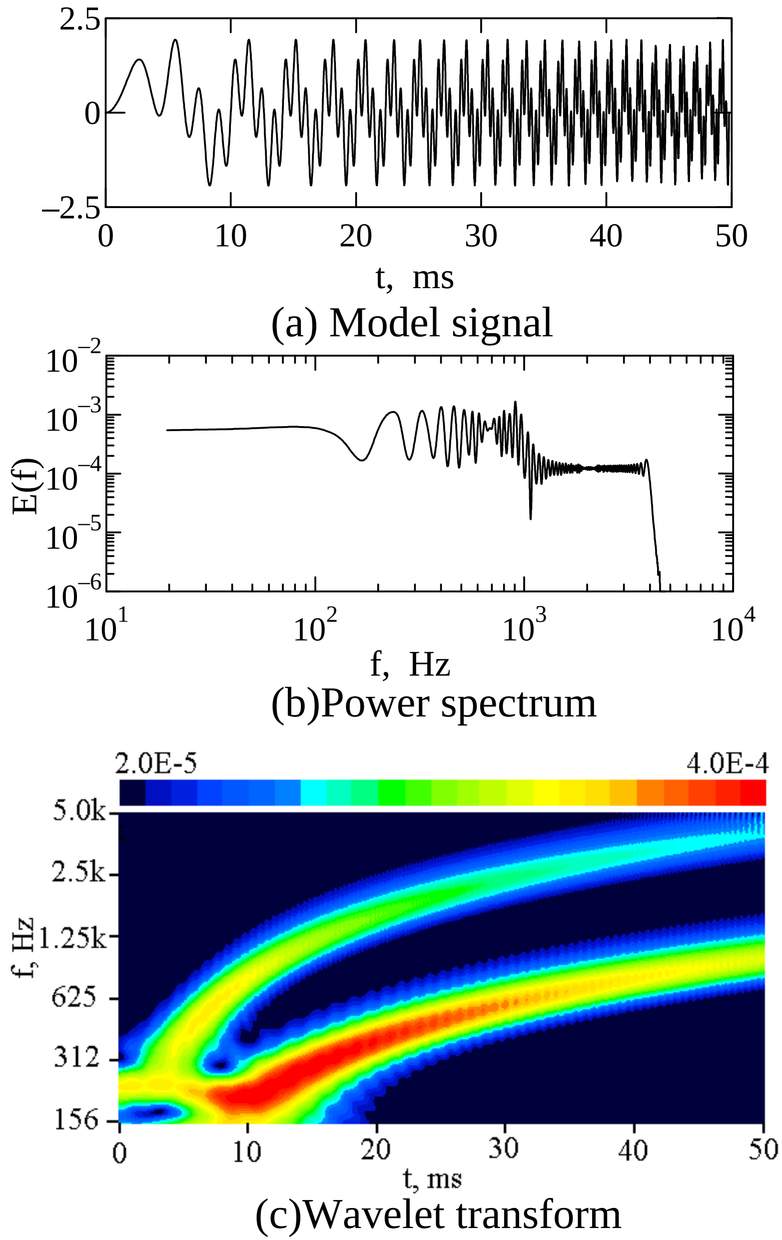


Fig.1 Continuance Wavelet Transform of Model Signal

3.1.2 Discrete Wavelet Transform (DWT)

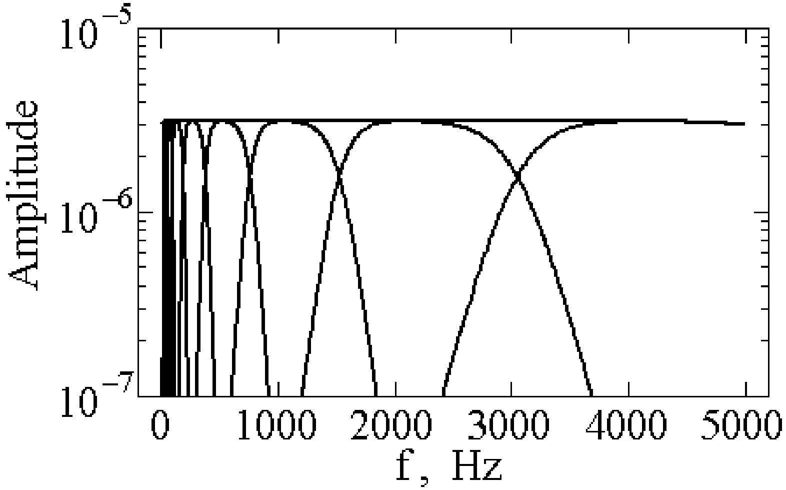
1)Definition of the DWT

$$d_k^j = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{j,k}(t)dt$$

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$$

$$a = 2^j$$

$$b = k2^j$$



(a) Basis of discrete wavelet Tansform

It is octave analysis so bases are not overlap (Non-redundancy)

2)Fast algorithm Based on MRA (by Mallat)

$$d_k^{j-1} = \sum_k a_{l-2k} c_k^j$$

$$c_k^{j-1} = \sum_k b_{l-2k} c_k^j$$

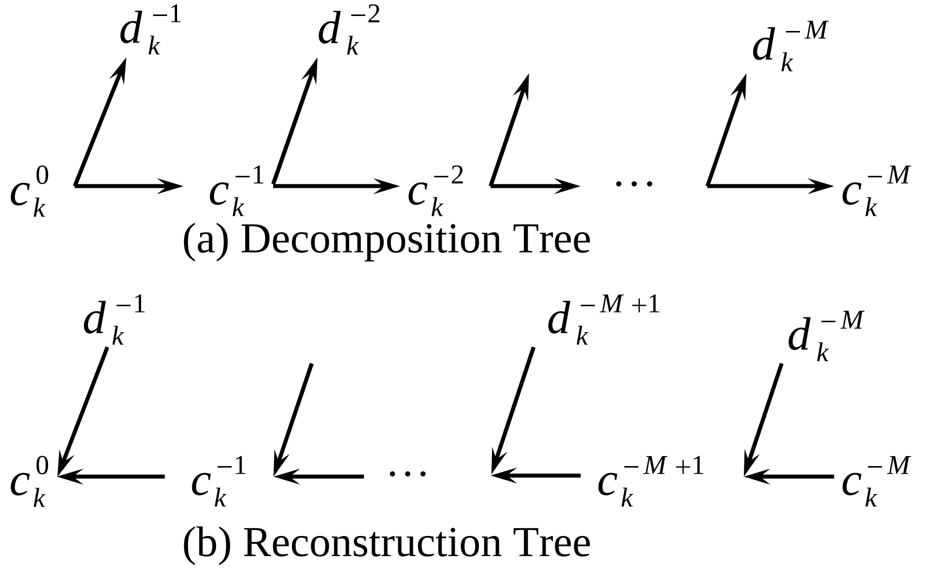


Fig.2 Tree algorithm of MRA

3) Features of the (Real) DWT

Advantages:

- **Good compression of signal energy.**
- **Perfect reconstruction with short support filters.**
- **Non-redundancy. Very low computation cost, order-*N* only.**

Problems:

- Severe shift dependence.
- Poor directional selectivity
  in 2-D,3-D etc.

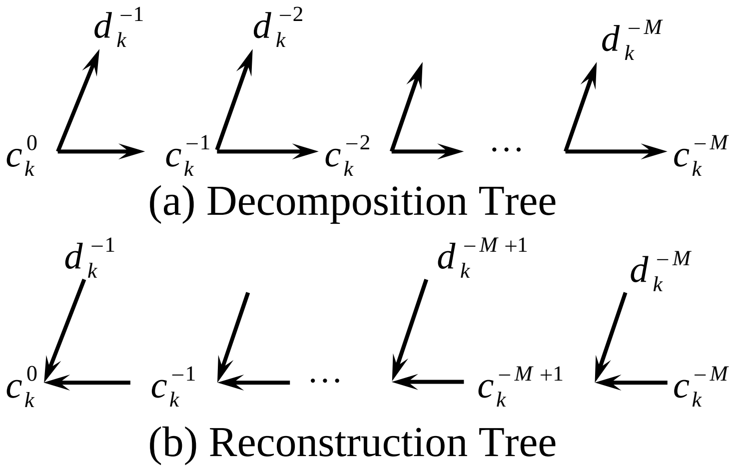


Fig.2 Tree algorithm of MRA

The DWT is normally implemented with a tree of high pass and low pass filters, which proposed by Mallat (fast algorithm).

4) Example of the DWT

$$d_k^j = \int_{-\infty}^{\infty} f(t)\overline{\psi}_{j,k}(t)dt$$

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^j t - k)$$

j: Level
k: Time

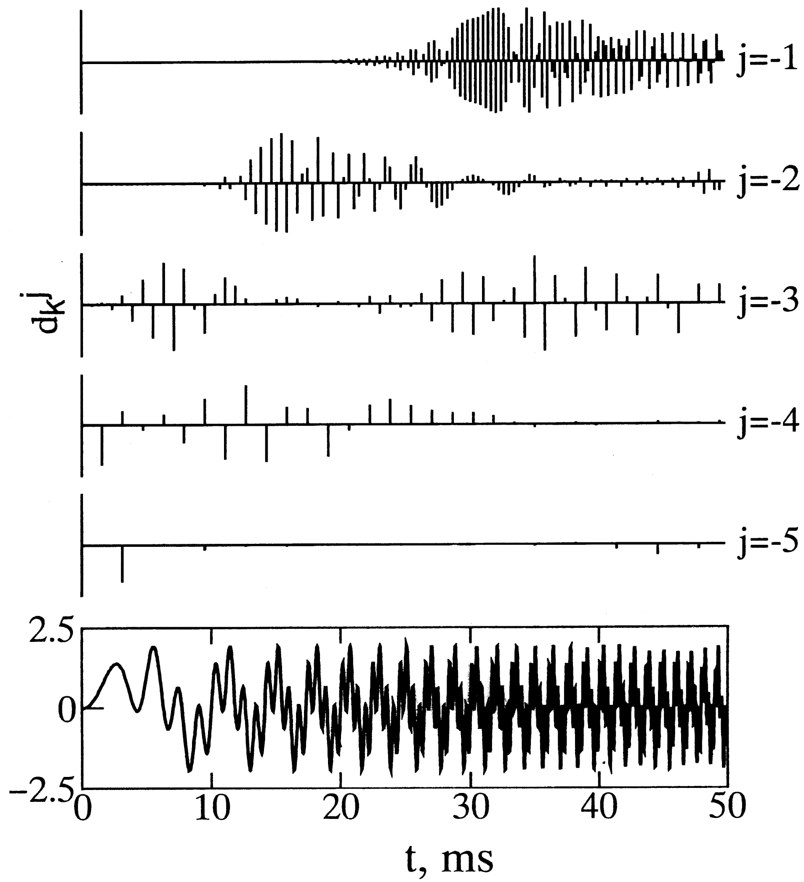


Fig.3 Discrete Wavelet Transform of Model Signal

3.2 Selection of MW for the CWT

3.2.1 Condition of MW selection

$$w(a,b)=\int_{-\infty}^{\infty} f(t)\overline{\psi}_{a,b}(t)dt$$

$$\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother Wavelet (MW)
a : Scale (1/a Frequency)
b : Time

Condition of The MW (admissibility condition):

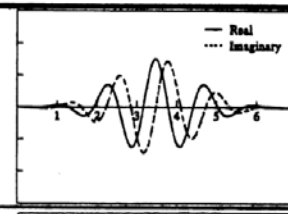
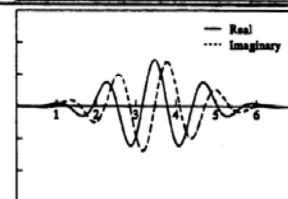
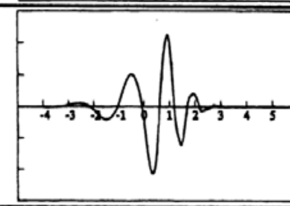
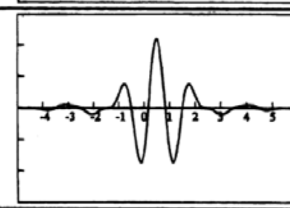
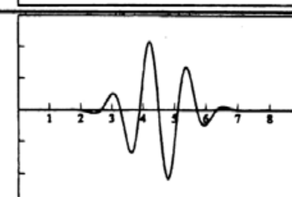
$$C_\psi=\int_{-\infty}^{\infty}\frac{|\hat{\psi}(\omega)|^2}{|\omega|}d\omega<\infty$$

This condition can be sampled as following Equation:

$$\boxed{\int_{-\infty}^{\infty}\psi(t)dt=0}$$

So many MW have been proposed and how do select them be comes problem.

