

2. Signal Type and Analysis Method

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1. Introduction and Digital, Sampling Proposition

1.1 Introduction

1.1.1 In the car production

1.1.2 Measuring System

1.1.3 Sensor

1.1.4 Signal Processing

1.2 Digital and Sampling Proposition

1.2.1 Analog and Digital Signal

1.2.2 Signal Sampling

1.2.3 Signal quantification

1.2.4 Signal's expressing method

1.3 Example of Abnormal extraction

Continuance wavelet transform

Abnormal extraction

Important points of Last lesson

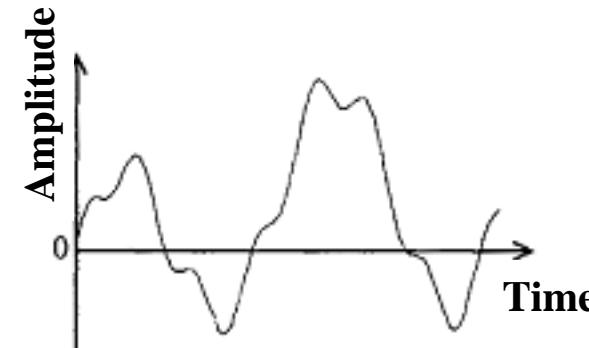
(1) Classification from signal type

Signal Type

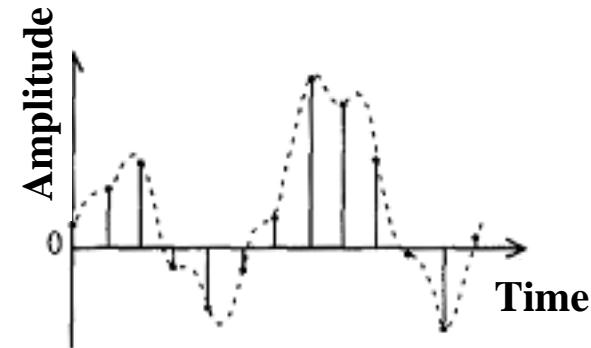
- Analog Signal
- Sampling signal
- Multi-value signal
- digital signal

Digital nature

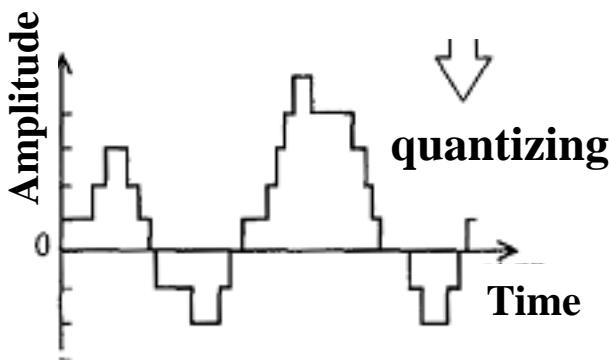
Quantification:
Amplitude digital
Sampling:
Time digital



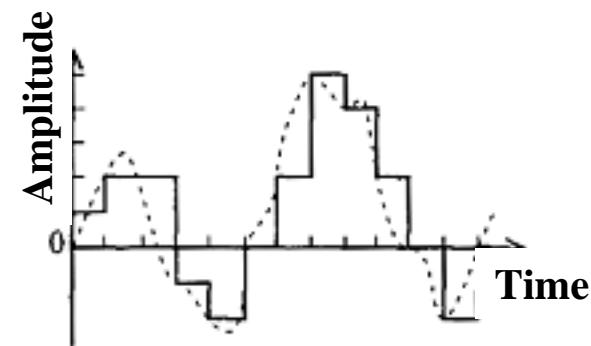
Analog Signal



Sampling signal

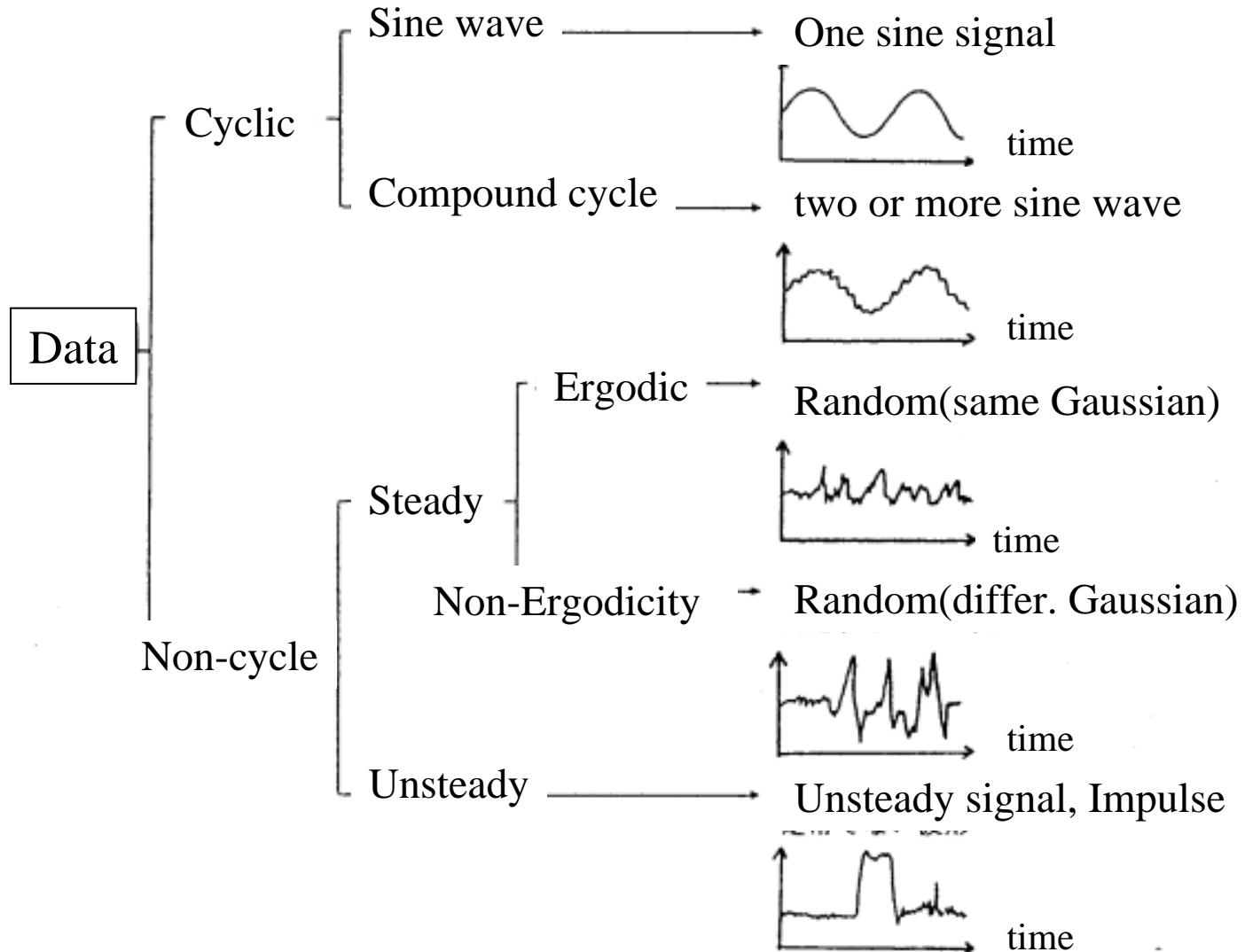


Multi-value signal



Digital signal

(2) Classification from signal nature



Typical Signal Analysis Methods

Signal Type and Comparing Analysis Methods

Wave \ Method	Statistics Method	Statistics Model	Spectrum Analysis (FFT, MEM)	STFT, WT
Sine wave	○	○	○	○
Compound cycle	○	○	○	○
Steady(Ergodic)	○	○	○	○
non-Ergodicity	×	×	×	○
Unsteady	×	×	×	○

*Method adapted for unsteady signal analysis:

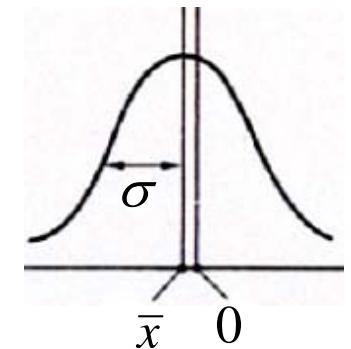
- 1) Short Time Fourier transform, STFT
- 2) Wavelet Transform,WT

2.1 Stationery Signal analysis Methods

2.1.1 Statistics Method

(1) Gaussian Distribution and parameters

Data (x_1, x_2, \dots, x_n) follows Gaussian distribution



Primary moment: $m_1 = \frac{1}{n} \sum_{i=1}^n x_i$ (Average \bar{x})

Secondary moment: $m_2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$ (Variance σ^2)

3rd moment: $m_3 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^3$ (Symmetry)

4th moment: $m_4 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^4$ (sharpen)

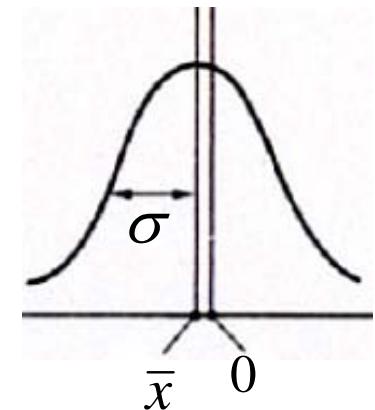
Gaussian distribution: $m_1 = 0, m_2 = 1 = \sigma^2, m_3 = 0, m_4 = 3\sigma^4 = 3$

(2) Evaluation of different from Gaussian distribution

Gauss distribution: $m_i = \begin{cases} m_i = 0, & i = 2n - 1 \\ m_i = (2n - 1)\sigma^{2n}, & i = 2n \end{cases} \quad n = 1, 2, 3, \dots$