Example of the MW

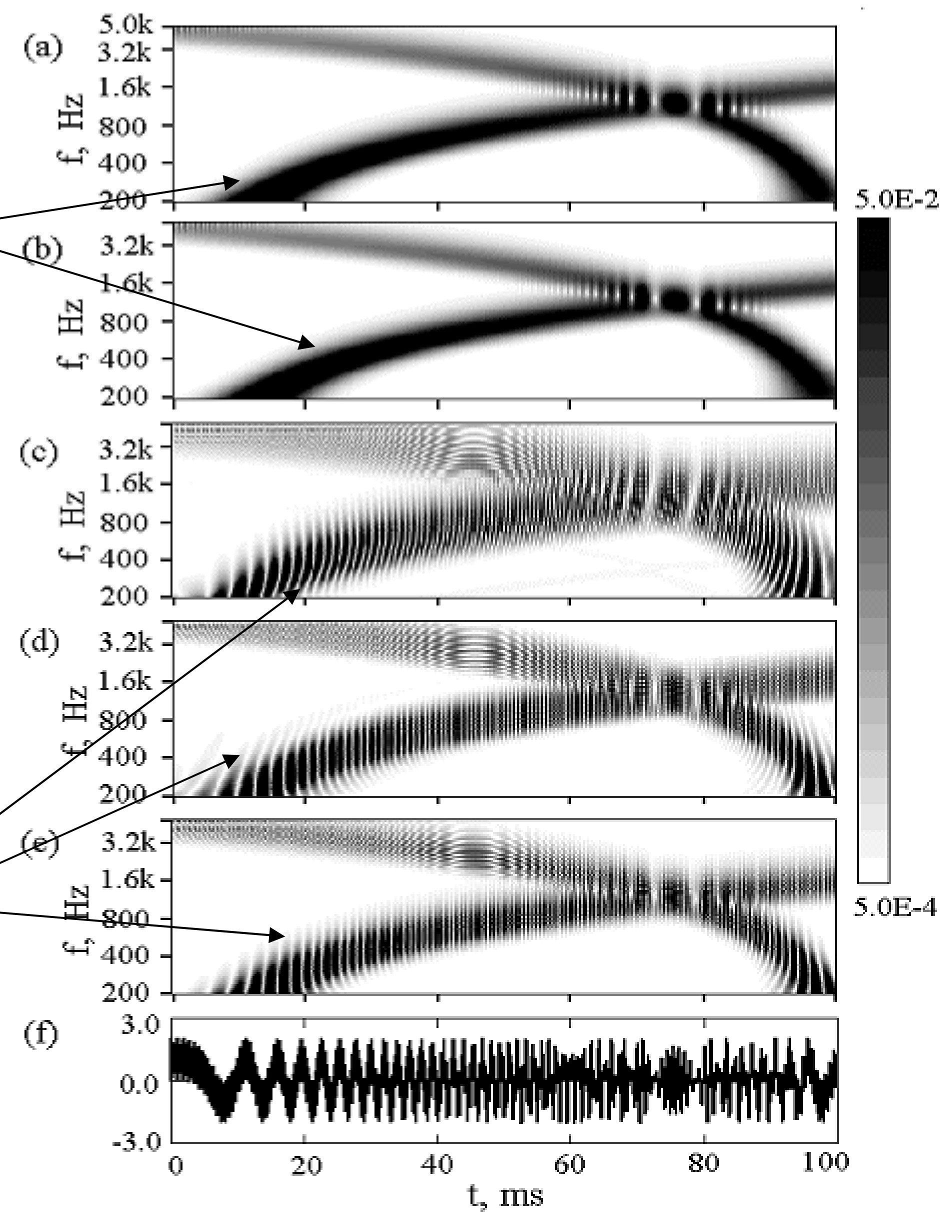
Mother Wavelet	Composition	Characteristic
Gauss type	$\psi(t) = exp(\frac{-t^2}{2})[exp(i\Omega t) - exp(-\frac{\Omega^2}{2})]$ $\Omega = 2\pi\xi$	 $\Delta t\Delta\omega$=0.5001
Gabor	$\psi(t) = \pi^{-1/4}(\omega^*/\gamma)^{1/2}exp[-(t\omega^*/\gamma)^2/2 - i\omega^* t]$ $\omega^* = 2\pi f^*$ $\gamma = \pi(2/ln2)^{1/2} \approx 5.336$	 $\Delta t\Delta\omega$=0.5002
Daubechise(6)	$\psi(t) = \sum_{k=0}^{2N-1}(-1)^k \bar{p}_{-k+1}\phi(2t-k)$ $\begin{cases} \phi(t) = \lim_{n\to\infty}\phi_n(t) \\ \phi_n(t) = \sum_{k=0}^{N} p_k\phi_{n-1}(t-k), n = 1, 2, \cdots \\ \phi_0(t) = N_2(t) \end{cases}$	 $\Delta t\Delta\omega$=0.9055
Meyer	$H(\omega) = \begin{cases} exp(1/\omega^2), & \omega > 0 \\ 0, & \omega \le 0 \end{cases}$ $G(\omega) = H(4\pi/3-\omega)/[H(\omega-2\omega/3)+H(4\pi/3-\omega)]$ $\phi(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{i\omega t}d\omega$ $\psi(t) = 2\phi(2t-1) - \psi(t-1/2)$	 $\Delta t\Delta\omega$=0.6040
Spline(5)	$N_m(t) = \frac{t}{m-1}N_{m-1}(t) + \frac{m-t}{m-1}N_{m-1}(t-1), t \in \boldsymbol{R}$ $N_{2m}(k) = \frac{k}{2m-1}N_{2m-1}(k) + \frac{2m-k}{2m-1}N_{2m-1}(k), k \in \boldsymbol{Z}$ $q_n = \frac{(-1)^n}{2^{m-1}}\sum_{l=0}^{m} C_l^m N_{2m}(n+1-l), n = 0, \cdots, 3m-2$ $\psi(t) = \sum q_n N_m(2t-n), n = 0, \cdots, 3m-2$	 $\Delta t\Delta\omega$=0.5010

Analysis results (1)

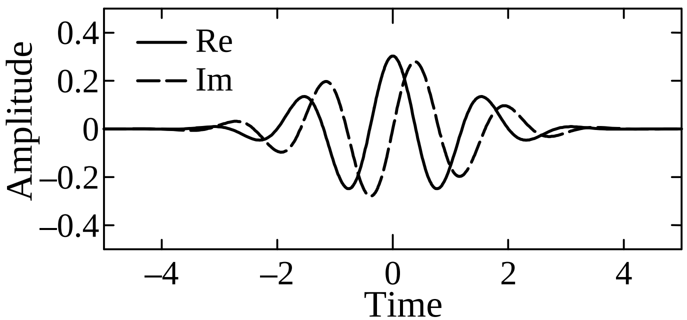
Continuous pattern

As is sown in example:
Analysis results are difference if we use different MW. It is because the MW has different feature, for example, Complex type or Real Type. For Real type, it has symmetric property or no.

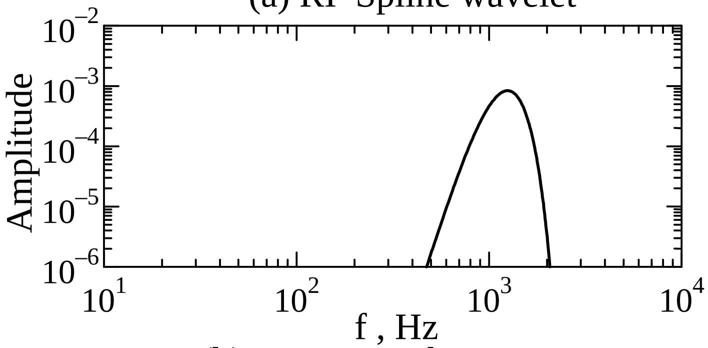
Stripe pattern



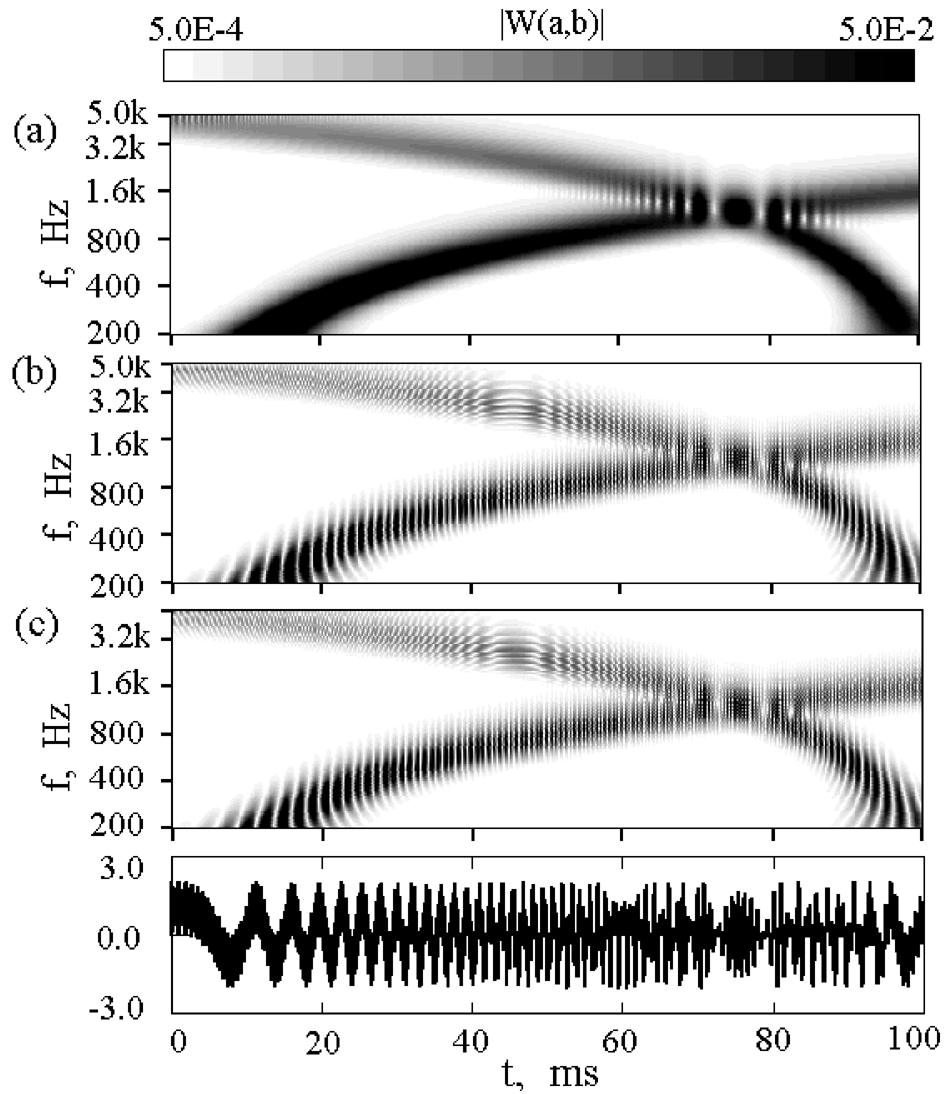
Analysis results (2)



(a) RI–Spline wavelet



(b)Frequency characteristic

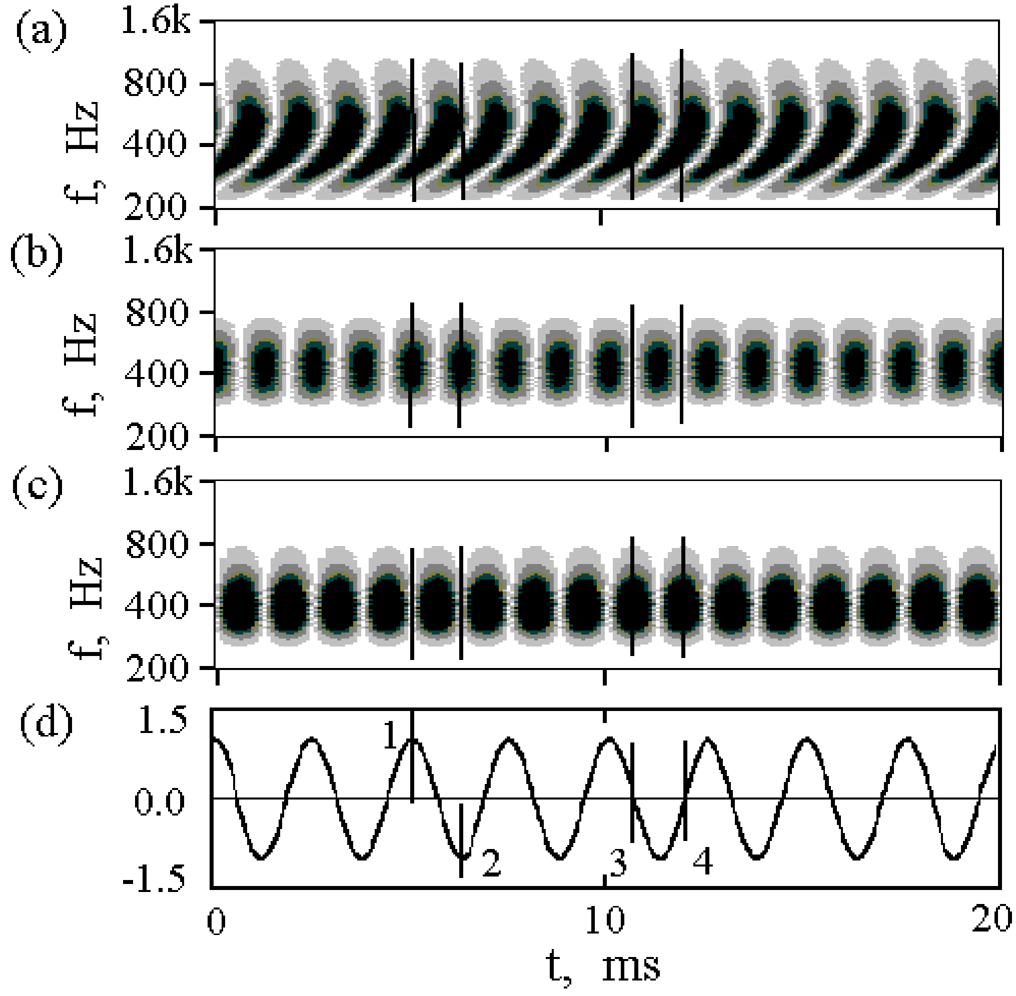


What is reason of non-continuance pattern?

Real MWs Phase (symmetric)

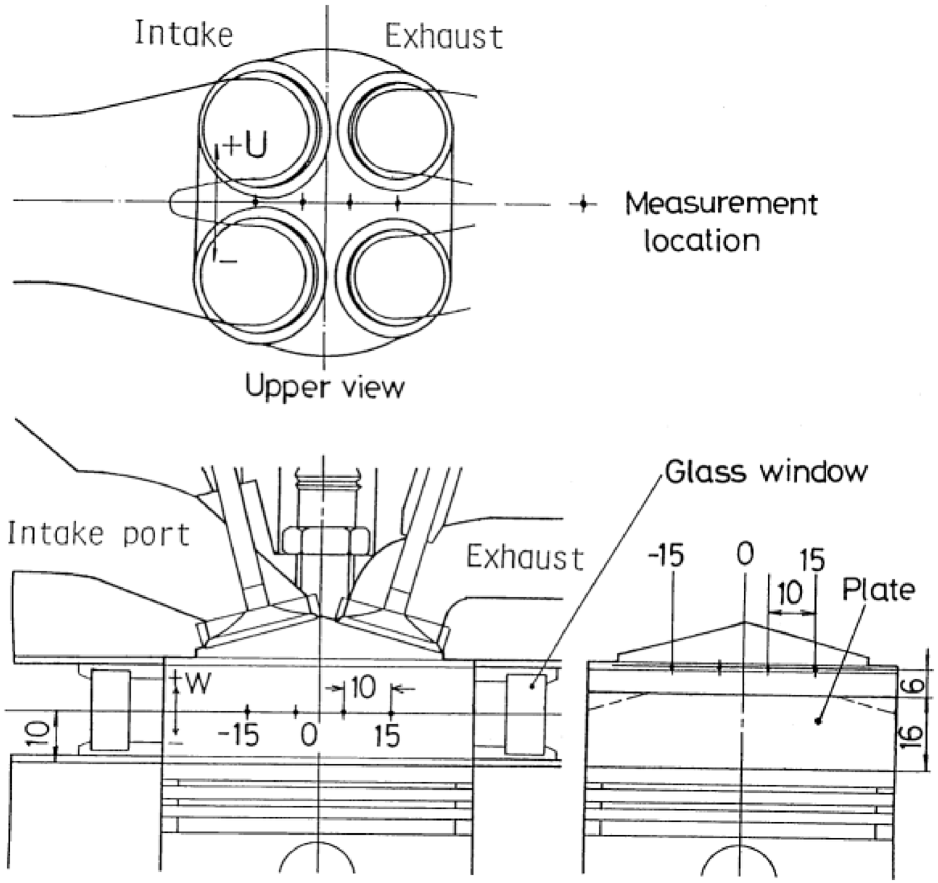
Analysis results (2)

Difference results can be obtained from different MW which has different symmetric property



How do select the MW becomes Problem, although MW is chosen freely.

3.2.2 Example of flow turbulence analysis



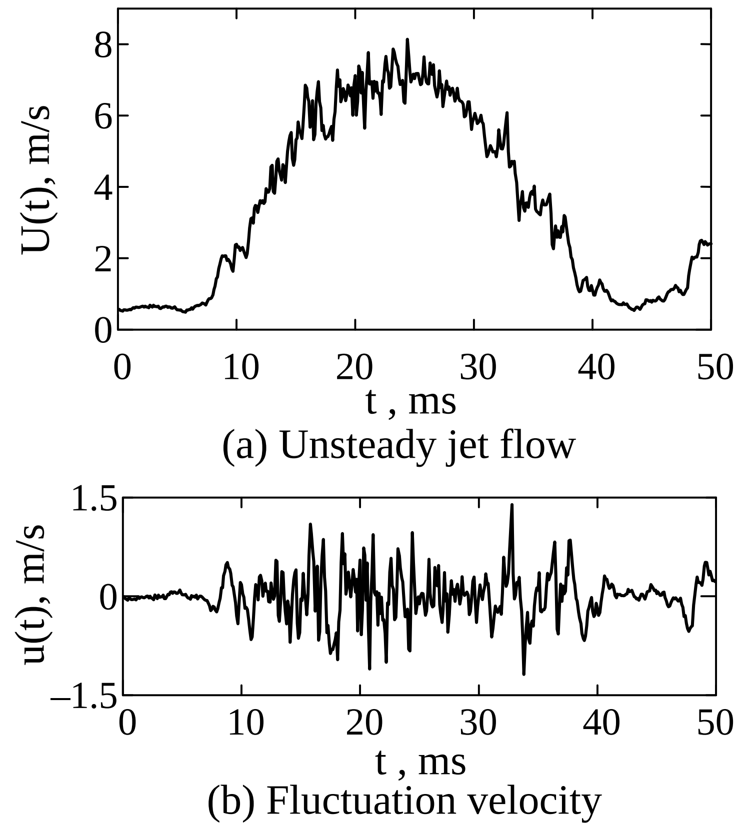


Fig.8 Example of unsteady jet flow U(t) and its fluctuation velocity u(t)

Result obtained by CWT Using different MW

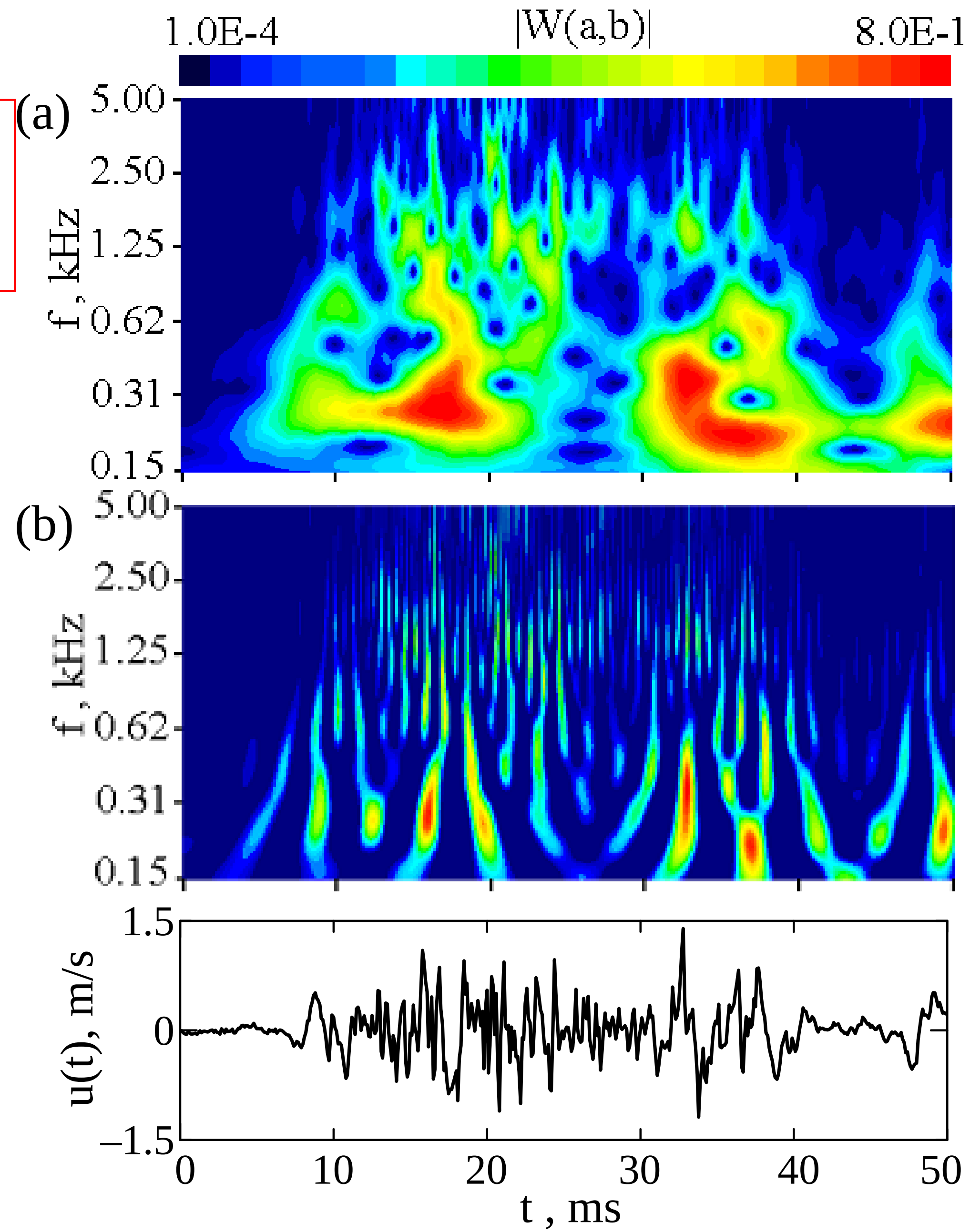


Fig. Results obtained by CWT, where (a) is RI-Spline wavelet, (b) is m=4 Spline wavelet. Signal is turbulent flow with swirl in the cylinder of the engine.