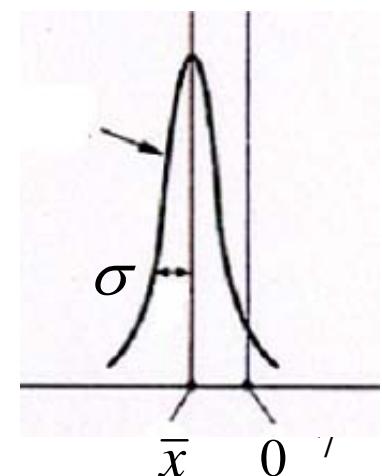
1. Average: $\bar{x} = m_1 = \frac{1}{n} \sum_{i=1}^n x_i \neq 0$

Grade of the center shifts



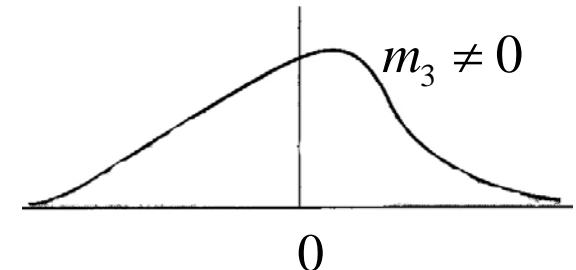
2. Variance : $\sigma^2 = m_2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \neq 1$

Grade of variation(large or small)



3rd moment : $m_3 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^3 \neq 0$

Symmetry is collapsed



3. Distortion (normalized 3rd moment) :

$$\beta_1 = m_3 / \sigma^3$$

Grade of symmetry collapse

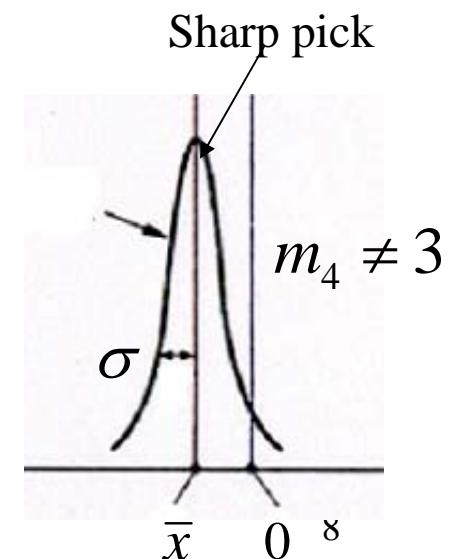
4th moment : $m_4 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^4 \neq 3$

The peak is sharp

4. Sharpens (normalized 4th moment) :

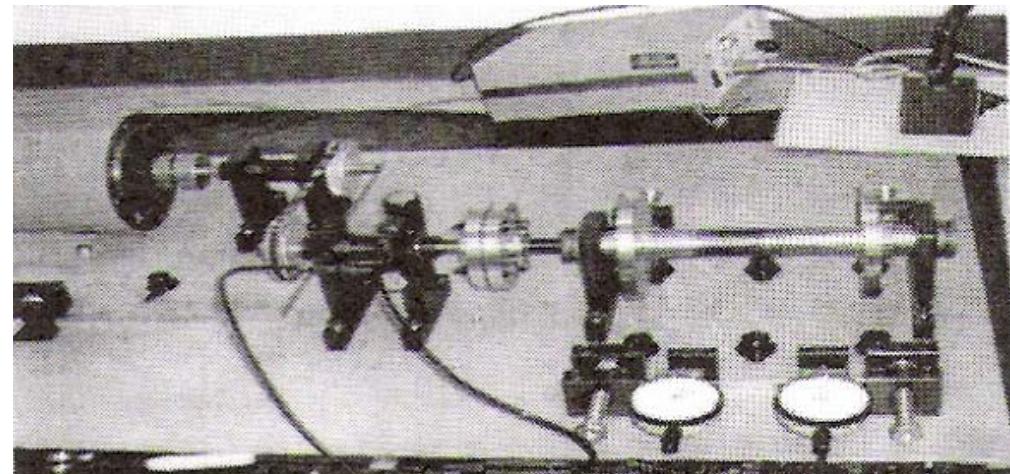
$$\beta_2 = m_4 / \sigma^4 - 3$$

Grade of peak sharpen

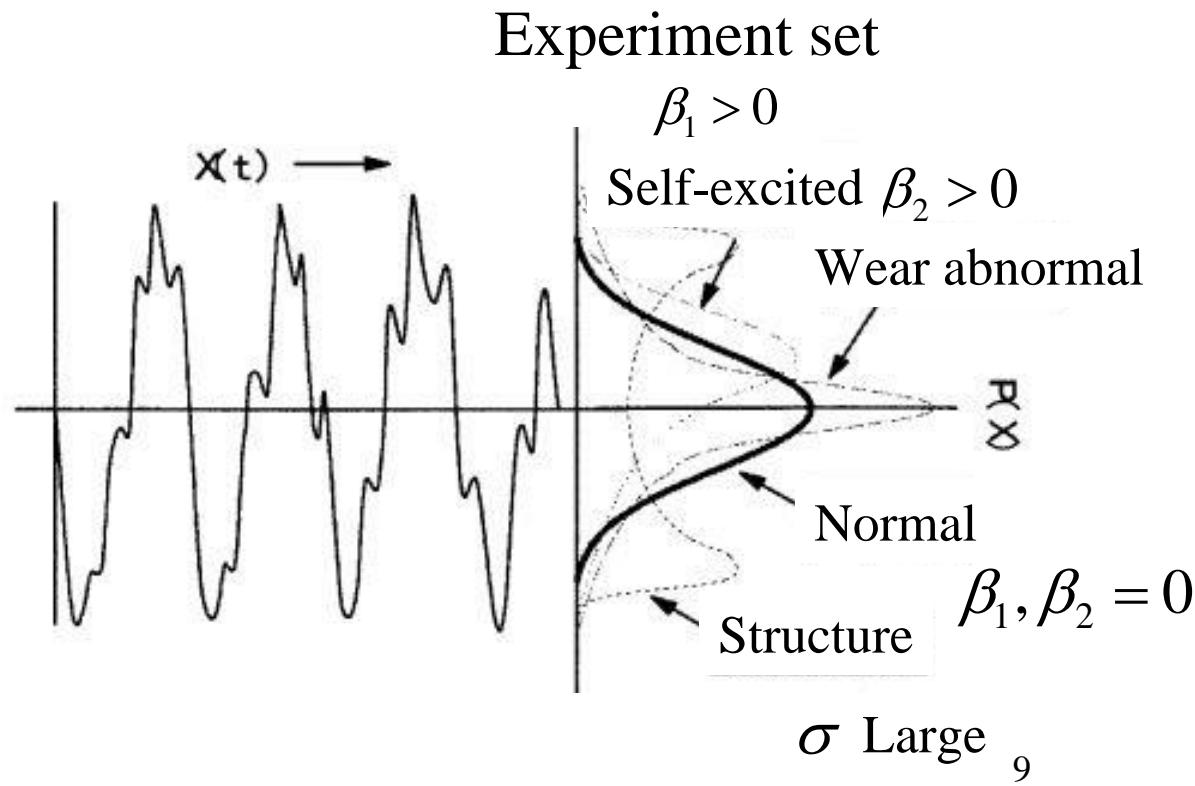


(3)Application

State evaluation of a rotation system's bearing



An Vibration sensor is attached in the place of an bearing, and vibration is measured. The signal obtained can be considered as a stationary signal.



2.1.2 Statistic Model

(1) Linear prediction model (AR model)

Linear prediction model is an auto-regressive model which predicts the future using the past data.

$$x_n = \sum_{k=1}^p a_k x_{n-k}$$

Prediction value x_n in Time n

$$\begin{cases} n & : \text{time } t = n\Delta\tau \\ \Delta\tau & : \text{Sampling interval} \\ a_k & : \text{Coefficients} \\ x_{n-k} & : \text{Past data} \end{cases}$$

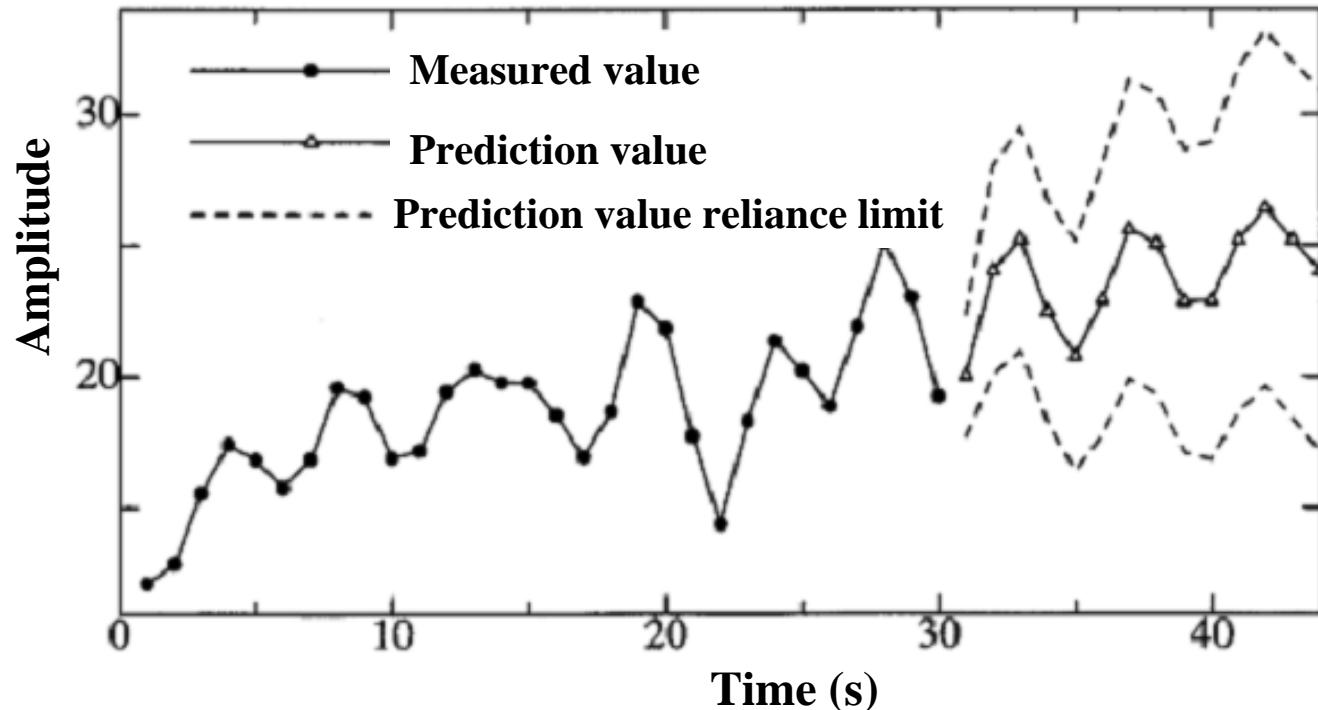
How calculate?