3.3 Fast Algorithm for the CWT

3.3.1 Fast Algorithm in frequency domain

$$w(a,b) = \int_{-\infty}^{\infty} f(t)\overline{\psi_{a,b}}(t)dt$$

$$= \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)\overline{\psi}(\frac{t-b}{a})dt$$

It is convolution integral, so it can be changed to next equation:

$$w(a,b) = a^{1/2} \int_{-\infty}^{\infty} X(f)\overline{\hat{\psi}}(af)e^{i2\pi fb}df$$

By using this equation, CWT can be did fast.

Comparing calculation cast

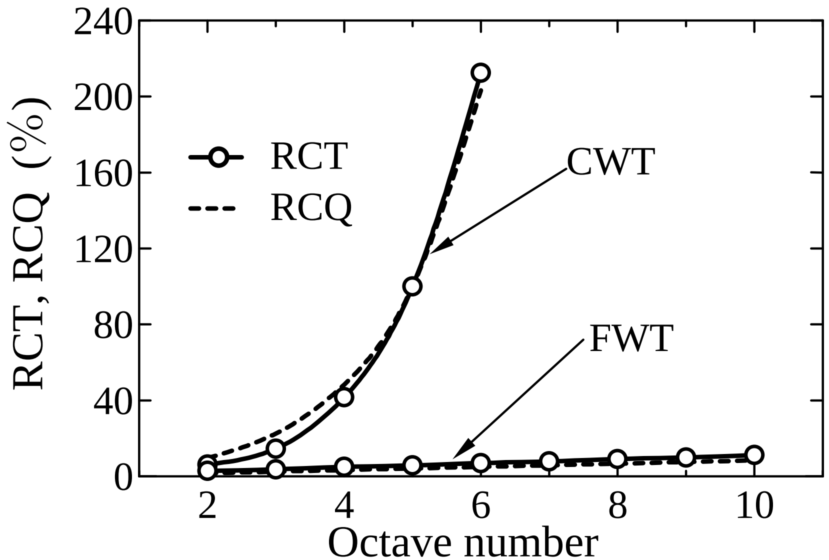


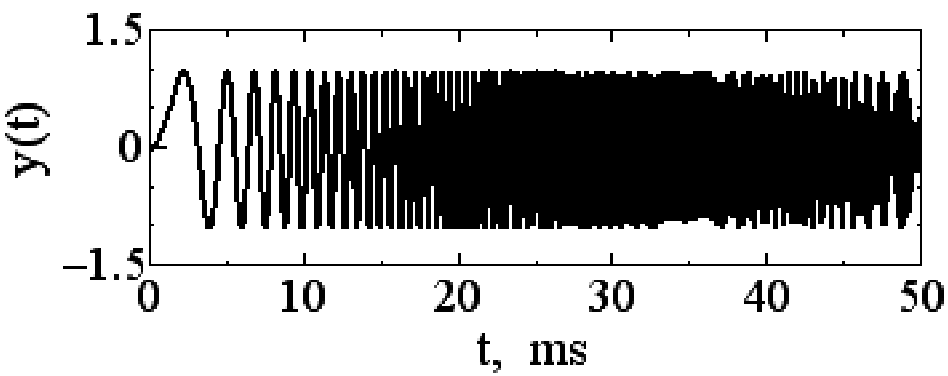
Fig. 9 RCT and RCQ change with octave number

$$CQ_{WT} = MTL(2^N - 1)$$

$$CQ_{FWT} = MNT\ (1 + \log_2 T)$$

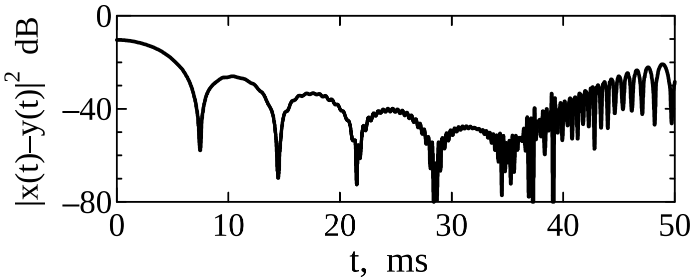
Fig. 9 shows RCT and RCQ (Ratio of Computation Quantity) which change with the increase in the number of analysis octaves, and Both RCT and RCQ are expressed with the ratio that setting the value of WT in five octaves as 100.

For comparing calculation accuracy, next signal has been used. The feature of the signal is that its frequency change with time, so it is suitable for test signal analysis method.

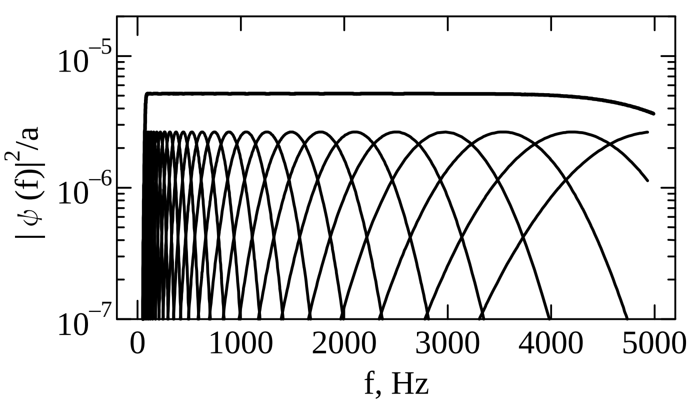


(a) Reconstructed signal

Comparing with conditional algorithm

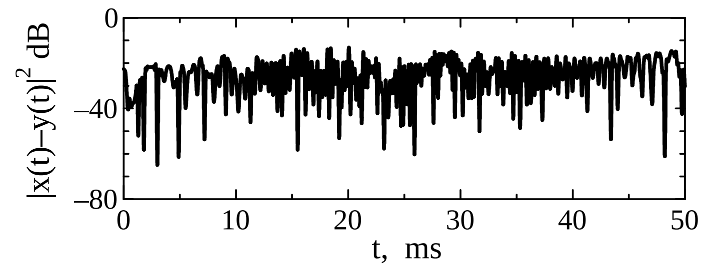


(a)Reconstructed error by CWT

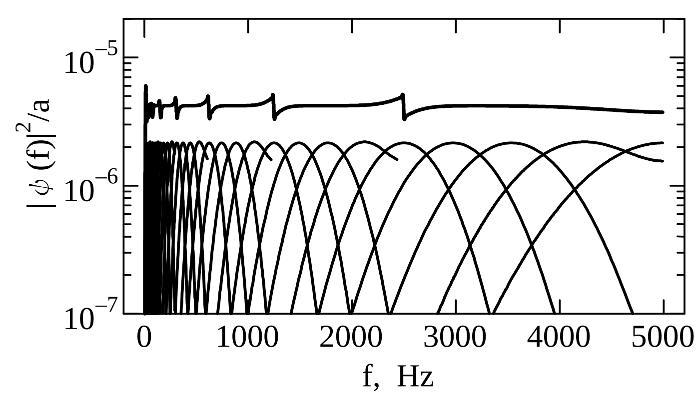


(b) Basis of CWT for 6 octave, 4 divided

Fig. 4 RE by using WT



(a) Reconstructed error by FWT

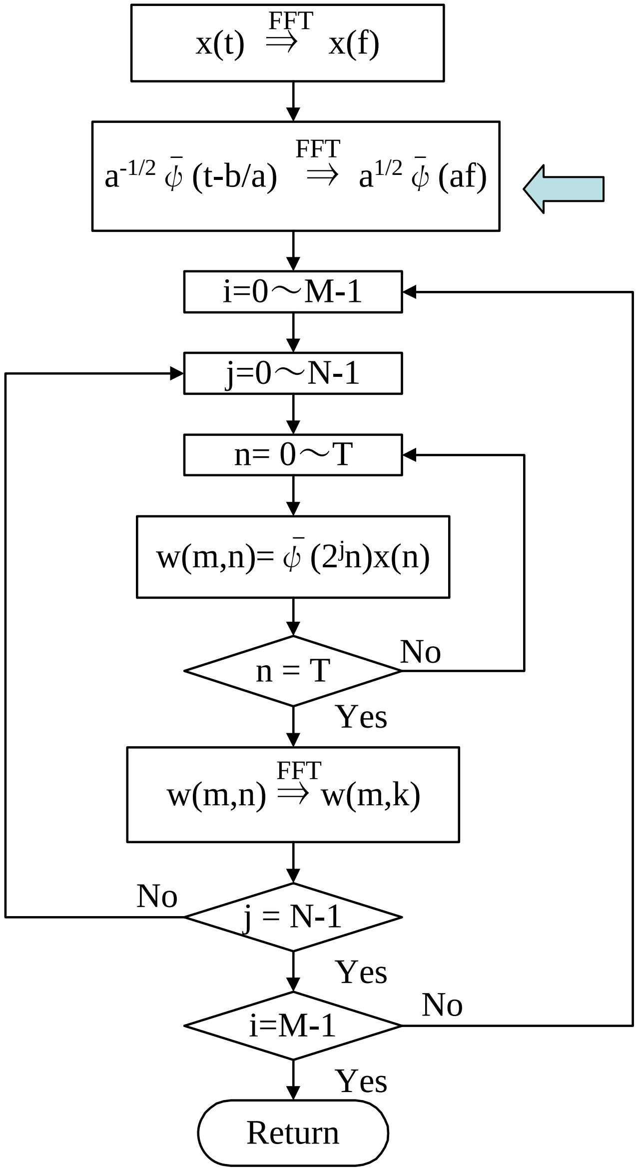


(b) Basis of FWT for 10 octave, 4 divided

Fig. 5 RE by using FWT

But calculation accuracy is not good by comparing with conditional method.

3.3.2 Improvement Fast Algorithm

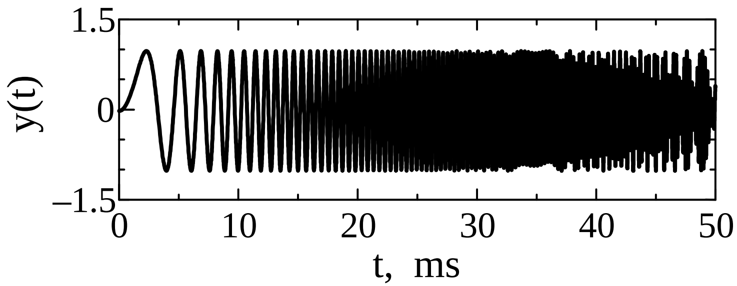


Improving calculation accuracy

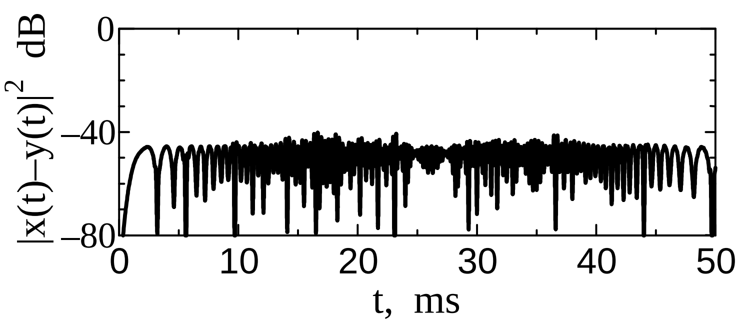
Reason: number of MW is small.
For improving calculation accuracy, we need using long MW with large number.

1) The length of data is doubled four and FFT of MW is performed.
2) It uses taking out one fourth of the obtained data.

Improving Calculation Accuracy

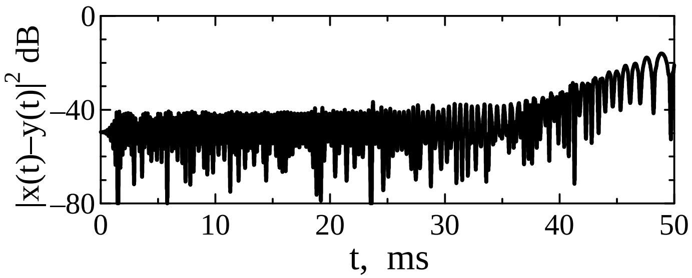


(a) Reconstructed signal

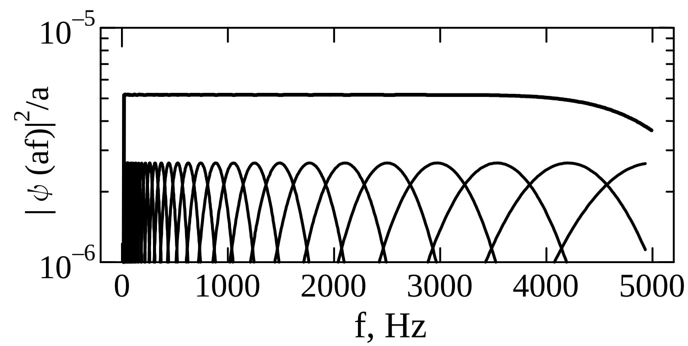


(b) Recontructed error in dB

Figure 7 RE by using FWTH



(a) Reconstruction error improved



(b) Basis of FWT

Fig. 6 RE by using FWT based CBFA

Best calculation accuracy has be obtained

3.3.3 Example of EEG analysis

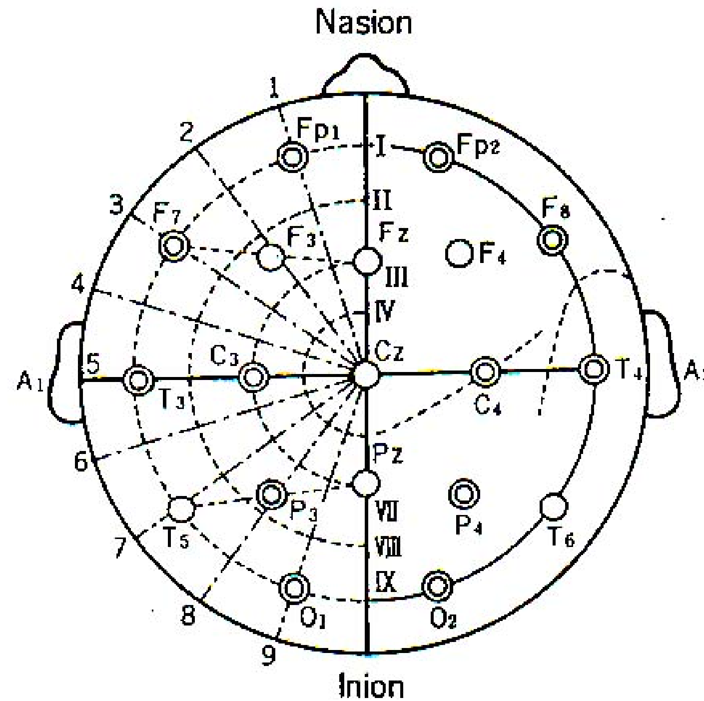


Fig. 10 Measured points of EEG by use Ten-twenty electrode system

Example of EEG waves

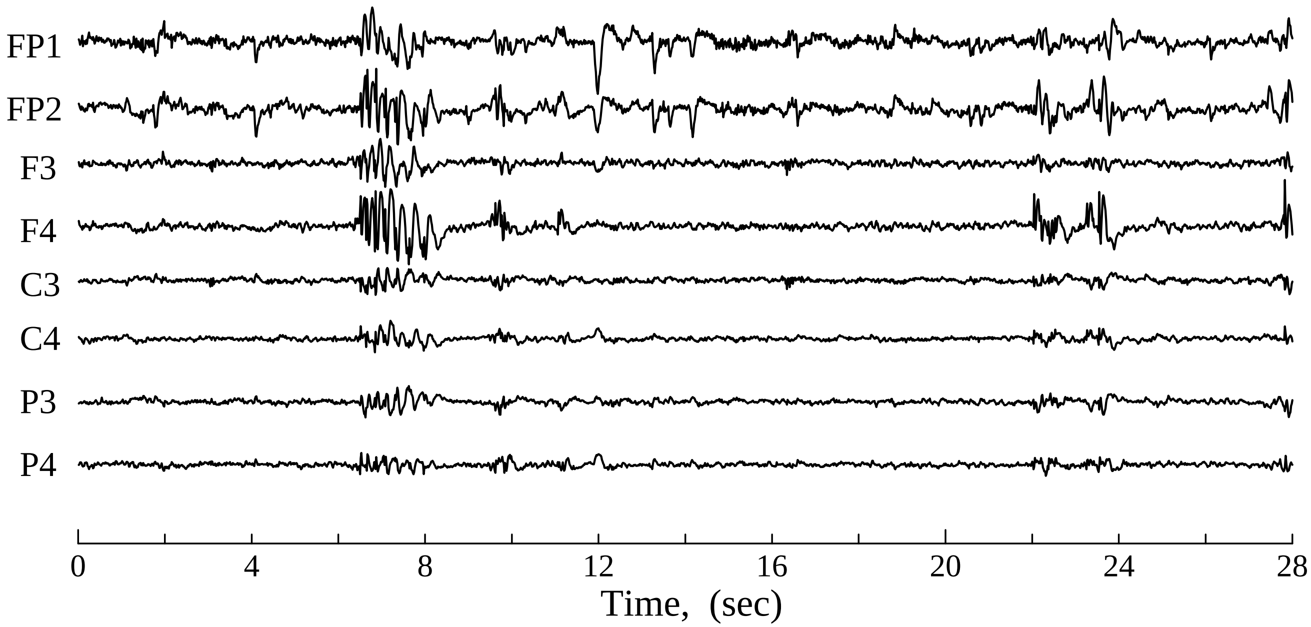
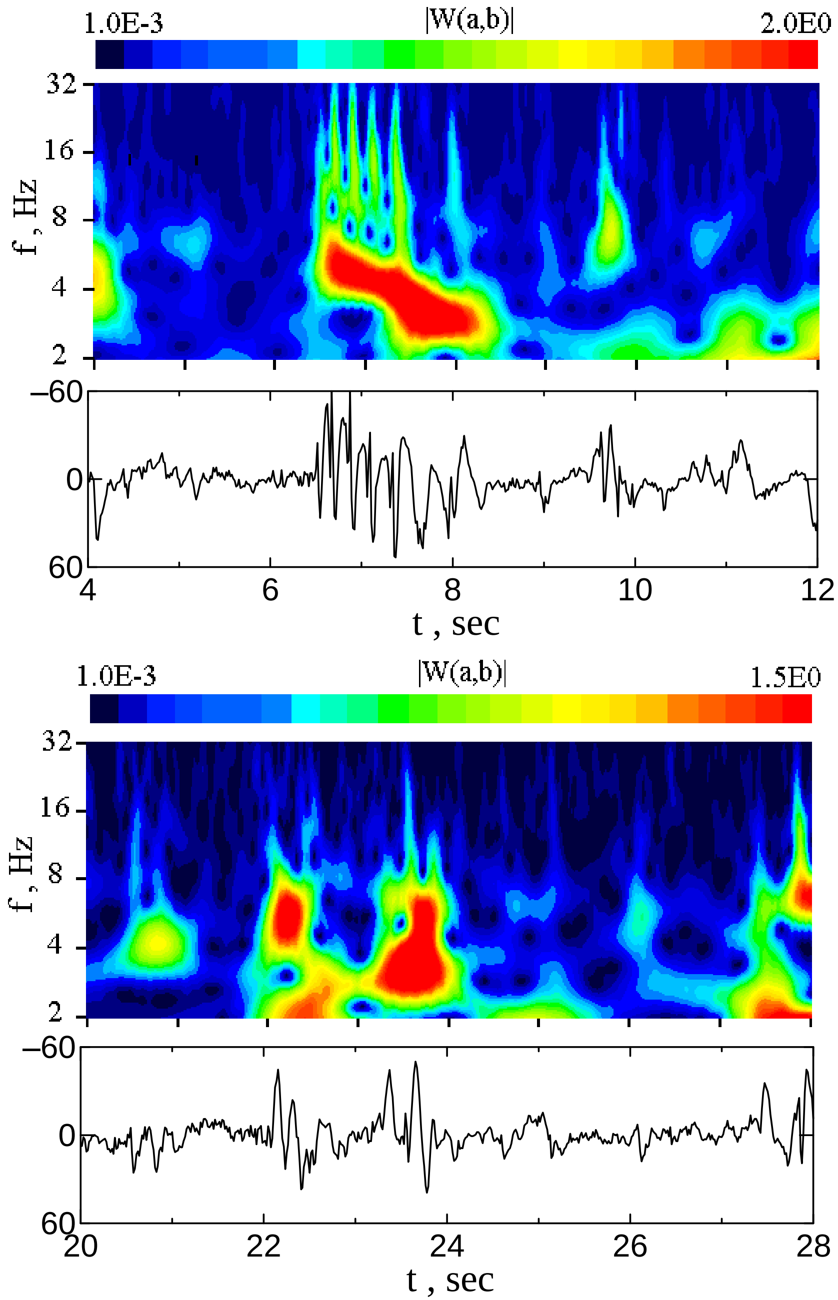


Fig. 11 Time series of EEG

Figure shows EEGs of a 14-year-old girl recorded when she was sitting in a chair and opening her eyes. The data sampling frequency is 64Hz . The spike and wave complex (SWC) around 3Hz are recorded near 6-8, and 22-24 sec. It is strong in the incipit and its generation source is in the depths.



EEG Analysis

Defined waves in EEG as:

δ: 2-4 Hz
θ: 4-8 Hz
α: 8-13 Hz
β: 13-30Hz

Computation time:

FWT is only 17% of CWT

Figure shows wavelet transform of FP2 using the FWT, where the ordinates denote frequency, transverse time and the amplitude |w(a,b)| is shown as the color label. Frequency range was chosen as four octaves and each octave was divided into 48 voices for clarity.