(AR auto-regression)

(2) Prediction by the statistics model

AR model (Auto-Regression) :

$$x_n = \sum_{k=1}^P a_k x_{n-k} \pm e_n$$



2.1.3.Spectrum Analysis

(1)Fourier Analysis

$$X_N(k) = \sum_{n=0}^{N-1} x_N(n) e^{-ik\omega_0 n}$$

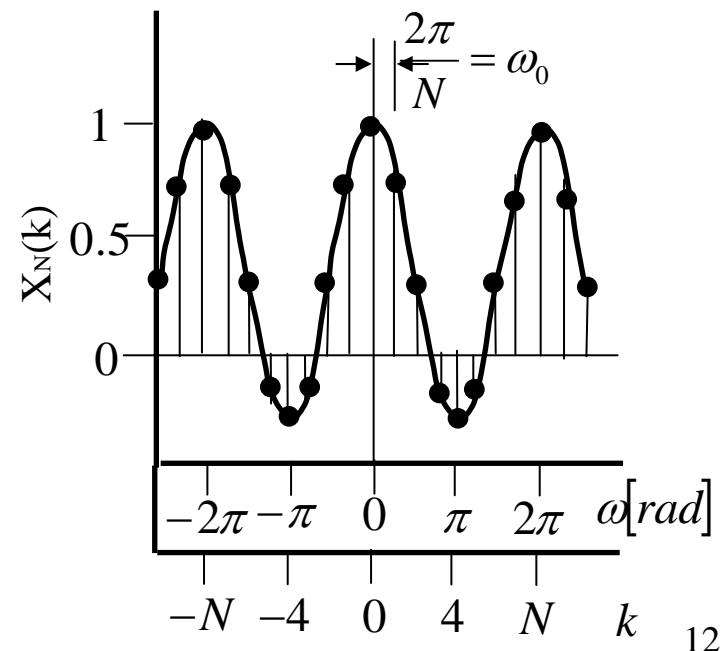
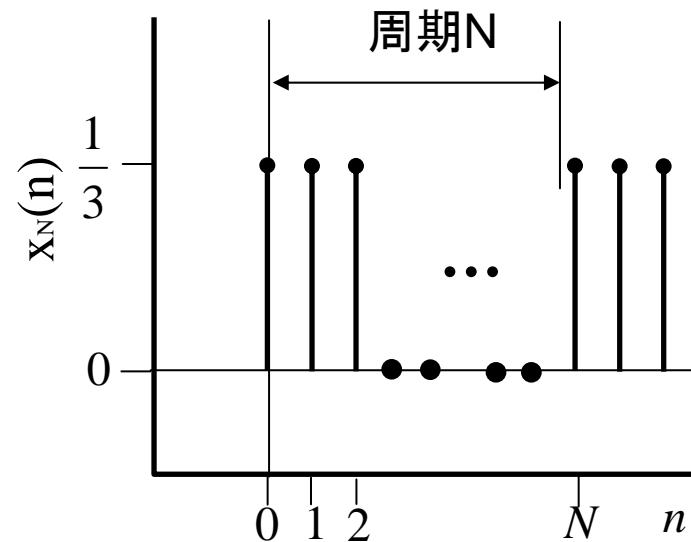
$$x(n) = \frac{1}{2N} \sum_{k=-N/2}^{N/2} X_N(k) e^{i\omega_0 nk}$$

$$\omega_0 = \frac{2\pi}{N}, \quad \omega = k\omega_0$$

Power Spectrum:

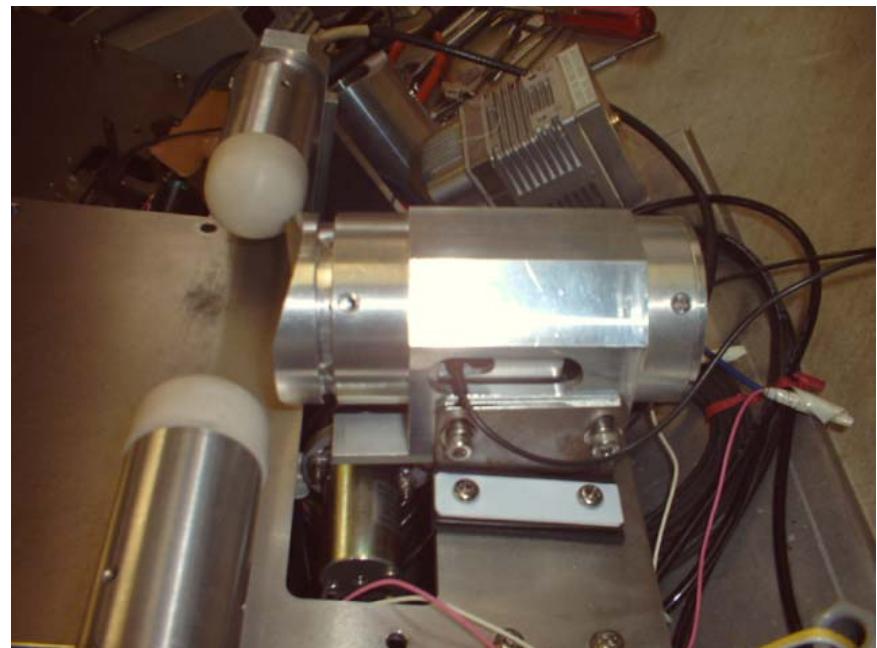
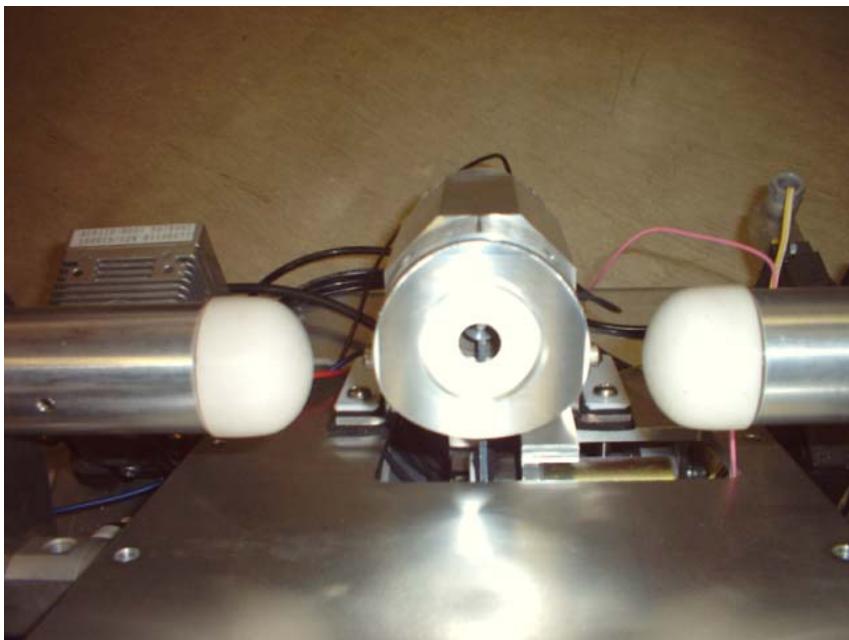
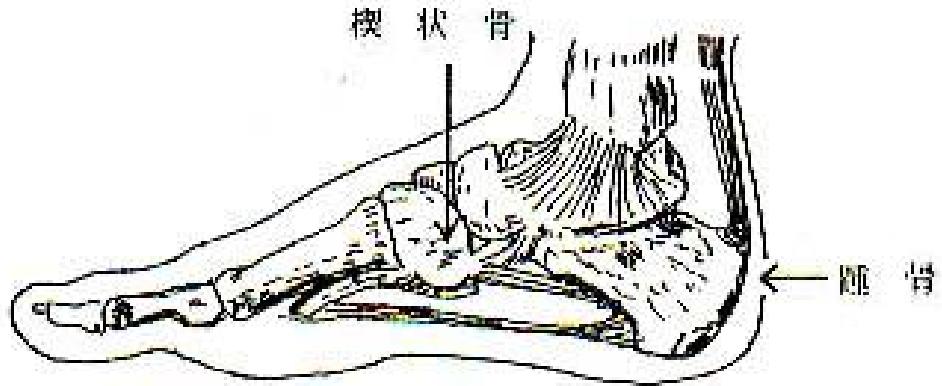
$$E(\omega) = \frac{1}{N} |X_N(\omega)|^2$$

$$f = \omega / 2\pi [Hz]$$

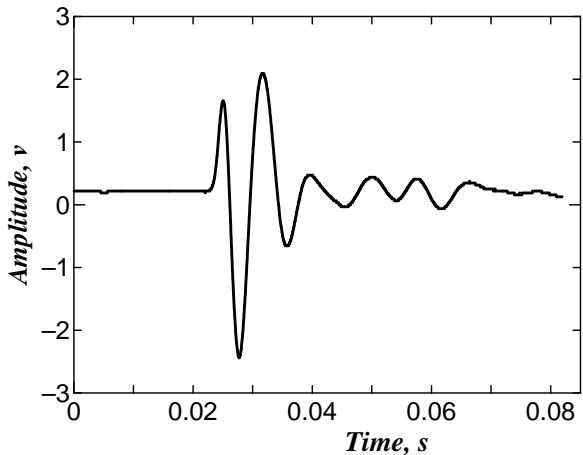


Example to the diagnosis of the bone illness by the Fourier analysis

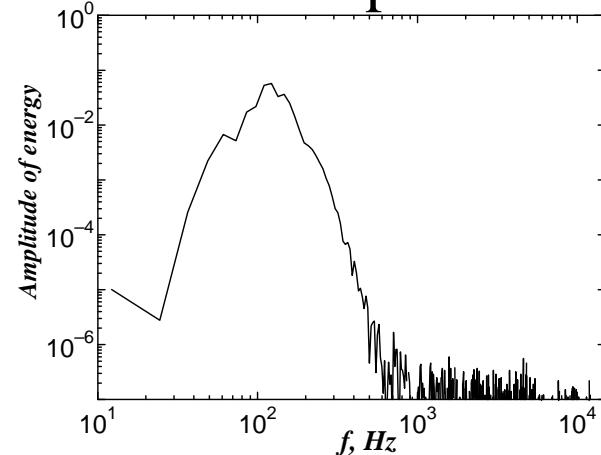
Acceleration sensor



Vibration signal



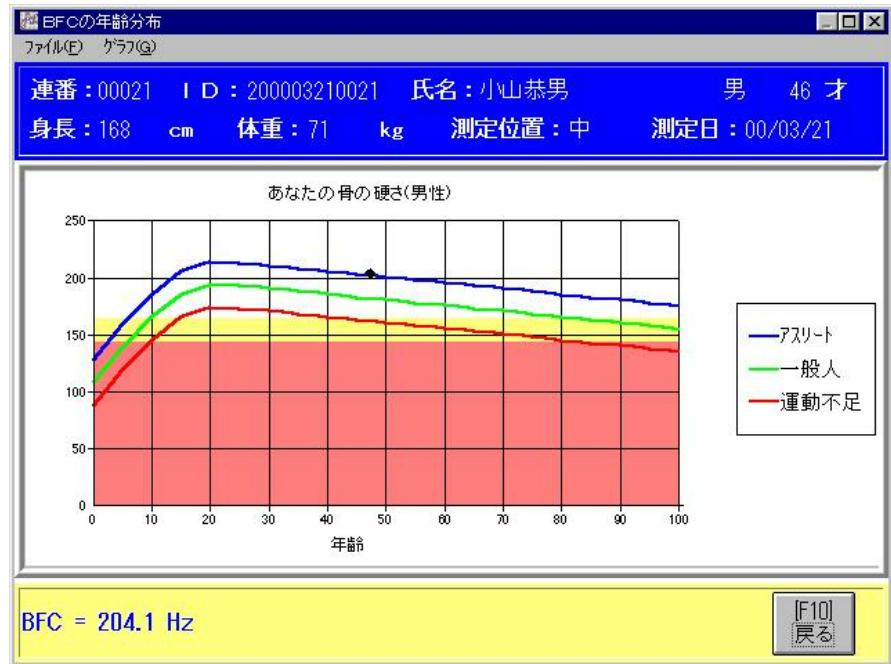
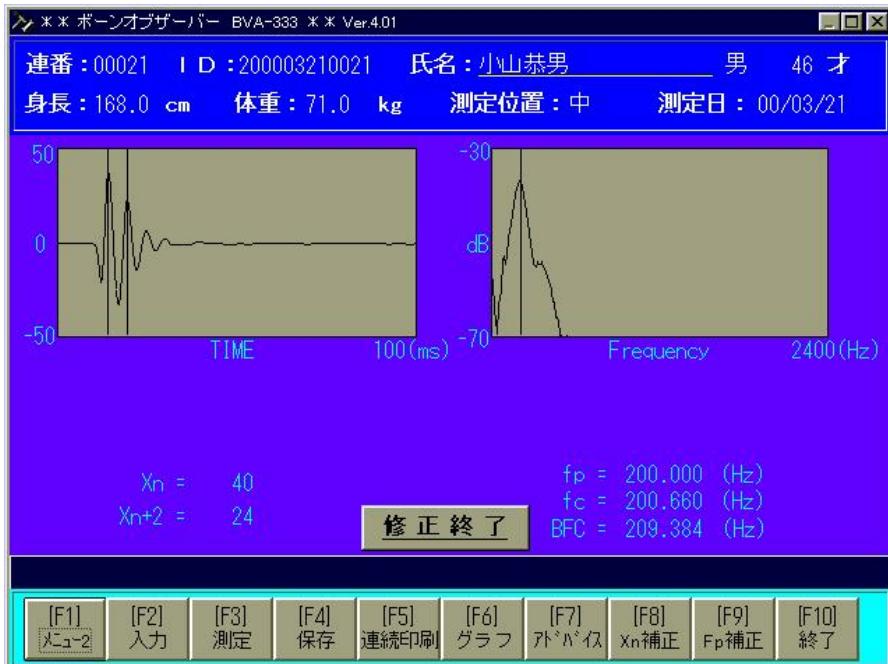
Power spectrum



Menu screen

$$E = 4\rho(lf_c)^2$$

E : Bone's elastic coefficient



(2) Maximum Entropy Method (MEM)

$$x_n = \sum_{k=1}^p a_k x_{n-k} + \sigma \xi_n$$

$\begin{cases} \sigma^2 : \text{Variance of error} \\ \xi_n : \text{white noise with } \sigma = 1 \end{cases}$

Prediction error

Power Spectrum:

$$E(f) = \frac{2\sigma^2 \Delta \tau}{\left| 1 - \sum_{k=1}^p a_k z^{-k} \right|^2}$$

$$z = e^{2\pi i f \Delta \tau}$$

(2)Algorithm of Bang

Maximum Entropy Method (MEM)

What's entropy ?

Entropy is a parameter showing the amount of information, and is defined as following formula.

$$H(x) = - \sum_i p(x_i) \log_2 p(x_i)$$

$$p(x_i) = \frac{|x_i|^2}{\|x\|^2}, \quad \|x\| = \sqrt{\sum_i |x_i|^2}$$

Where, $p(x_i)$ is the occurrence probability of x_i

Entropy maximum means generating all matters in the same probability

For example, if occurrence probability of x_i is set to 1, $H(x) = 0$ will be obtained.

Calculation of coefficient a_k

$$E(f) = \frac{P_m \Delta \tau}{\left| 1 + \sum_{k=1}^M a_k z^{-k} \right|^2} \approx \sum_{k=-M}^M \phi_k z^k$$

$\phi_k = E[x(k)x(k-i)]$: Correlation of the date $x(k)$

M: Degree of Approximation-equation

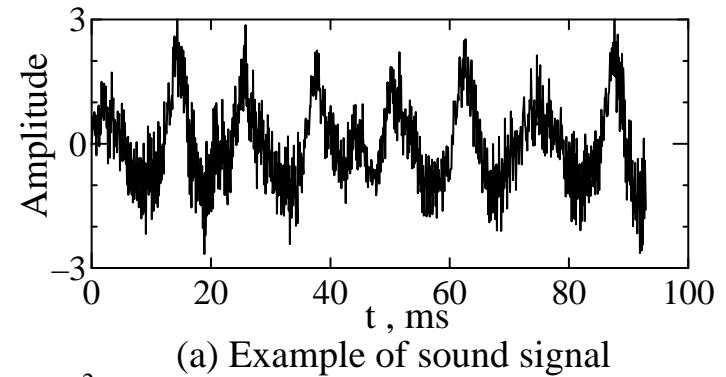
That is, Fourier transform of the autocorrelation can be shown in the series development of the M degree, which is certain extrapolation and has maximum Entropy

a_k can be found by following linear relation :

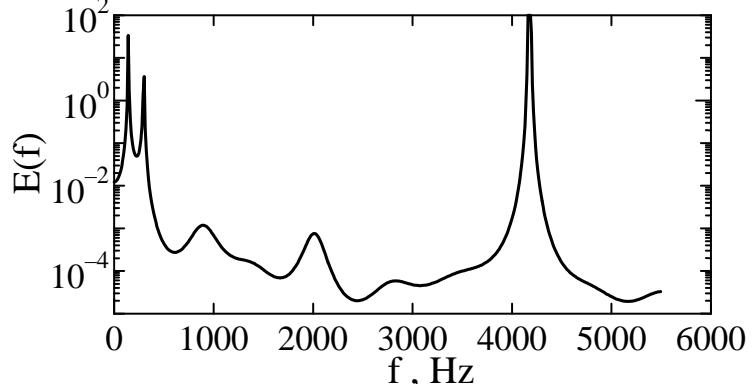
$$\begin{bmatrix} \phi_0 & \phi_1 & \phi_2 & \dots & \phi_m \\ \phi_1 & \phi_0 & \phi_1 & \dots & \phi_{M-1} \\ \phi_2 & \phi_1 & \phi_0 & \dots & \phi_{M-2} \\ \vdots & & & & \\ \phi_M & \phi_{M-1} & \dots & \dots & \phi_0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} P_M \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

(2)Frequency analysis by MEM

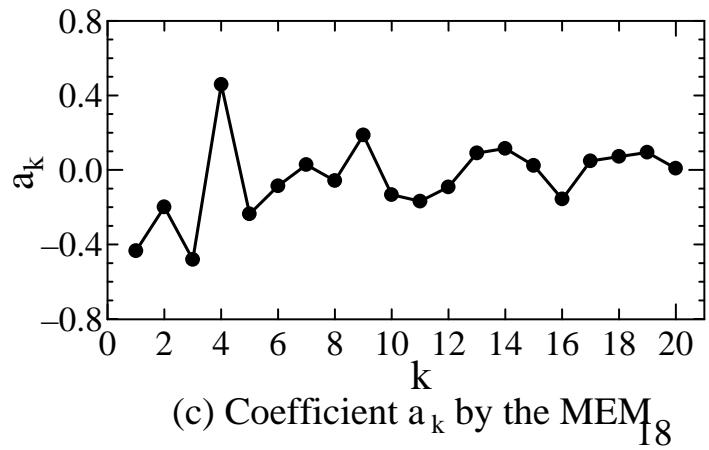
Example of analysis result of “PEE—“ Muffler whistling sound has been shown in Figure, where (a) shows wave of "PEE -- " sound, sampling frequency is 11kHz and data number is 1024; (b) is the power spectrum obtained by MEM; (c) is the coefficient a_k showing the feature of a signal.