δ, θ, α, β Waves Change with Tim

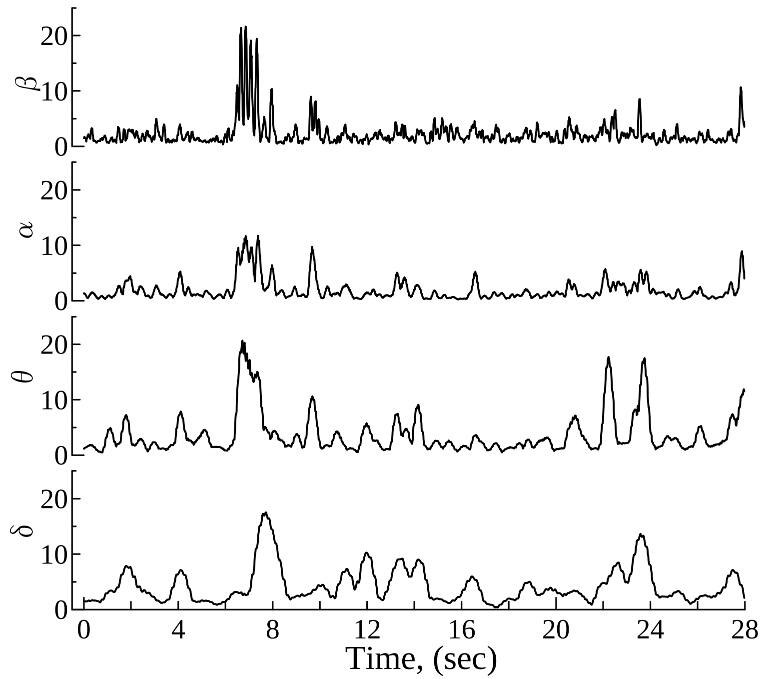
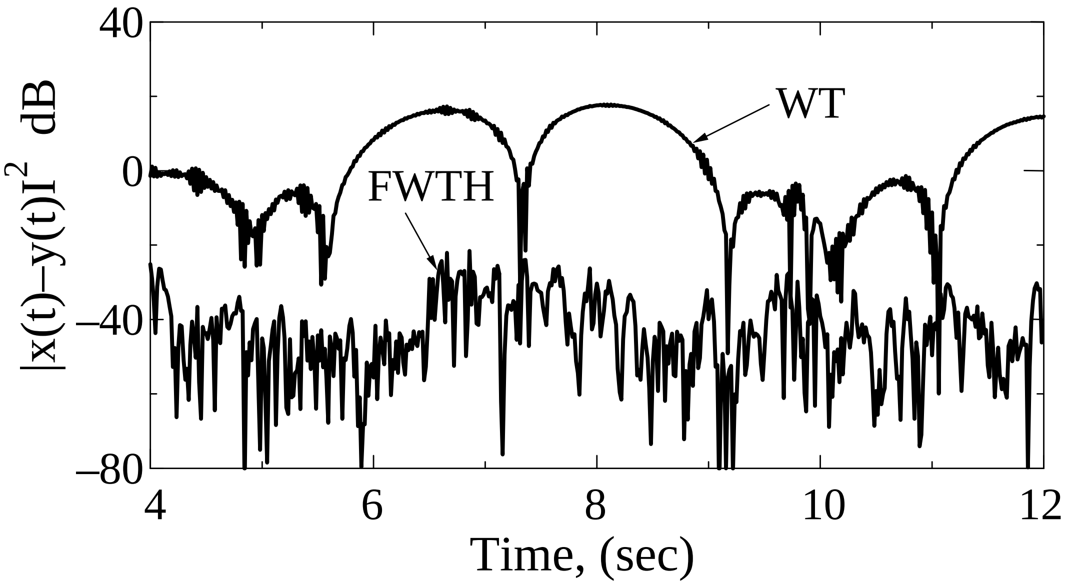


Fig. 13 δ, θ, α, β waves of EEG in point FP2

As is shown in Fig.13, we can observe the active change of the δ, θ, α, β waves before and after SWC occurred between 6-8 sec. The same phenomenon was observed before and after SWC occurred between 22-24 sec.

Comparing Calculation Accuracy



Average RE

FWTH: -44.0dB

WT: 0.69dB

Fig.14 RE of EEG in point FP2 from 4 to 12 sec by using FWTH and WT

As shown in Fig. 14, the accuracy obtained with FWTH is higher than WT. The average value of the RE of FWTH between 4-12 sec is -44dB and WT is 0.69dB. Therefore, we may conclude that our approach proposed in this study is effective for EEG analysis in real time with high accuracy and is furthermore useful for general signal processing.

3.4 Constructing new RMW

Original Signal s(t)

(1) Hanning Window

(2) Normalize

(3) Fourier Transform

$$rmw(t) \rightarrow \widehat{rmw}(\omega)$$

(4) Hilbert Transform

$$\hat{Z}(\omega) = \begin{cases} 2\widehat{rmw}(\omega) & \omega > 0 \\ \widehat{rmw}(\omega) & \omega = 0 \\ 0 & \omega < 0 \end{cases}$$

(5) Zeroise Phase

$$\hat{Z}(\omega) = \begin{cases} \sqrt{\hat{Z}(\omega)^2_{real} + \hat{Z}(\omega)^2_{imaginary}} & \text{real} \\ 0 & \text{imaginary} \end{cases}$$

(6) Inverse Fourier Transform

$$SCRMW(t) \leftarrow \hat{Z}(\omega)$$

Symmetric Complex RMW

$$SCRMW(t)$$

Real signal Mother Wavelet

Condition $C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \quad \int_{-\infty}^{\infty} \psi(t)dt = 0$

$$\|\psi_R\| = 1$$

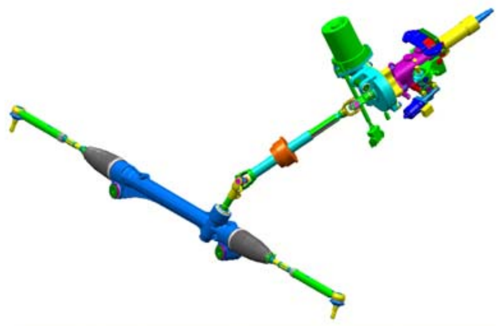
Complex RMW

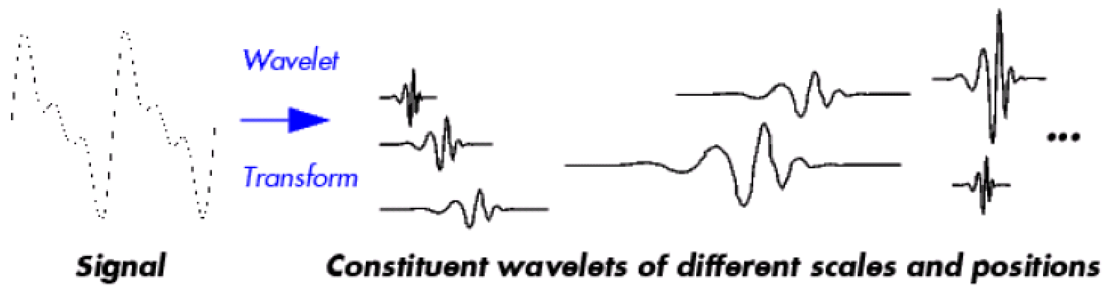
Phase imformation

Symmetric Complex RMW

It’s suitable for unsteady signal

Correlation between Noise and Vibration



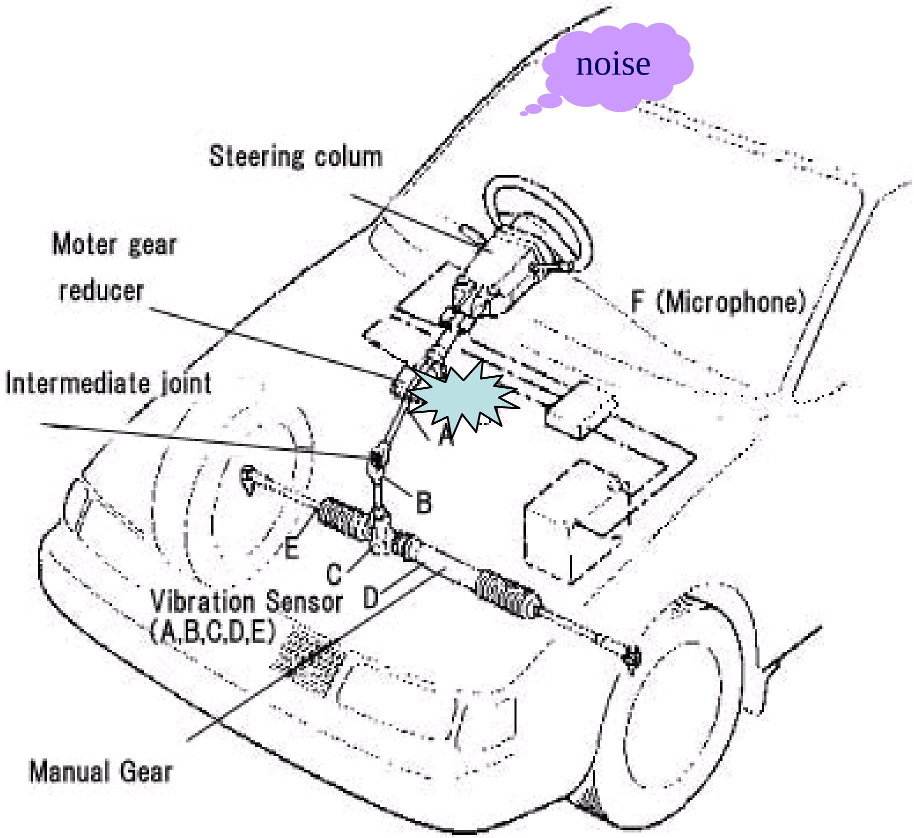


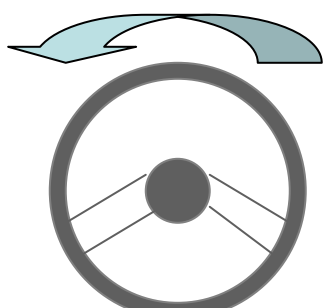
Wavelet Instantanious Correlation Value (WICV)

$$R(b) = \left|W_{(a=1,b)}\right| = \int_{-\infty}^{\infty} f(t)\psi_R(t)dt$$

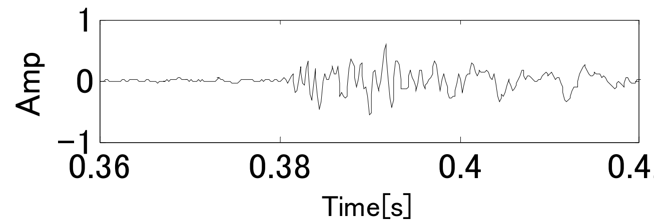
WICV → Instantaneous Correlation (time,strength) → Identify the noise sources