(a) Example of sound signal



(b) Power spectrum of the sound signal



(c) Coefficient a_k by the MEM

Character of MEM

Signal:

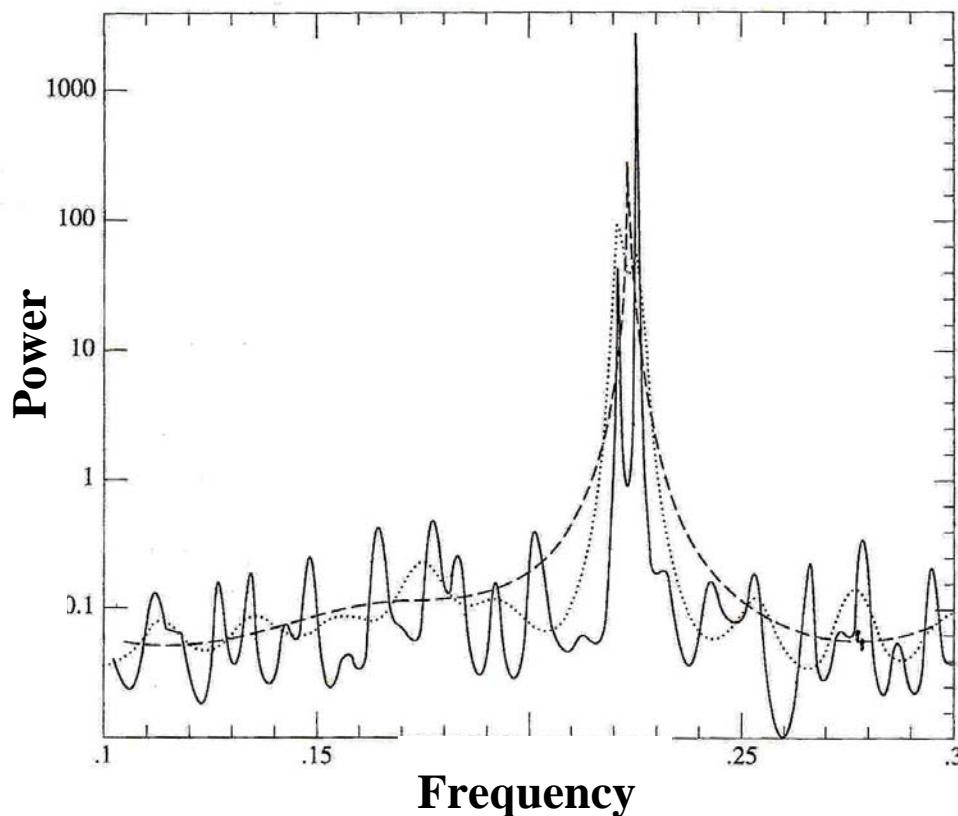
- 1) Two sin waves that frequency came close very much and with same power white noise
- 2) Number of the samples are 512

Results:

Solid line: $M=150$

Dotted line: $M=40$

Dashed line: $M=20$



Characters:

- 1) Analysis result depends on number coefficient M and false peaks are easy to appear for analysis result when there are too many coefficients.
- 2) Frequency Interval is free and can show details than Fourier transform

2.2 Non-stationary Signal Analysis Method

Method: **1.Statistic method (Gaussian Distribution),**
2.Statistic Model (AR Model),
3.Spectrum analysis (FFT, MEM)

They are effective to a stationary signal and unsuitable for a non-stationary signal. It is because, the information about time has been lost in signal analysis.

*Method adapted for unsteady signal analysis :

- 1) Short Time Fourier transform, STFT
- 2) Wavelet Transform,WT

2.2.1 Short Time Fourier transform(STFT)

Fourier transform loses information about the time. So Short-time Fourier transform is suggested to have both information of time and of frequency.

$$\text{STFT} : X(m, e^{i\omega}) = \sum_{n=-\infty}^{\infty} w(n-m) x(n) e^{-i\omega n} \quad w(n)(\cos(\omega n) + i \sin(\omega n))$$

Window function (Hanning, width: $2L$)

$$w(t) = \begin{cases} \frac{1}{2}(1 + \cos(\frac{\pi t}{L})), & |t| \leq L \\ 0, & |t| \geq L \end{cases}$$

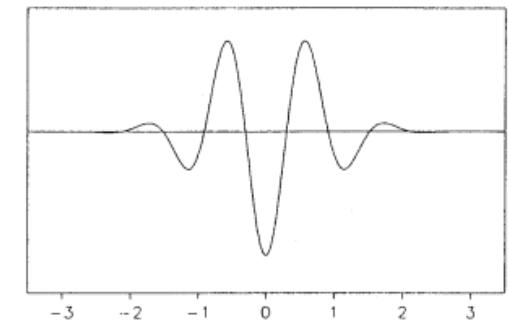


图 3.1.1 $\text{Re } G_{6,2\pi}^s, \alpha = 0.2925$

Inverse-STFT:

$$x(n) = \frac{1}{2\pi} \frac{1}{win(n)} \int_0^{2\pi} X(m, e^{i\omega}) e^{i\omega n} d\omega$$

$$win(n) = \sum_l w(n-l)$$

Power spectrum: $E(t, f) = \frac{1}{2\pi} X(m, e^{i\omega})^2$

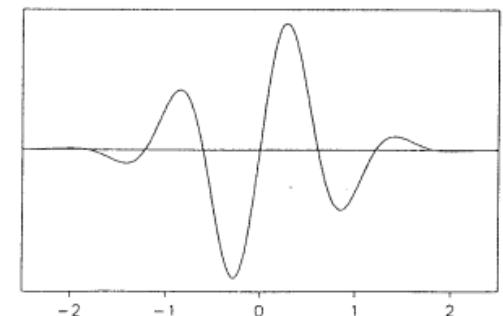
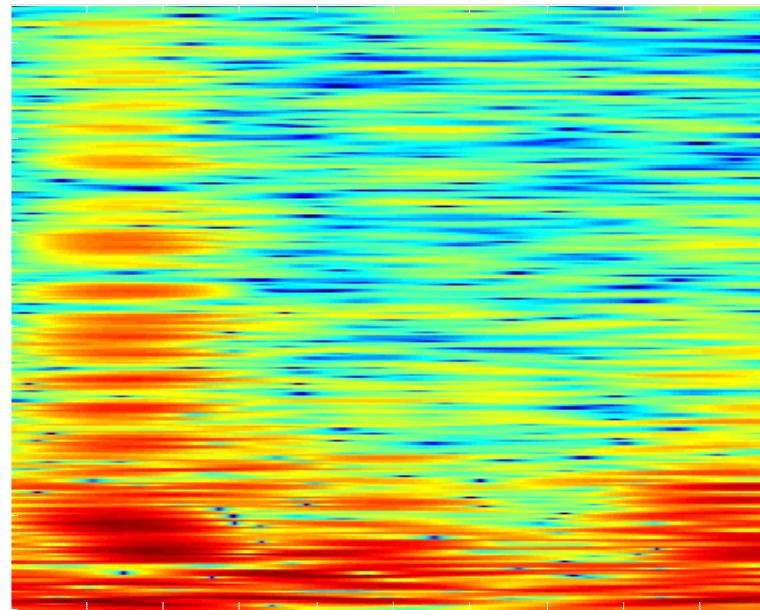
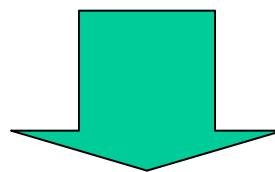
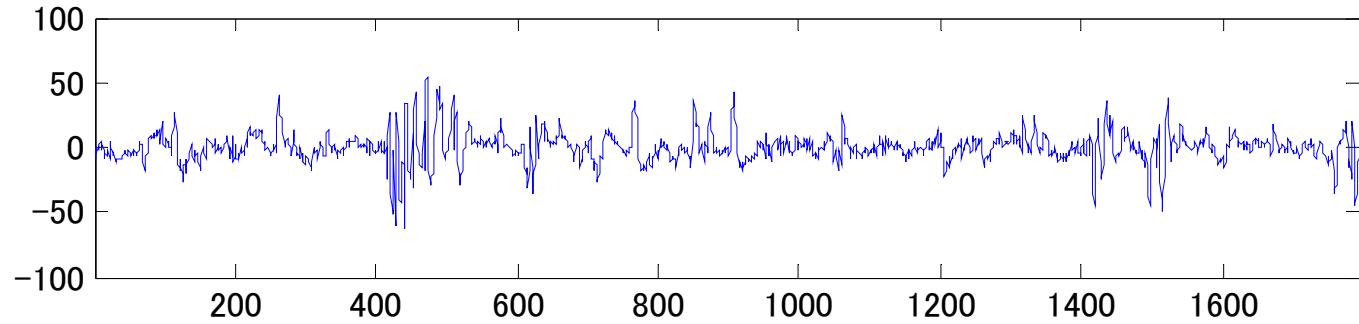


图 3.1.2 $\text{Im } G_{6,2\pi}^s, \alpha = 0.2300$

Characteristics



STFT

2L=512

2.2.2 Continuance Wavelet Transform (CWT)

$$w(a,b) = \int_{-\infty}^{\infty} f(t) \bar{\psi}_{a,b}(t) dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

$\psi(t)$: Mother Wavelet
 a : Scale ($1/a$ Frequency)
 b : Time

$$\psi(t)$$

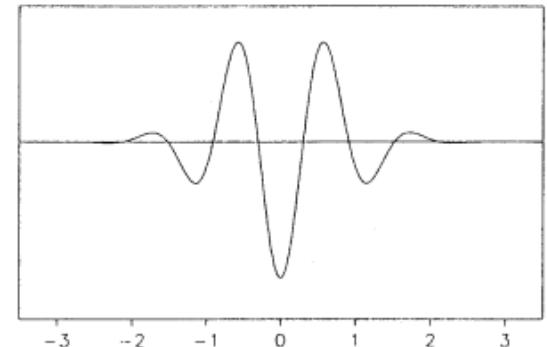


図 3.1.1 $Re G_{0,2\pi}$, $\alpha=0.2925$

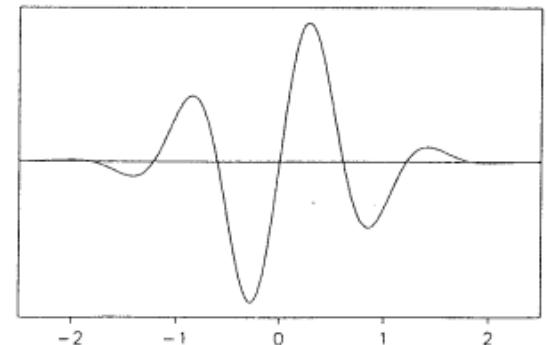
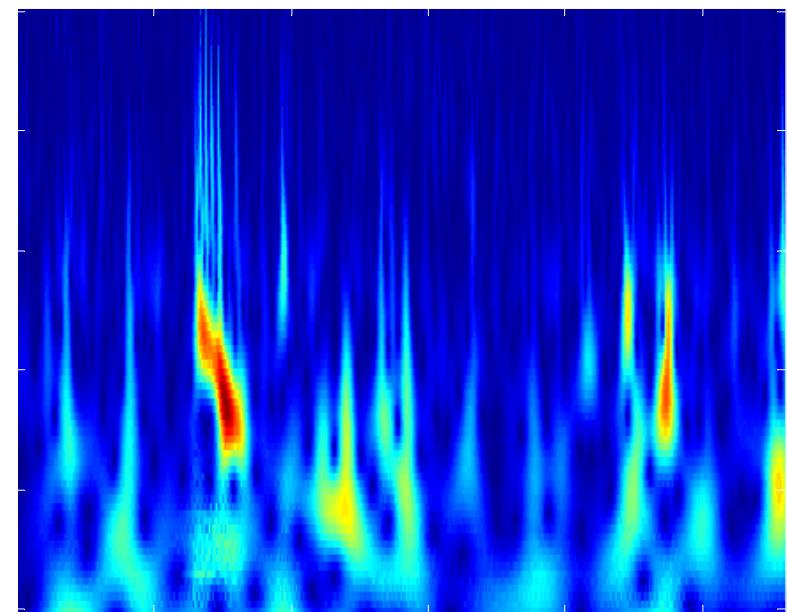
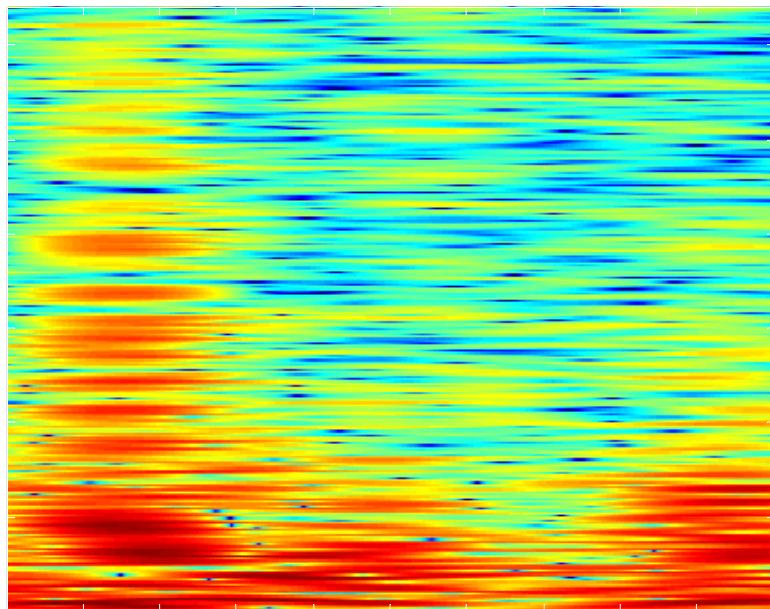
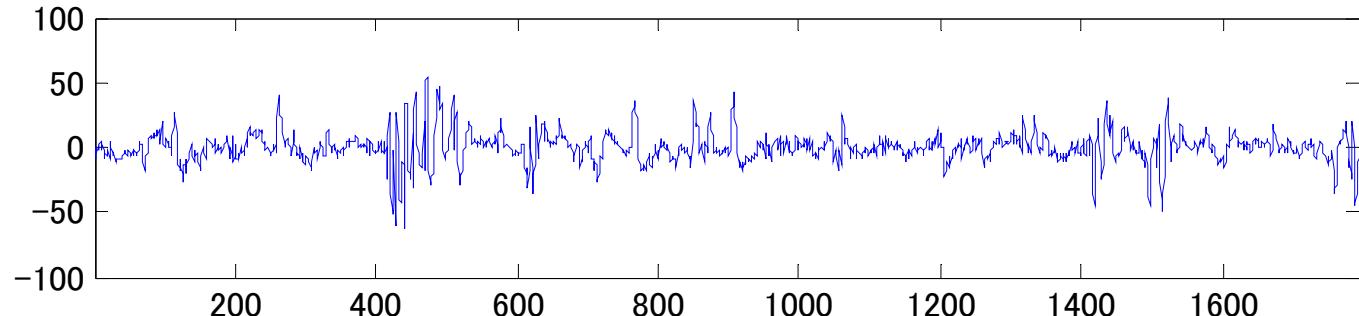


図 3.1.2 $Im G_{0,2\pi}$, $\alpha=0.2300$

Characteristics



Why?

What's difference?

STFT:

$$X(t, f) = \int_{-\infty}^{\infty} w(t - \tau) x(\tau) e^{-i 2 \pi f \tau} d\tau$$

CWT:

$$w(a, b) = \int_{-\infty}^{\infty} f(t) \overline{\psi}_{a,b}(t) dt$$

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

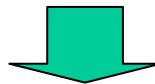
$$w(t) e^{-i 2 \pi f t} \approx \psi(t) \neq \psi\left(\frac{t-b}{a}\right)$$

2.3 Example: sound source separation

The demand for speech signals as a user interface for robotic applications, home electric appliances, and cellular phones has increased.

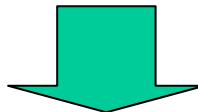


Cellular-phone



Problems :

- Sound recognition performance decreases when other sound sources (noise) exist
- In a real environment, there are two or more sound sources, which the location is unknown and the observed signals are mixture of the sounds with noise.

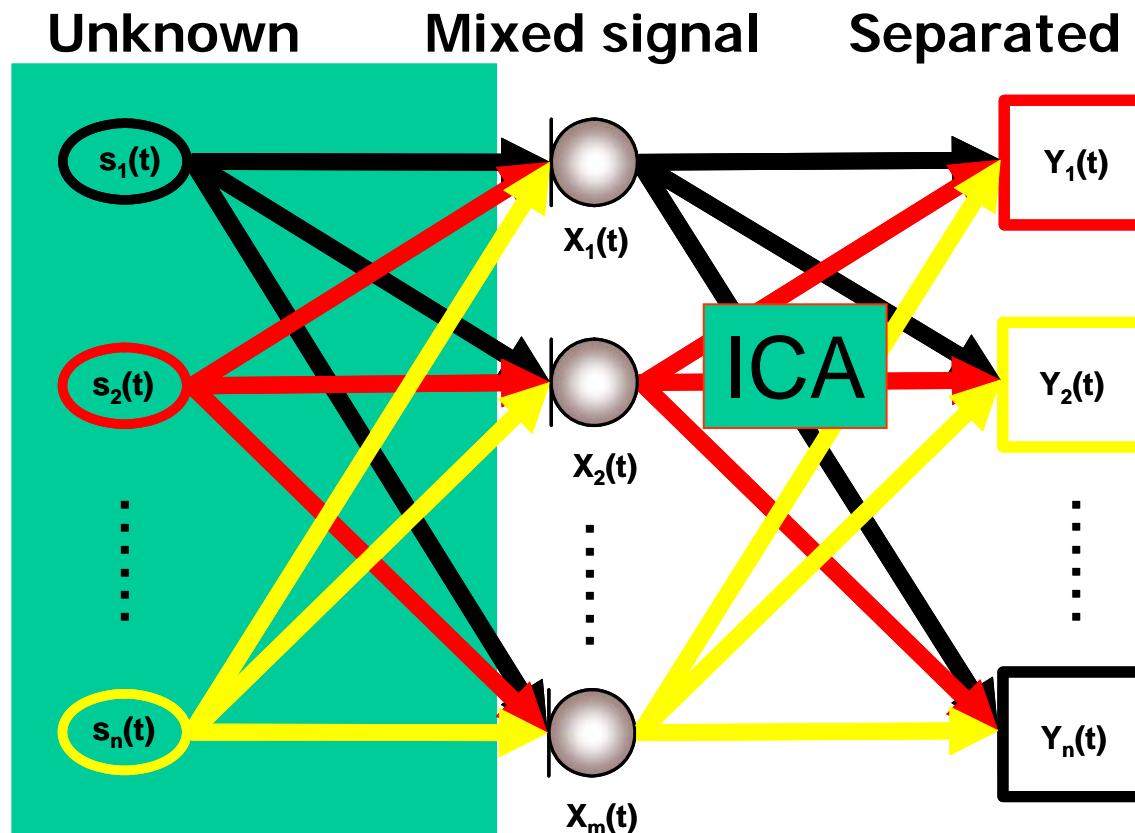


Development of signal processing system:

Blind Source Separation by Independent Component Analysis (ICA)

What is ICA

The ICA is a statistical method which guesses an original signal from the mixture signals even if the original signal and the transfer function are unknown under the assumption of statistical independence.



Problems of Traditional Method

Traditional ICA in Time-Frequency domain:

- Short Time Fourier Transform (STFT)+ICA

For each frequency component, the window width is fixed. So it is impossible to choose an optimal window for each frequency component.