Application on Impact Noise in Case 1

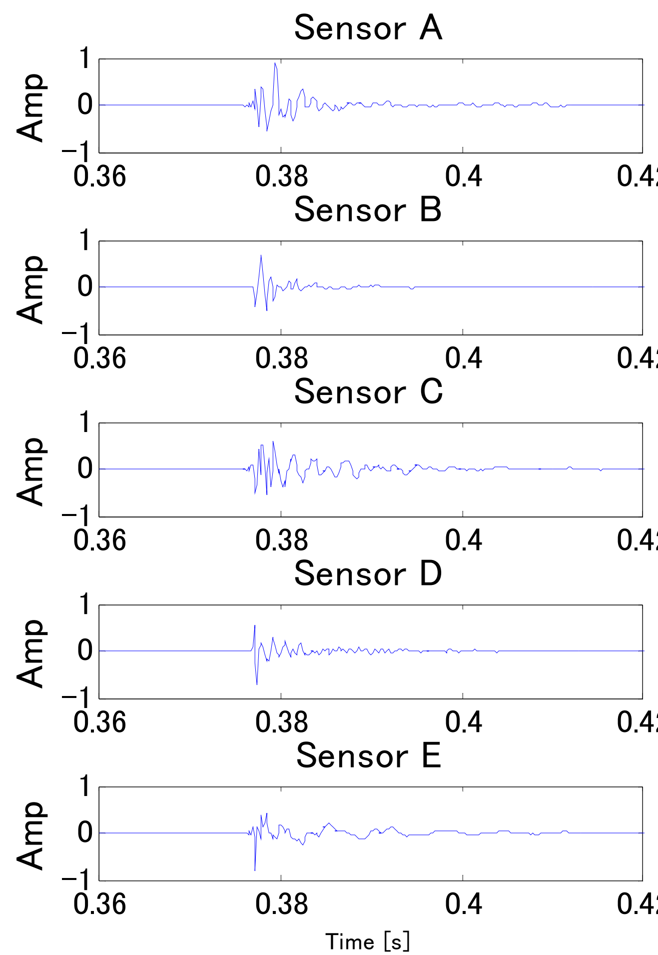




Steering wheel



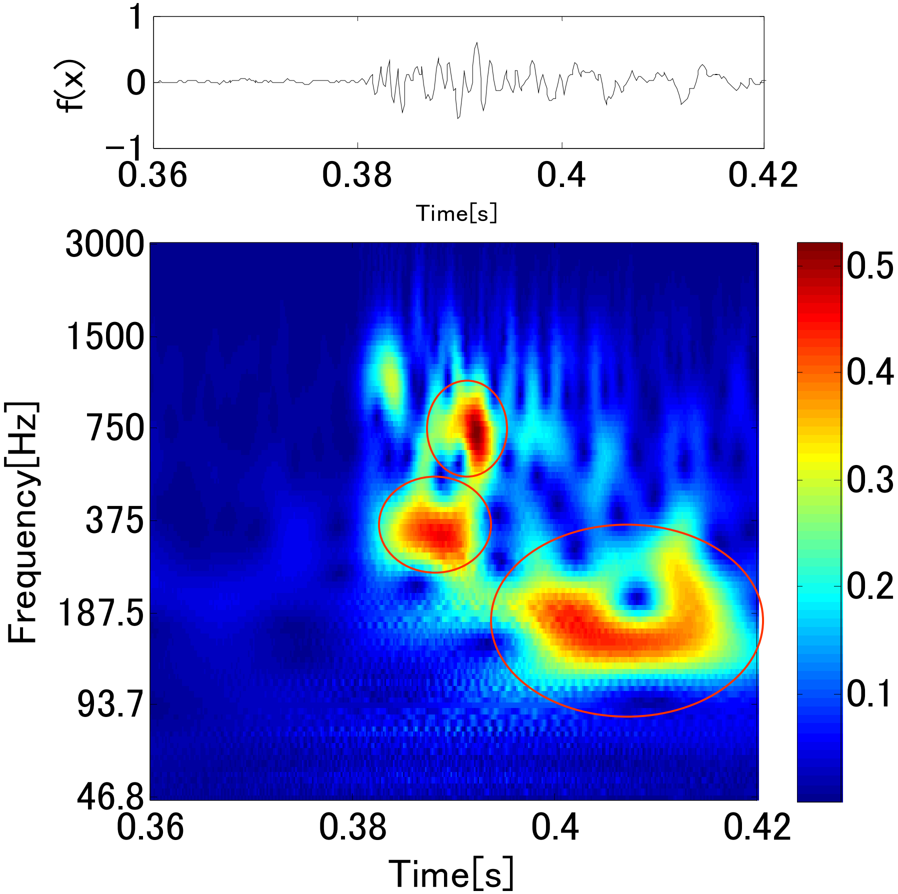
Noise



Vibration

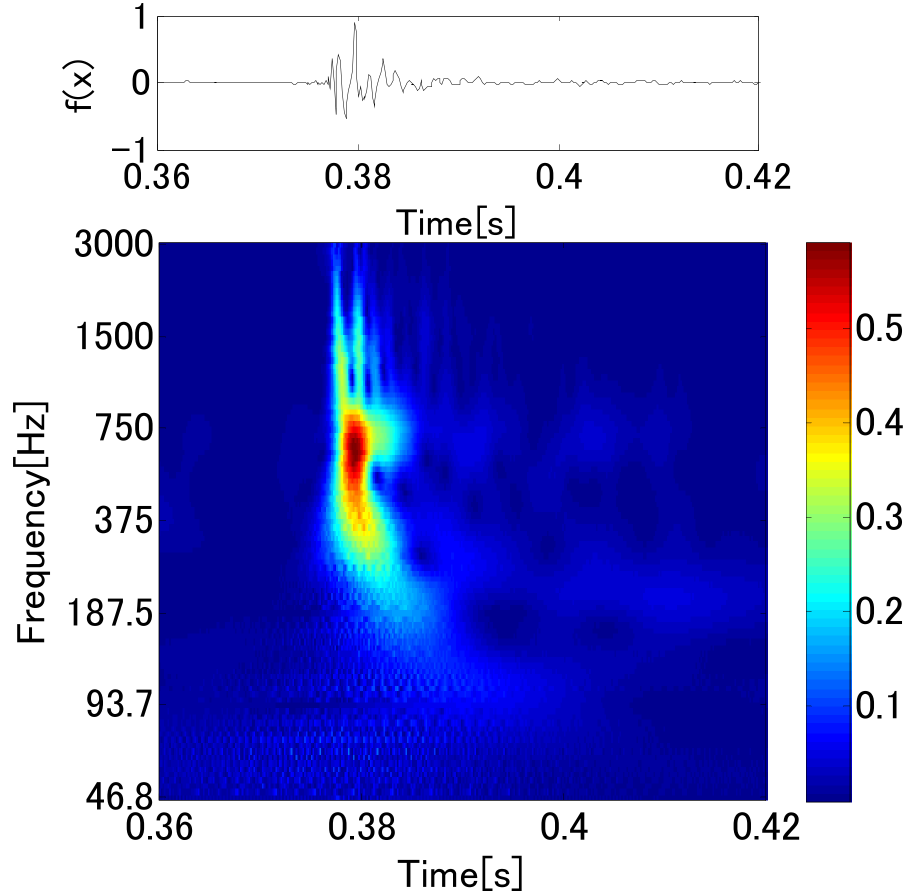
Example of CWT

Impact Noise



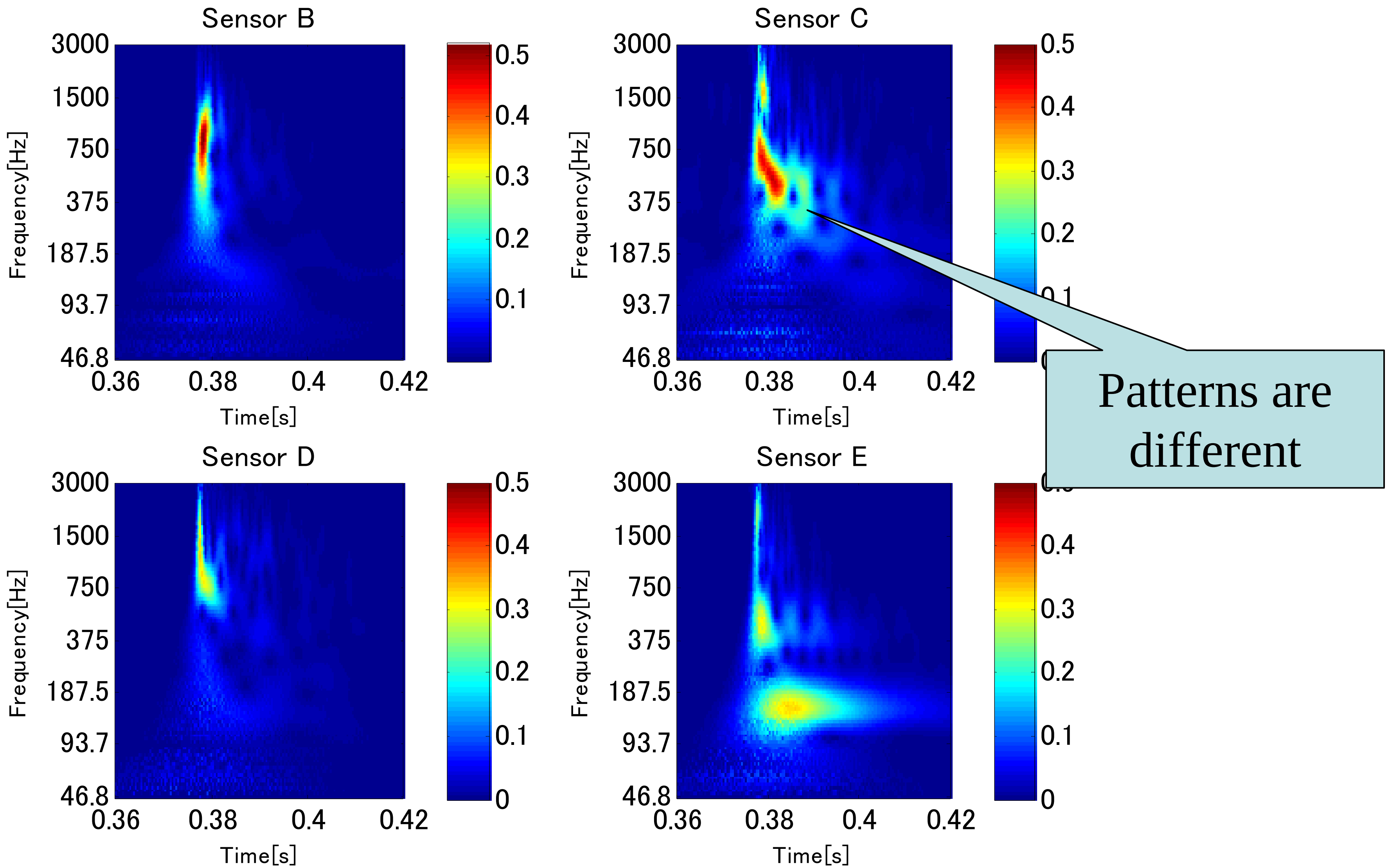
CWT of noise using Gabor function

Vibration on Sensor A



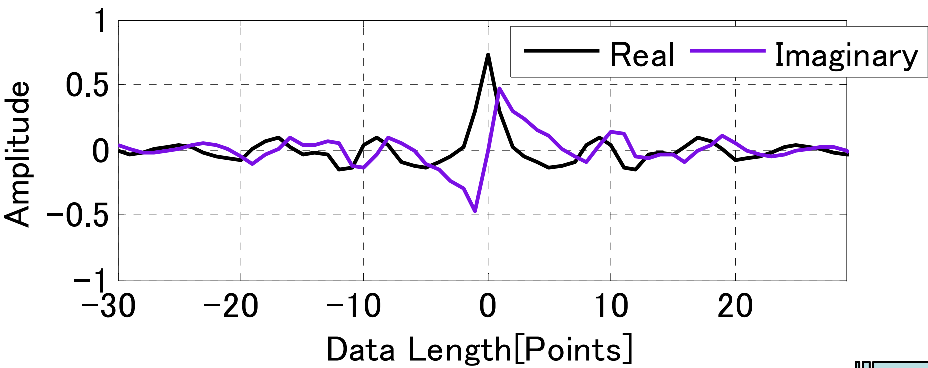
CWT of vibration using Gabor function

Example of CWT



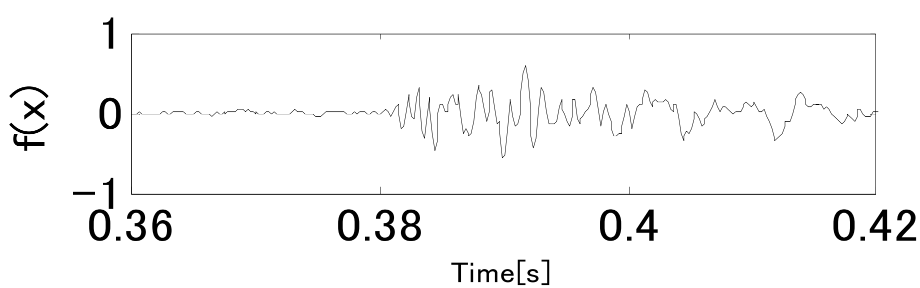
CWT of vibration signals using Gabor function