- Discrete Wavelet Transform (DWT)+ICA

DWT that was carried out by Mattal's fast algorithm also has a drawback of lacking shift invariance

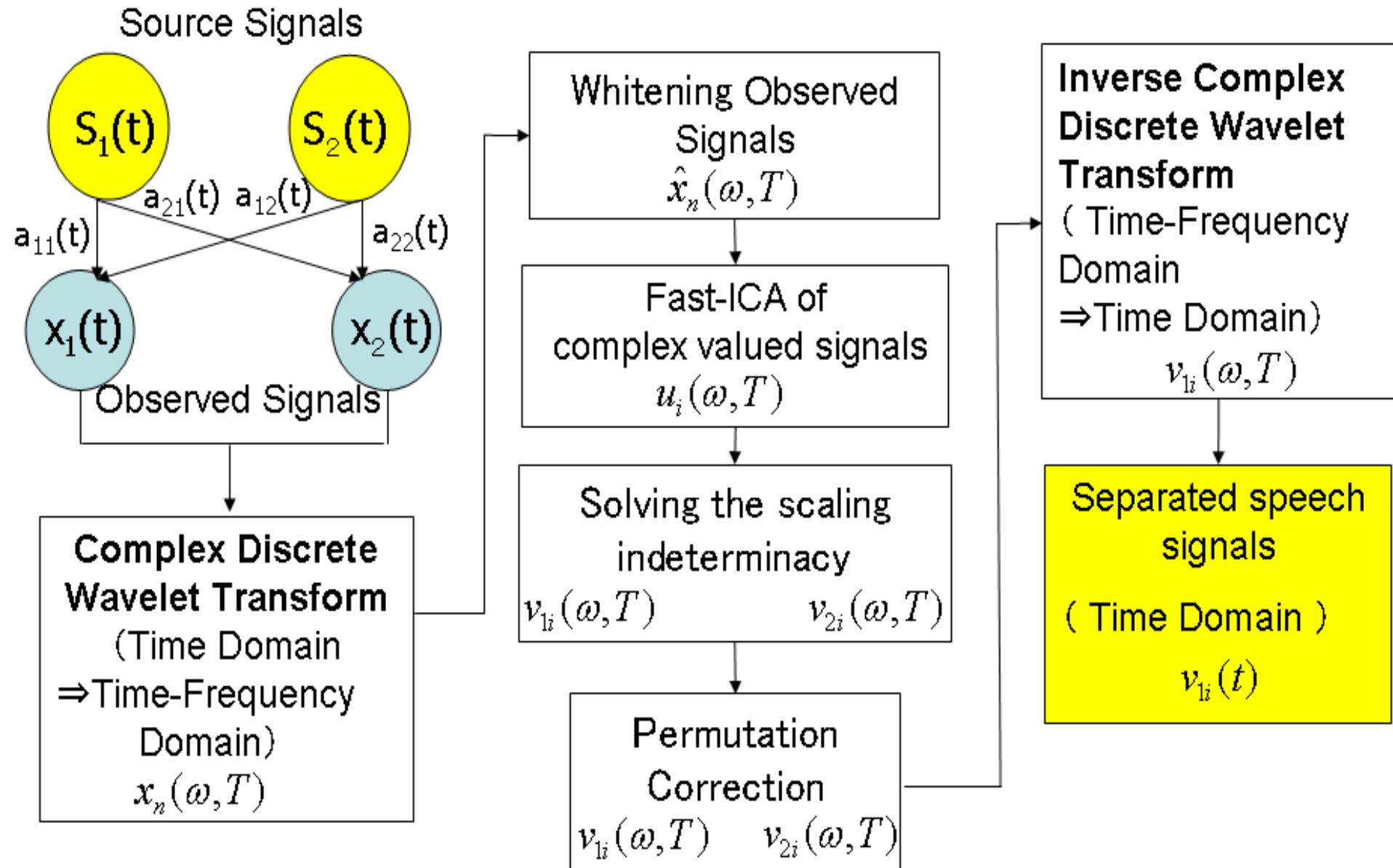
Proposed Method:

- Complex Discrete Wavelet Transform (**CDWT**) + **ICA**

To solve the problems of STFT and DWT



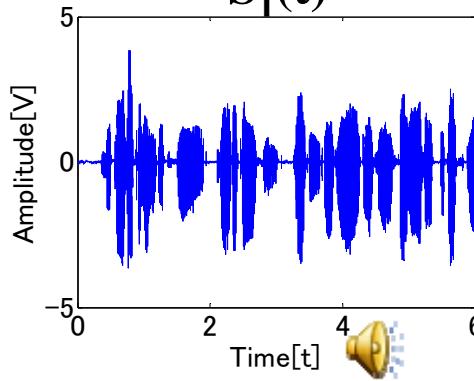
Flowchart of CDWT + ICA



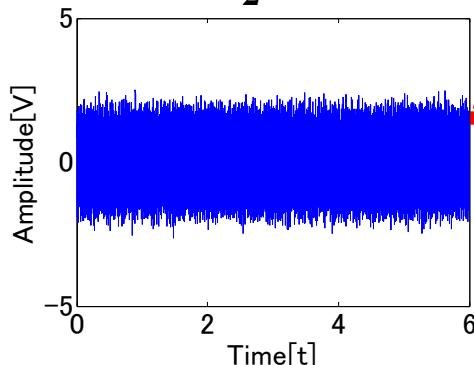
Separated results from Noise

Source

$S_1(t)$

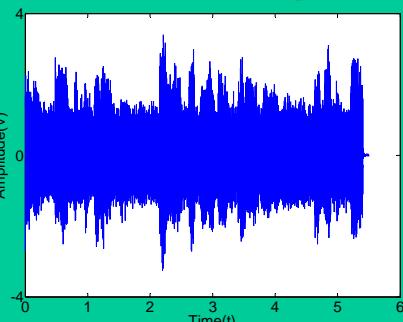


$S_2(t)$



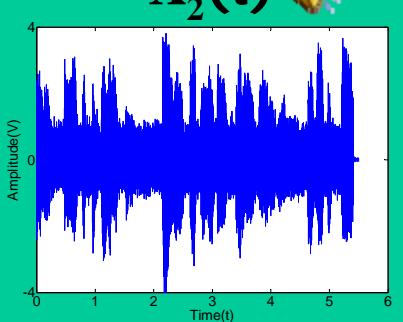
Signal(Mic.)

$X_1(t)$

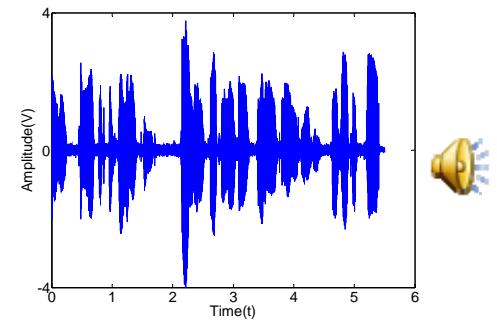


CDWT + ICA

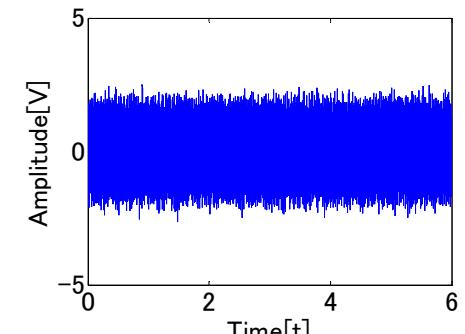
$X_2(t)$



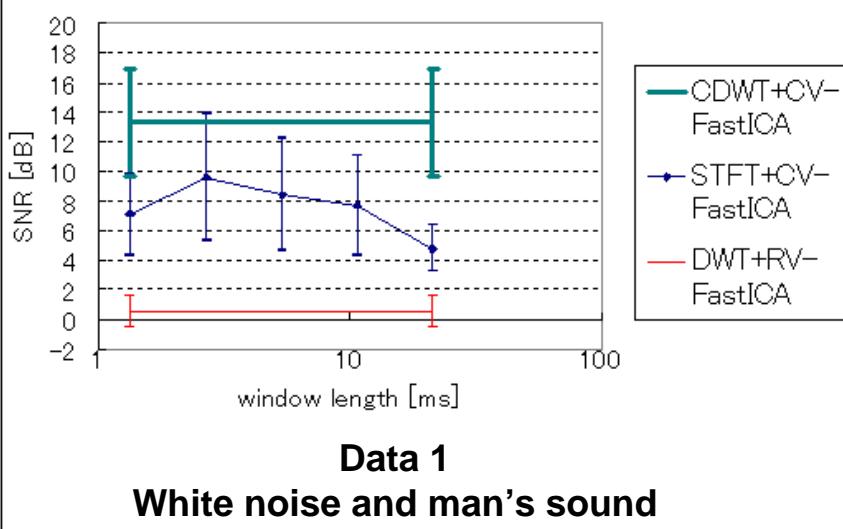
Separated Signal(1)



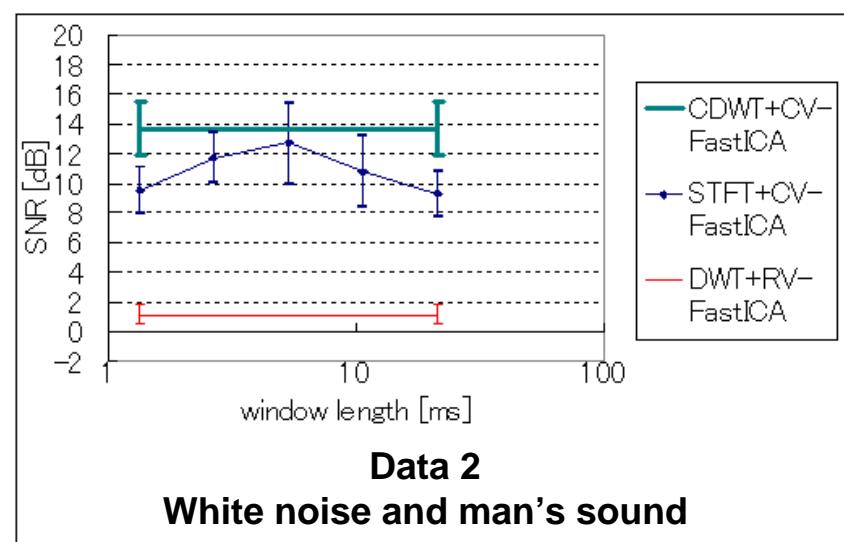
Separated Signal(1)