Wavelet Instantaneous Correlation (WIC)



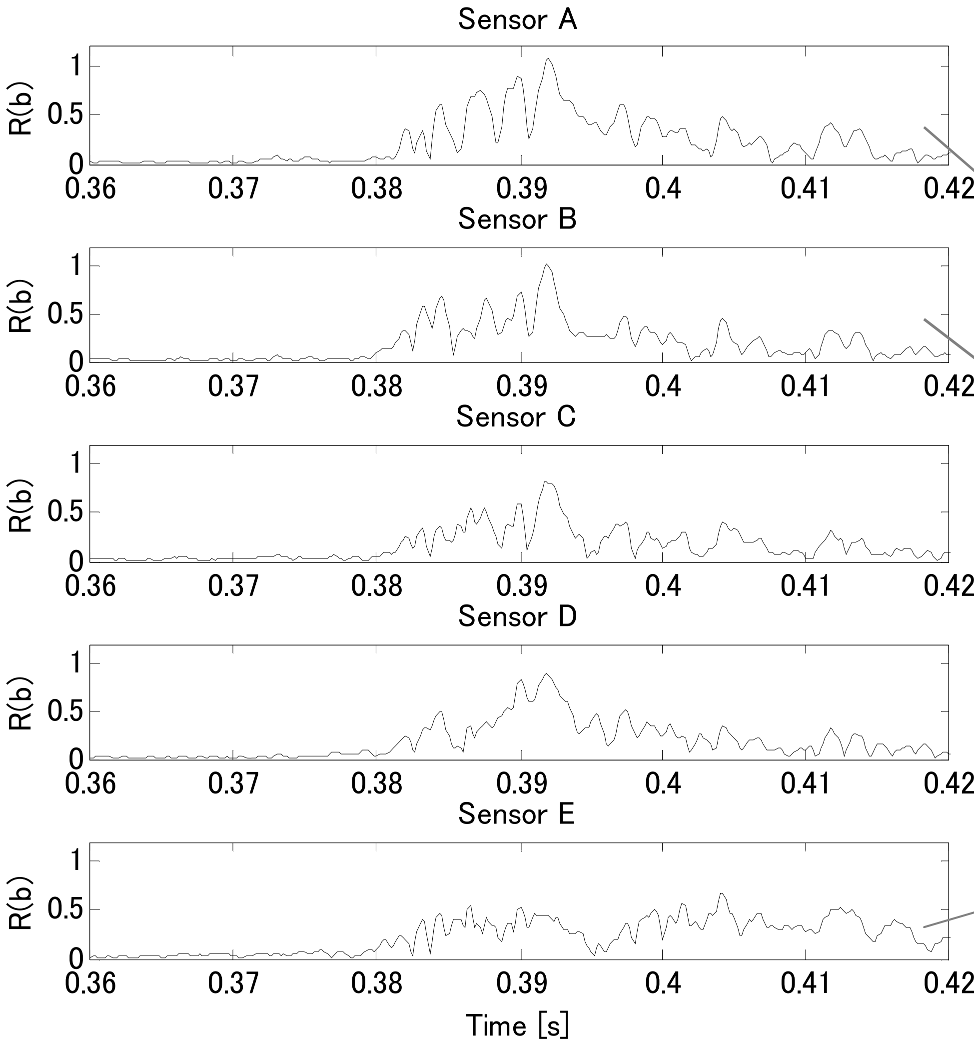
Scale 0:same frequency characteristic

$$R(b) = \left|W_{(a=1,b)}\right| = \int_{-\infty}^{\infty} f(t)\psi_R(t)dt$$

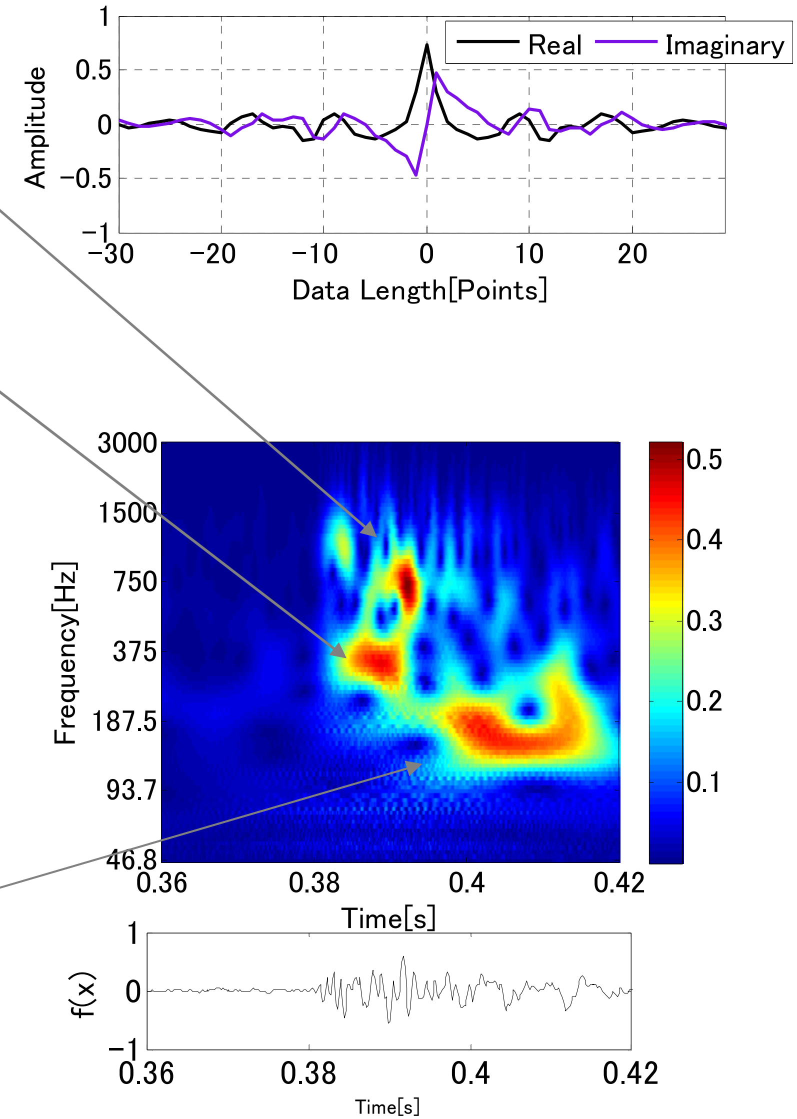


WIC using SCRMW constructed the vibration

Results of WICV R(b)

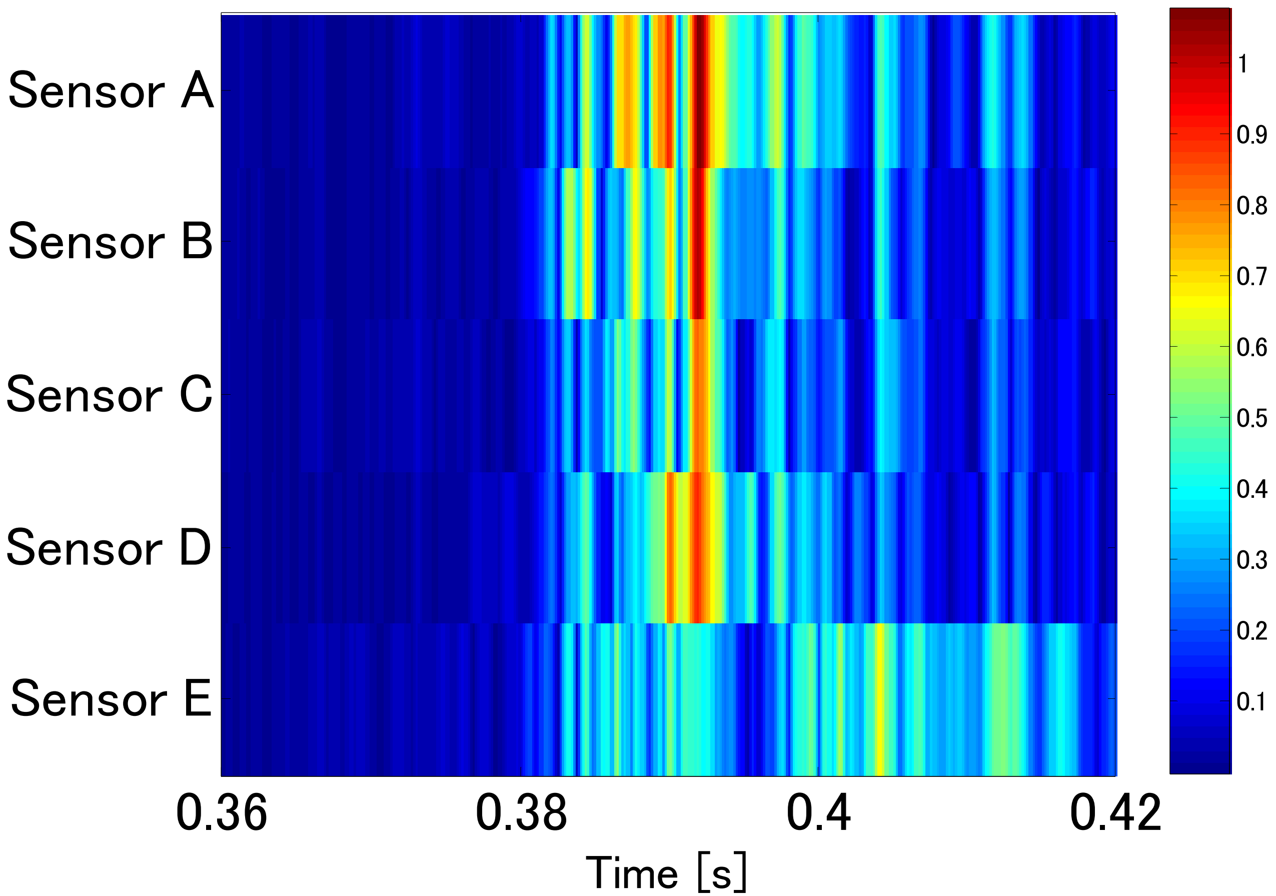


WICV R(b) of the noise by the SCRMW constructed from vibration signals



CWT of noise signals using Gabor function

Results of WICV R(b)



WICV R(b) of the noise by the SCRMW constructed from vibration signals

3.Contiunance Wavelet Transform

3.1 Introducing Wavelet Transform

3.1.1 Continuance Wavelet Transform (DWT)

3.1.2 Discrete Wavelet Transform (DWT)

3.2 Selection of MW for the CWT

3.2.1 Condition of MW selection

3.2.2 Example of flow turbulence analysis

3.3 Fast Algorithm for the CWT

3.3.1 Fast Algorithm in frequency domain

3.3.2 Improvement Fast Algorithm

3.3.3 Example of EEG analysis

3.4 Constructing new RMW

Wavelet Instantaneous Correlation (WIC)