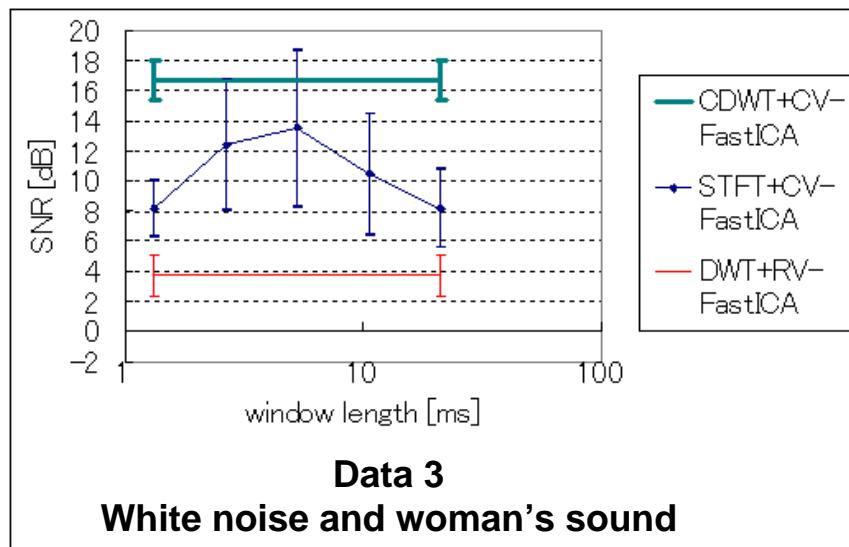
Results and comparison



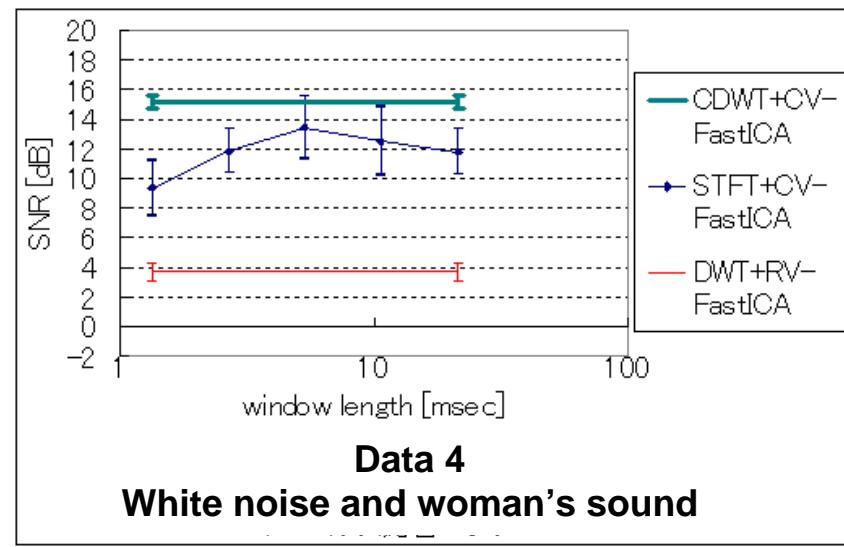
Data 1
White noise and man's sound



Data 2
White noise and man's sound

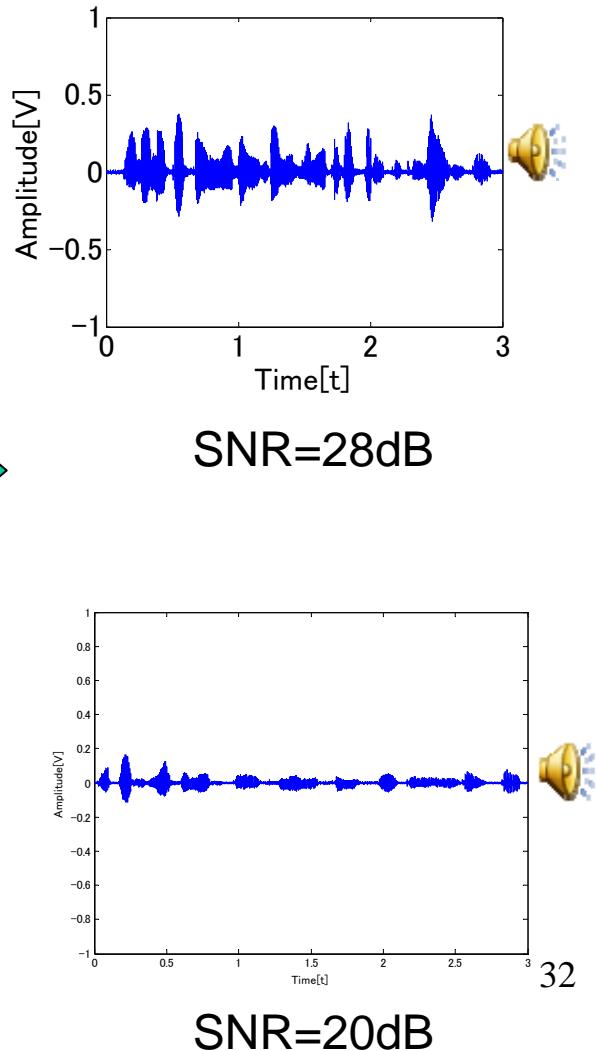
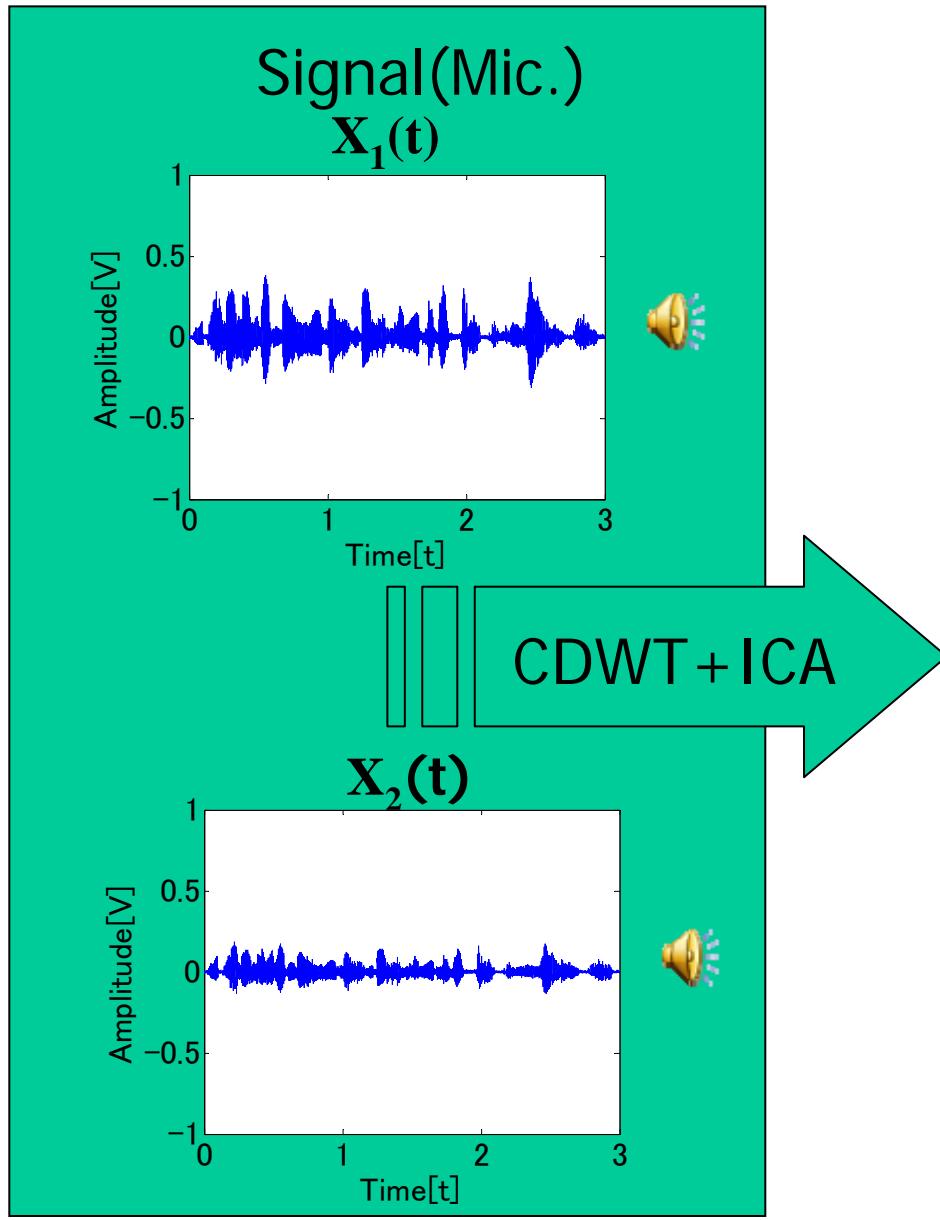


Data 3
White noise and woman's sound



Data 4
White noise and woman's sound

Sound source separated Results



2. Signal Type and Analysis Method

2.1 Stationery Signal analysis Methods

2.1.1 Statistics Method

(1) Gaussian Distribution and parameters

(2) Evaluation of different from Gaussian distribution

2.1.2 Statistic Model

2.1.3.Spectrum Analysis

(1)Fourier Transform

(2) Maximum Entropy Method (MEM)

2.2 Non-stationary Signal Analysis Method

2.2.1 Short Time Fourier transform(STFT)

2.2.2 Continuance Wavelet Transform (CWT)

2.3 Example: sound source separation

Sound source separated By ICA and CDWT